Unemployment rates and population changes in Spain*

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Abstract

This paper discusses the long run effect of changes in the age distribution of Spanish population on the unemployment rate, disaggregated by sex and age segments in the light of cointegration theory given the non stationarity of the series. Four main results are obtained. First, empirical analysis does not provide a clear scheme concerning the long run relationships between population variables and the specific unemployment rates for different groups. Second, as a first approximation one can detect the existence of, at least, one long run equilibrium relationship in all sex-age groups, except for the ones including the middle aged and oldest unemployed female workers. Third, a more thorough analysis enables us to justify the existence of such long run relationships for the youngest and middle aged male workers. One cannot argue, however, a joint evolution of population variables and the unemployment rates associated with female workers and to the oldest male workers. Fourth, the short run dynamics of unemployment rates for the youngest (male and female workers) and the middle age male workers is affected by the transitory deviations from these long run relationships.

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1 Introduction

This paper analyzes the effects of population age structure on unemployment rates disaggregated by age and sex in Spain for the period 1976:3-1998:4.

Two seemingly unrelated issues such as the unemployment rate and its high degree of persistence in developed countries and the influence of demographic factors on economic activity have received considerable attention in the economic literature.

Concerning the former, the focus generally centers on the aggregate unemployment rate, because references dealing with disaggregate unemployment rates by sex and age are scarce. With respect to the latter, there is also an extensive literature, largely motivated by the increasing ageing of the population in western economies. For instance, one can find numerous references dealing with the influence that the population structure exerts on education, pension and health expenditures, public expenditures in general and even public revenues. Social security economics is by far the best example of this particular line.

However, the existing literature on these two issues (unemployment and demographics) considered together is much less extensive, though there are some references along this line. For instance, Welch (1979), Berger (1985) and Fair and Dominguez (1991) have studied the effects of the US age structure of population on the labor market; Zimmermann (1992) and Schmidt (1993) focus on the effects of changes in the age composition on the unemployment rates for specific age segments and sex in Germany.

Focusing on the Spanish economy, Bover and Arellano (1995) discuss the main factors leading to an increase in women’s participation rate in Spain during the 1980’s; Castillo and Jimeno (1996) find that changes in the active population do not contribute significantly to explain the Spanish unemployment rate. There are, however, no empirical works on population ageing as a relevant factor to explain the high unemployment rates among some groups of population in Spain and, in particular, among the youngest. Limiting its scope to a specific group, Ahn et al. (2000) may be considered an exception, suggesting the existence of a positive relationship between the relative size of young to old population and their associated unemployment rates.

The present article follows this line of research. Specifically, our purpose is to analyze the influence that changes in the age distribution of population may exert upon unemployment rates, disaggregated by age and sex. The article may be considered, therefore, as an extension of Zimmermann (1992) and Schmidt (1993).

1 The population pyramid is changing its structure dramatically. There are two main reasons in the Spanish case. First, a remarkable fall in birth rates since the 1960s: from 21.98 births per thousand in 1964 to 9.37 per thousand in 1997. And, second, the progressive increase in the life expectancy of individuals [see INE (2000) and Fernández Cordón (1998)].

2 Unemployment rates in Spain are among the highest in OECD countries, especially for younger workers [see OCDE (1994) and Ahn et al. (2000)].
The method followed here consists of three stages for each age segment and sex. In the first stage we search for unit roots in the series. To this end we show the results of some tests [Dickey and Fuller (1981), Phillips and Perron (1988), Kwiatkowski, Phillips, Schmidt and Shin (1992), Perron (1990), Perron and Vogelsang (1992) and, finally, Clemente et al. (1998)]. After implementing that sequence of tests, and detecting at least one unit root in all the series, in the second stage we estimate the possible cointegrating relationships between demographic variables and unemployment rates. We use two approaches for this purpose: the procedure in Engle and Granger (1987), and the one suggested by Johansen (1988). Finally, we implement several tests relative to the cointegrating vectors and short run adjustment coefficients. More precisely, we study whether the long run behavior of the whole system depends on all the variables. We also test, in a multivariate setup, the stationarity of the series, and test whether some of the variables do not adjust in the short run to guarantee the long run equilibrium path.

We obtain four main results. First, the sample evidence does not give precise information about the existence of long run relationships between population variables and unemployment rates, disaggregated by sex and age. Second, as a first approximation the analysis suggests the presence of such relationships in all age segments, except for middle aged and oldest female workers. Third, after running several tests, however, a joint evolution of demographic variables and unemployment rates can be justified only for the youngest and for middle aged male workers. Fourth, the short run dynamics of the unemployment rates of the youngest workers (male and female) and the male workers in the middle aged segment is affected by transitory deviations from these long run relationships.

The rest of the paper is organized as follows. In Section 2 a theoretical model is presented to formalize the effects of changes in the age structure of population upon disaggregate unemployment rates. Section 3 defines the variables in the paper. The analysis of stationarity and the later study of the cointegrating relationships is carried out in Section 4. Moreover, we run tests concerning cointegrating vectors and weighting matrices, the latter capturing the adjustment coefficients of each variable towards the long run equilibrium. Section 5 concludes and draws the main conclusions.

2 The theoretical model

In this Section, a simple model is presented to formalize the effects of changes in the age structure of population upon unemployment rates. It is a career phase model in essence, and it thus captures the fact that every worker goes through successive career or professional stages during his/her working life.3 [See Welch (1979) and Schmidt (1993).]

3This approach is quite similar to that of the life cycle model, because the stage or phase the worker is in is determined by his/her age.
The basic idea behind career phase models is that, at any moment in the individual’s working life, the worker goes through a transition process between two phases. In our case, the phase is characterized by the age segment to which the individual belongs. Assuming that these phases are associated with different degrees of expertise in the labor market (the older the worker, the greater his/her expertise, for instance), one can interpret the transition process in terms of acquiring knowledge to increase human capital stock. In particular, we consider three stages in the worker’s life: young workers (from 16 to 29 years, both included), who we denote by \( j = 1 \); middle aged workers (from 30 to 44), denoted by \( j = 2 \); and elder workers (45 and above), denoted by \( j = 3 \).\(^4\) That makes a total of six groups of workers: three age segments for both sexes, the latter denoted by \( i = h \) for men, and \( i = m \) for women.

Stated briefly, the model assumes that when the worker starts the \( j \)-th stage (at the age of \( x_j \)), he/she leaves the previous stage behind, \( j - 1 \) [except, of course, when \( j \) is the first one], and he/she also starts the transition to the next stage, \( j + 1 \) [except, of course, when \( j \) is the last one]. More precisely, when in \( j \)-th stage, the worker devotes a (decreasing with age \( x \)) share \( p_j^i(x) \) of his/her time to this stage; the rest of the time, in an (increasing) share \( 1 - p_j^i(x) \), is devoted to accumulating human capital to access the following stage, \( j + 1 \). Formally, \( p_j^i(x) \) is such that \( dp_j^i(x)/dx < 0 \), for \( x \in [x_j, x_{j+1}] \), \( p_j^i(x_j) = 1 \), and \( p_j^i(x_{j+1}) = 0 \). [See Figure I].

\(^4\)We are certainly aware that, first, these age segments are not necessarily homogeneous and, second, that alternative partitions might also be considered [see, e.g., Ahn et al. (2000)]. One has to set a limit to identify some reasonable groups, however: by specifying these three age segments the model captures those workers who have recently entered the labor market; those who have attained a certain level of qualification; and those with the highest level of expertise in the labor market. Schmidt (1993) considers seven age segments and notes that this excess disaggregation, along with the small sample size, might explain the rejection of the existence of long run relationships between the size of the cohorts and the unemployment rates for some of those age segments.
Thus, the number of workers offering labor services in group \( ji \) (i.e., their labor supply) is obtained as:

\[
M_j^i = \int_{x_{j-1}}^{x_j} [1 - p_{j-1}^i(x)]m^i(x)dx + \int_{x_j}^{x_{j+1}} p_j^i(x)m^i(x)dx, \tag{1}
\]

for \( j = 1, 2, 3, \ i = h, m \), where \( m^i(x) \) stands for active population of age \( x \) and sex \( i \).\(^5\) Notice that both individuals in the \( j \)-th age segment and individuals in the previous age segment \( j - 1 \) offer their labor services to the \( j \)-th segment. The former do it in a share \( p_j^i(x) \), whereas the latter in a \( 1 - p_{j-1}^i(x) \) share.

Production is represented by an aggregate production function \( Y = f(N,Z) \). \( N \) denotes the total amount of labor, assumed to be a function of all groups of workers in the economy, i.e., \( N = g(N^h_1, N^m_1, N^h_2, N^m_2, N^h_3, N^m_3) \), with \( N_j^i \) standing for the number of workers of sex \( i \) in the \( j \)-th age segment. Different groups of workers are allowed to exhibit different productivities; otherwise, \( g(\cdot) \) would be the sum of all the \( N_j^i \)'s. \( Z \) denotes all other production factors, fixed in the short run and representing the state of the economy.

Suppose that for given wages for the three age segments \( w_j \) (\( j = 1, 2, 3 \)) [to be determined later on], and for a given \( Z \), firms choose the labor inputs corresponding to the six worker groups, \( N_j^i \), so that they solve the following problem:

\[
\max_{\{N_j^i\}} f(N,Z) - \sum_{j=1}^3 w_j (N_j^h + N_j^m) \\
\text{s.t.} \quad N = g(N^h_1, N^m_1, N^h_2, N^m_2, N^h_3, N^m_3).
\]

\(^5\)By construction, for stage \( j = 1 \), the first term on the right hand side of (1) must be identically zero. That is also the case for the second term in the last stage \( [j = 3 \text{ in our case}] \).
We are implicitly allowing for different wages for different age segments [see, for instance, Welch (1979) and Berger (1985)], although wages are assumed to be the same for both sexes.\(^6\) Nothing precludes the model from allowing for sex wage discrimination, however.

From that maximization problem, we obtain a system of first order necessary conditions from which labor demand functions for each group are derived:

\[ N_{ij}^{*} = h_{ij}(w_1, w_2, w_3, Z), \quad j = 1, 2, 3, \quad i = h, m, \]  

(2)

which depend on wages for all age segments and on economic activity.

Having obtained the number of employed workers and the number of individuals offering their labor services, we can derive the unemployment rates associated with each group:

\[ U_{ij} = \frac{M_{ij} - N_{ij}}{M_{ij}}, \quad j = 1, 2, 3, \quad i = h, m. \]  

(3)

Therefore, the \(U_{ij}\)'s are determined by the wages associated with all age segments, the distribution of the active population (by age and sex), and the level of economic activity, \(U_{ij} = f_{ij}[w_1, w_2, w_3, m^i(x), Z]\).

To close the model, suppose finally that there is a union that, representing the whole labor force in the economy, sets the wages for all workers. The union is assumed to have preferences defined over wages and employment for all groups and represented by a utility function \(V\). This way, the setting of the wage by the union will require that \(V\) be maximized subject to firms’ labor demand for every group:

\[
\max_{[w_j]} V(w_1, N_1^h, N_1^m, w_2, N_2^h, N_2^m, w_3, N_3^h, N_3^m) \\
\text{s.t.} \quad N_{ij}^{*} = h_{ij}(w_1, w_2, w_3, Z), \quad j = 1, 2, 3, \quad i = h, m. 
\]  

(4)

The first order necessary conditions for that problem are given by:

\[
\frac{\partial V}{\partial w_j} + \sum_{k=1}^{3} \sum_{i=h,m} \frac{\partial V}{\partial N_{ij}^k} \frac{\partial N_{ij}^k}{\partial w_j} = 0, \quad j = 1, 2, 3. 
\]  

(5)

Finally, from (5) we obtain optimal wages as functions of economic activity alone, \(w_j^* = w_j(Z), \quad j = 1, 2, 3\). The same result can be also found in Schmidt (1993).

In sum, the specific unemployment rate for each group is determined by economic activity level and the age structure of the active population \([U_{ij}^* = \ell_{ij}^*[m^i(x), Z] \)]. In our case, we use the GDP growth rate [which we denote by \(Y\)] as a proxy for \(Z\).\(^7\) We follow

\(^6\)Welch (1979) and Berger (1985) find significant effects of the age structure of population upon wages, and show that elderly workers (with a larger expertise) earn higher wages than the younger workers who have entered the labor market recently.

\(^7\)Alternatively, one could use the aggregate unemployment rate [Schmidt (1993)]. Both variables are related, however. The relation, of course, is given by Okun’s law.
Zimmermann (1992) when specifying the age structure and approximate it by means of two variables: the relative mean age and the relative size of adult population [which we denote by $E_i$ and $T_i$, respectively, and define more precisely in the next Section]. An alternative specification would consist in describing this age structure by means of the cohort sizes. This way, one would take into account both the age and the population size [see Schmidt (1993)]. Thus, the unemployment rate for every sex and age segment is characterized by:

$$U_j^i = \ell_j^i(E_i, T_i, Y), j = 1, 2, 3, i = h, m.$$  

Now, our concern is to confirm whether such rates are really determined by their fundamentals, at least in the long run, for the Spanish economy.

Before leaving the Section, two assumptions implicit in the model should be noted. First, we ignore the presence of migratory flows and therefore assume that migration has not been a determining factor for the Spanish age structure of population. Second, the transition process between the phases in the individual’s career is exogenous and, therefore, cohort size independent.

### 3 Definition of variables

The data sources used are the Active Population Survey [Encuesta de Población Activa] and the Quarterly National Accounts [Contabilidad Nacional Trimestral], both collected by the National Statistical Institute [Instituto Nacional de Estadística]. The data are quarterly and the sample size extends from 1976:3 to 1998:4. The variables are defined below:

- **Unemployment rates**, $U_j^i$, previously defined in (3). Remember that $j = 1$ refers to workers between 16 and 29, $j = 2$ refers to workers from 30 to 44, and $j = 3$ refers to workers of 45 and over; $i = h$ refers to male and $i = m$ refers to female workers.

- **Demographic variables**. Two demographic variables are used to characterize the age distribution of population:
  
  - The relative size of adult population, $T_i$. People between 16 and 39 (both inclusive) are considered young and people between 40 and 65 (both also inclusive) adults, where 65 is the legal retirement age [see Bailén and Gil (1996)]. $T_i$ is defined as the ratio of the size of the second group of workers to the first one. The rest of the population is left out due to its negligible participation in the labor force.
  
  - The relative mean age of adult population, $E_i$. It is defined as the ratio of the mean age among adults to the mean age among the young.

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*The author does not assure, however, that cohort sizes are an accurate measure to explain the unemployment rate specific to each age segment.*
• **Growth rate of GDP, Y.** The quarterly growth rate of real GDP, base 1986. Including this variable allows us to capture the influence of the business cycle on unemployment rates.\textsuperscript{9}

The variables are represented in Appendix I. The evolution of disaggregate unemployment rates by sex and age shows fluctuations similar to the ones in the aggregate unemployment rate. While all are countercyclical, some are more pronounced [the aggregate rate, the male rates and the rate of the youngest female workers]. The unemployment rates of the older than 30 female workers ($U_{2m}$ and $U_{3m}$) show some divergence. Therefore it will not be surprising to obtain different results for these two groups.

It is also noticeable that an analysis of unemployment series shows, first, higher rates for female workers than for male workers and, second, higher rates for the youngest than for the oldest. We believe that these differences motivate the need to decompose the aggregate unemployment rate by age and sex.

Finally, concerning the population variables, the pattern in the series ($T_i$, $E_i$) is similar in both sexes. For instance, $T_i$ shows, in general, a decreasing trend until the mid-90s, reflecting the pressure exerted by those born during the baby boom. From about 1994, however, $T_i$ suffers a change in its behavior, showing smooth stability or even a slight increase. The smaller number of births since the late 70s begins to exert its effect on the youth population as this generation reaches working age.

Likewise, one can infer a smooth negative relationship between the relative size of adult population and the unemployment rates for the period 1976-1998. The unemployment rates for each specific age segment, in general, tend to increase during the first half of the sample (from 1976 to about 1986) and to decrease from the mid-90s. The opposed pattern is shown in the relative size of adult population, both male and female: a decrease at first and a change later in its behavior during the second half of the 90s. Therefore, one can expect a negative sign for the effect of $T_i$ on $U_j$.\textsuperscript{8}

Finally, it should be noticed that none of the series seems to have a constant mean. Therefore, they are expected to be non-stationary or generated by a stochastic process with (at least) one unit root.

\textsuperscript{9}As mentioned before, this is the essence of Okun’s Law. Empirical evidence for the Spanish case supports a correlation coefficient of -0.7 between changes in (aggregate) unemployment rate and in GDP growth. [See Mankiw (1997), p. 47].
4 Methodology

4.1 Stationarity analysis of the series

4.1.1 ADF, PP and KPSS tests

Depending on how one specifies the null hypothesis, two strategies may be followed to test for the order of integration of a time series. The first strategy refers to tests proposed by Fuller (1976), Dickey and Fuller (1981) [ADF test], Phillips (1987) or Phillips and Perron (1988) [PP test]. In all these tests the series has a unit root under the null hypothesis. The second strategy, on the contrary, tests the null of the stationarity of the series. These tests are motivated by the low power of the ADF and the PP tests to detect the presence of unit roots in the long run frequency. That is the case of the test constructed by Kwiatkowski, Phillips, Schmidt and Shin (1992) [KPSS test].

Table I shows the values of the ADF, PP and KPSS test statistics associated with the series in levels.10

| INSERT TABLE I AROUND HERE |

All series in levels follow similar patterns. The unemployment rates (U), demographic variables (T and E) and GDP growth rate (Y) show evidence in favor of at least one unit root. Therefore, any transitory shock will have permanent effects.11 As expected, in view of the profile followed by the series [see Appendix I], the null hypothesis of unit root is rejected in no case when DFA and PP tests are used. The null hypothesis of stationarity is also widely rejected when KPSS test is applied. Only when middle-aged unemployed male workers (U₂) and GDP growth rate are studied, is there some uncertainty about this possible non stationarity, because the KPSS test statistics do not reject the null of absence of a unit root. This lack of rejection is, nevertheless, quite marginal, as the values of the statistics are almost equal to the critical values tabulated by Kwiatkowski, Phillips, Schmidt and Shin (1992).

After concluding that the series are non stationary, the next step is to test whether they are I(1) or I(2). Thus, the previous tests are implemented once more, the endogenous variables being the first differences of the series this time. The results are shown in Table II for the ADF, PP and KPSS tests.

| INSERT TABLE II AROUND HERE |

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10 The method followed in choosing the lag truncation parameter \( p \) is the one suggested by Campbell and Perron (1991), Hall (1990), Perron (1990) and Perron and Vogelsang (1992).

11 The non stationary pattern of aggregate unemployment in the Spanish economy, or hysteresis, has been studied by Leslie et al. (1995), Alogoskoufis and Manning (1988), Andrés (1993), and Dolado and Lopez Salido (1996), among others.
Discrepancy in the results does not allow us to conclude about the order of integration of all variables. There exists evidence of stationarity of the first differences of the demographic variables ($E$, $T$), the GDP growth rate ($Y$), the unemployment rates of elderly workers regardless of their sex ($U^h_3$ and $U^m_3$), and middle aged female workers ($U^m_2$). This implies that all relevant information about the present and the future evolution of the series is summarized in their past behavior.

The results are not that conclusive, however, for the unemployment rates of workers in the first age segment, both male and female ($U^h_1$ and $U^m_1$), and male workers in the second age segment ($U^h_2$). Although the $ADF$ test does not reject the presence of a second unit root in these series, the $PP$ and the $KPSS$ tests do reject the second unit root in favor of only one unit root. Therefore, we conclude that in these cases ($U^m_1$, $U^h_1$, and $U^h_2$) there is not enough sample information to ascertain the order of integration.\(^\text{12}\)

In short, the implementation of this sequence of tests allows us to conclude the non stationarity of the series we are considering. The conclusion is not that immediate, though, when one has to ascertain their order of integration. It seems that the unemployment rates for some groups may have not one unit root but two. However, such evidence may arise because of some overlooked structural change throughout the sample period.\(^\text{13}\)

That is the result in Dolado and López Salido (1996), who show evidence in favor of a change in the mean of the first differences of unemployment rates after the first quarter of 1986, when Spain joined the EEC. Andrés, Molinas and Taguas (1991) also allow for a structural change in the first difference of the aggregate unemployment rate in 1986.

The $ADF$ test previously implemented has low power when the data generating process is trend stationary and the trend contains a structural change. In other words, the tests are biased in favor of not rejecting the null hypothesis of unit root. Therefore, when the hypothesis is not rejected, it makes sense to test for structural change [see Perron (1989, 1990), and Montañés and Reyes (1998)]. Some tests consider the presence of a single change in the sample period [see Perron (1989, 1990), Perron and Vogelsang (1992), Banerjee et al. (1992), Christiano (1992), Zivot and Andrews (1992)]. Others, however, allow for two changes [see, e.g., Lumsdaine and Papell (1997), Clemente et al. (1998)].

\(^\text{12}\)Zimmermann (1992) found that, regardless of age and sex of individuals, German unemployment rates had a unique unit root. Likewise, he showed the non stationarity of the demographic variables, the evidence being much clearer for the relative mean age than for the relative mean size of the population.

\(^\text{13}\)The sample period, 1976:3-1998:4, includes recessions and expansions which have helped to change the pattern of unemployment in Spain. During the late 1970s and early 1980s the Spanish economy went through a recession followed by an expansion beginning in 1986. From that moment onwards, until approximately 1991, the unemployment rates fell substantially, changing the trend once again.
4.1.2 Structural change

Following Dolado and López Salido (1996), we assume that the mean of the first differences of the unemployment rates have suffered only one change, during the first quarter of 1986. To test for the presence of unit roots we make two alternative assumptions concerning the nature of the period $T_B$ in which the structural change takes place; this depends on whether the change in the mean is assumed to be gradual [innovational outlier model, Table III], or instantaneous [additive outlier model, Table IV]. This distinction is important not only because the paths of transition are different but also because the statistical procedures of test differ. In both cases, we run the Perron (1990) test, which assumes that $T_B$ is known by the econometrician. We consider that $T_B = 1986 : 1$.14

INSERT TABLE III AROUND HERE

Notice that we have run the test only for $\Delta U^h_1$, $\Delta U^m_1$ and $\Delta U^h_2$, the variables whose order of integration we have not been able to determine.15 Our results differ from those obtained by Dolado and López Salido (1996) and Andrés, Molinas and Taguas (1991). They found only one unit root in the unemployment rate when allowing for a change in the mean of the first differences of the series. Assuming that $T_B = 1986 : 1$, the Perron (1990) test shows evidence in favor of the presence of one unit root with structural change in all series analyzed. The hypothesis that $U^h_1$, $U^m_1$ and $U^h_2$ are $I(2)$ is not rejected at the 5% significance level. Their results do not necessarily contradict ours for at least two reasons: the sample periods (1970-1994 vs. 1976-1998) and the series (aggregated vs. disaggregated by sex and age unemployment rates) are not the same.

INSERT TABLE IV AROUND HERE

The results obtained when implementing the additive outlier model and the innovational outlier model are similar: for the series studied, $\Delta U^h_1$, $\Delta U^h_2$ and $\Delta U^m_1$, we do not reject that they are $I(1)$.

As a first extension, one can consider that the structural change period $T_B$ is not known ex ante, so it has to be estimated and tested using the sample information [see the discussion in Perron and Vogelsang (1992), Zivot and Andrews (1992), Christiano (1992), Banerjee et al. (1992) and Clemente et al. (1998)]. They consider, likewise, the

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14 Considering that changes in unemployment are not immediate but require an adjustment processing, the first assumption is more reasonable than the second one. However, we shall present the results for both tests. So we are able to analyze their robustness considering different alternatives with regard to the nature of the structural change period.

15 Our main concern is not the presence of structural changes, but the order of integration in the series. For the rest of the series, the order of integration has already been determined using ADF, PP and KPSS tests.
possibility of this change being gradual or, alternatively, instantaneous. Following Perron and Vogelsang (1992), we obtain that the results concerning the presence of unit roots are similar to those derived previously using the Perron (1990) test: the first differences of the variables do not reject the presence of a unit root when a structural change is produced. However, they differ with respect to the structural change period: for instance, in the innovational outlier model, in no case is it 1986:1, but 1983:4 or 1984:3 instead. These estimated periods, however, are not supported by the graphs of the series. Visual inspection suggests that, if it exists, this occurred around 1986, when Spain joined the EEC.\footnote{The results are not included for reasons of space, but they are available on request.}

As a second extension, we may ask if the non-coincidence between the structural change period proposed beforehand and the estimated one is due to the existence of more than one change in the structure of the series. There would not be a contradiction in the results obtained from Perron (1990) and Perron and Vogelsang (1992) tests, if there were two changes in the mean of the first differences rather than one. García-Fontes and Hopenhayn (1996), e.g., refer to three phases (or two structural changes) in the aggregate unemployment rate in Spain.\footnote{The first one from 1976 to 1984; the second one from 1984 to 1991, and the last one from 1991 to 1993.}

Clemente et al. (1998) provide an adequate method to make a test of this type. The results, not shown in the paper (available on application to the authors), do not reject an order of integration equal to two in $U_1^h$, $U_1^m$ and $U_2^h$. Likewise, in all cases except for $\Delta U_1^h$, the first estimated period for the structural break, under the additive outlier model, coincides with the one estimated previously using the Perron and Vogelsang (1992) methodology.

A basic conclusion can be drawn from this Section. For some series ($E_h$, $E_m$, $T_h$, $T_m$, $U_2^m$, $U_2^h$, $U_3^m$ and $Y$) the hypothesis of integration of order 1 is not rejected. For the other series ($U_1^h$, $U_1^m$ and $U_2^h$), there exists uncertainty about their order of integration: the results of some tests suggest that they are stochastic processes with a unit root, whereas others point to their being processes with two unit roots (regardless of whether there are structural changes or not). The justification for this apparent contradiction might be found in the reduced sample period for the series analyzed.

### 4.2 Cointegration analysis

Once the non stationarity of the series has been analyzed, we focus on the study of the long run relationships between the unemployment rates for each group, demographic structure and GDP growth rate.

The interpretation of the results obtained from the estimated model by means of
more traditional econometric techniques must be done cautiously. Cointegration theory provides the appropriate framework for analysis because it facilitates the estimation of the long run parameters in an equation with non-stationary series. Likewise, the short run relationships can be studied by means of the error correction model. In other words, it also allows us to merge short run dynamics with the long run equilibrium.

Let us consider $U_{ij}^j$ the variable to be explained and $E_i$, $T_i$ and $Y$ the explanatory variables ($i = h, m$ and $j = 1, 2, 3$). Assuming a labor market where workers with different ages are not perfect substitutes, one may conclude the existence of a crowding-out effect among the cohorts. In other words: larger cohorts are affected by higher unemployment rates [see Ahn et al. (2000)]. So, for example, the largest youth population during the sample period (due to the baby boom of the 1960s) will exhibit the highest unemployment rate. One could expect, therefore, a negative relationship between the relative size of adult population and the unemployment rate. The same can be also inferred from the visual inspection of the series.

This relationship is positive in Zimmermann (1992), because the definition of the demographic variables is exactly the contrary: young relative to adult population. Schmidt (1993) and Ahn et al. (2000) have also found that increases in the cohort size are associated with increases in the unemployment rates. We shall endeavor to confirm this hypothesis using Spanish economy data for 1976 : 3 - 1998 : 4.

We test the presence of long run equilibrium relationships among the variables using two alternative approaches: the one proposed by Engle and Granger (1987) and the one proposed by Johansen (1988). Next, we present both methodologies and their results.

### 4.2.1 Engle and Granger’s approach

Concerning the first one, and as for Zimmermann (1992) and Schmidt (1993), we estimate a regression model, for each specific group, among the levels of the variables:

$$U_{ijt} = b_0 + b_1 E_{it} + b_2 T_{it} + b_3 Y_t + \zeta_t,$$

for $j = 1, 2, 3$, $i = h, m$, and $t = 1, \ldots, T$. The $OLS$ estimator is adequate for this purpose, as it is super-consistent. Their probability distribution is not, however, a normal distribution. Therefore, the $t$ and $F$ statistics used for the tests of individual and joint significance, respectively, do not have standard distributions. So the common inference procedures are not appropriate. The election of the regressors cannot be based, therefore, on the usual tests of significance. The results are shown in Table V and VI for male and female workers respectively.

**INSERT TABLES V AND VI AROUND HERE**
An OLS regression therefore enables us to estimate the long run relationship among the variables, ignoring the dynamic process. This, however, can only be justified if the residuals obtained are stationary. In this way, we analyze the residuals in the cointegrating regression, $\hat{\zeta}_t$.\(^{18}\) The augmented Dickey and Fuller test is one of the recommended procedures in Engle and Granger (1987) to study this feature of the residuals. The results are shown in the last row of Tables V and VI [see $ADF_{res}$].\(^{19}\)

The results obtained are contrary to expectations: the presence of a unit root in the residuals is not rejected in any of the models. Consequently, there is no long run equilibrium relationship between the unemployment rate, the variables representing the age structure of population and the GDP growth rate. In other words, the evidence of a long run joint evolution among the implicated variables cannot be assured. This conclusion can be applied to all the groups studied, regardless of their sex and age.

With a view to analyzing the robustness of these results, we considered some alternative specifications in the study of this kind of relationship. In particular, we estimated some models which include only one variable representing the demographic structure: either the age or the population size. As a third alternative, we have added a trend to the equation (6). [See Schmidt (1993) and Ahn et al. (2000)]. The same conclusion is obtained in all cases: the absence of long run equilibrium relationships between the demographic variables, the unemployment rate of each specific group and the GDP growth rate. It seems that these series, therefore, do not follow a common pattern in the long run.

These results can be interpreted in several different ways. First, there is uncertainty about the order of integration of the series in some cases (see $U_1^h$, $U_1^{un}$ and $U_2^h$). The estimated models, nevertheless, consider that all series are $I(1)$ and the existence of cointegration is conditioned to stationary residuals. This could be different if some series were $I(2)$, as might be the case with the three series mentioned above. If so, we would be including variables with different orders of integration and we would be trying to explain an $I(2)$ variable by using $I(1)$ variables.

Second, the fact that we do not detect any kind of cointegration does not imply that the series are not related in the long run through an equilibrium relationship: some of the variables omitted in the regression might affect the equilibrium, so that the error term would make the residual non-stationary.

Third, another justification may be found in some variables of the model, particularly

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\(^{18}\)We test the null hypothesis that $\hat{\zeta}_t$ has a unit root to the alternative that the series is stationary. This requires the estimation of an auxiliary regression as: $\Delta \hat{\zeta}_t = a_1 \hat{\zeta}_t + \sum_{i=1}^p a_i \Delta \hat{\zeta}_{t-i} + error_t$.

\(^{19}\)The critical values firstly tabulated by Fuller (1976) are not valid for this purpose: these were originally derived considering the variable to be analyzed as an observed and not an estimated one. Engle and Granger (1987), Engle and Yoo (1987) and Phillips and Ouliaris (1990), however, have provided the appropriate percentiles.
those which vary slowly. Variability of data referring to population variables is very low in small samples, which implies a lack of precision in the estimation.

Some counter-intuitive results led us to produce a second analysis with an alternative method of estimation. Johansen’s approach was suitable for this purpose.

### 4.2.2 Johansen’s approach

This approach, worked out by its author in several papers [see Johansen (1988, 1991a, 1991b) and Johansen and Juselius (1990, 1992)], suggests estimating simultaneously the space of cointegrating vectors, in a multivariate setup, using the maximum likelihood method. The approach admits the existence of multiple cointegration and allows for hypothesis testing on the model parameters.

Let \( X_t \) be a vector, \( X_t' = [U_{i,t} E_{t,i} T_{i,t} Y_t] \), whose dimension, \( k \), is given by the number of series in the model (in our case, \( k = 4 \)). We assume that \( X_t \) admits a representation of error correction model (ECM):

\[
\Delta X_t = \Pi X_{t-1} + \sum_{i=1}^{p-1} \Gamma_i \Delta X_{t-i} + \mu + \eta_t, \tag{7}
\]

where \( \mu \) is a vector of constants and \( \eta_t \) a \((k\)-dimensional) independent stochastic process, normally distributed, with zero mean and covariance matrix \( \Omega \). \( \Pi \), a \( k \times k \) matrix, is the long run impact matrix, and it is this matrix that provides information about the existence of cointegrating relationships among the series in \( X_t \). If \( \Pi \) has reduced rank equal to \( r \) \((r < k)\), we can conclude that there exist \( r \) long run equilibrium relationships, and \( \Pi \) can be written as \( \Pi = \alpha \beta' \). Then, \( \beta \) is a \( k \times r \) matrix that contains the \( r \) cointegrating vectors and \( \alpha \), with a similar order, includes the short run adjustment coefficients of each of the variables to the long run equilibrium.\(^{20}\)

The determination of the possible existence (and, where appropriate, the number) of some long run equilibrium relationships among the series in \( X_t \) is made using the trace, \( Tr \), and the maximal eigenvalue statistics, \( \lambda_{max} \). We carried out the tests assuming first that the model contains a deterministic linear trend \( (Tr, \lambda_{max}) \) and, later, assuming that this trend is not present \( (Tr^*, \lambda_{max}^*) \).\(^{21}\) Following Johansen (1991a), we use a statistic defined as \( [Tr^*(r) - Tr(r)] \) to test for the null hypothesis of absence of a linear trend.\(^{22}\) The results are shown in Tables VII (for male workers) and VIII (for female workers).

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\(^{20}\)We have checked that the assumptions in the model are satisfied: first, independence across time of the error term [we have used the Akaike (1973) and Schwarz (1978) information criteria, and the Ljung Box's test (1978)] and, second, normality of the errors. The latter is not essential because the maximum likelihood method is robust to the absence of this property [see Gonzalo (1994)].

\(^{21}\)Critical values for their asymptotic distributions have been tabulated by Johansen and Juselius (1990) and Osterwald-Lenum (1990, 1992).

\(^{22}\)To find the number of long run relationships among the variables, we also follow a graphical analysis of the possible cointegrating relationships. The representations are not included here for reasons of space.
We began by analyzing the possible long run equilibrium relationships among the young
est unemployed male workers $U^h_1$, the demographic variables $E^h$ and $T^h$, and the GDP growth rate $Y$, assuming that all variables are $I(1)$ [see Table VII]. The existence of four cointegrating relationships is not rejected, thus reflecting the stationarity of the four variables. This result differs from the one obtained in the previous section. There, we showed that $U^h_1$ contained at least one unit root. This may be the problem: $U^h_1$ has been included in the model as a stationary variable in first differences, but this might not be the case. Therefore, this result gives rise to a second analysis in which we consider that $U^h_1 \sim I(2)$. In this case, three of the four statistics ($Tr$, $\lambda_{\text{max}}$ and $\lambda_{\text{max}}^*$) do not reject the existence of two possible cointegrating relationships. A different conclusion is derived from the $Tr^*$ statistic, because $H_0^* : r \leq 2$ is rejected. This rejection is, nevertheless, marginal, as the values of the statistics are almost equal to the critical values tabulated (20.81 vs. 20.17, at the 5% significance level). So, we consider the existence of two long run relationships in a model without a linear trend.

When the unemployed male workers in the middle aged $U^h_2$ are included in $X_t$, the result is analogous to that obtained in i). Assuming that $U^h_2$ is $I(1)$ and analyzing the trace statistics, four cointegrating relationships are found. The values obtained using the maximal eigenvalue statistics do not provide the same result. However, these are almost equal to the critical values tabulated. We can think, therefore, that $r = 4$. This implies that $U^h_2$, $E^h$, $T^h$ and $Y$ are stationary, once again differing from the results in the previous section. Thus, in a second step, and as in i), we consider that the $U^h_2$ series is $I(2)$. In this case, we do not reject the existence of two cointegrating relationships in a model that does not contain a linear trend.

To finish with the unemployed male workers, we consider unemployed male workers over 44, $U^h_3$. There exists ambiguity when we choose the number of cointegrating vectors, because both testing procedures produce different results. Thus, under $Tr$, the presence of four cointegrating relationships in a model with a linear trend is not rejected. Under $\lambda_{\text{max}}$ and $\lambda_{\text{max}}^*$, however, and in the absence of a linear trend, only one is found. The final conclusion has been based on the maximal eigenvalue statistics. As a

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23 The exception arises from the analysis of $\lambda_{\text{max}}$ for the null of $H_0 : r \leq 2$. However, the value of the statistic is almost the critical value tabulated. The same occurs when we analyze $H_0^* : r \leq 3$ in a model without a linear trend.

24 With regard to the trend, some procedures can be used to detect its presence. First, by testing $H_0 : r \leq 2$ and $H_0^* : r \leq 2$. Neither of the two hypotheses is rejected, which does not imply the presence of a linear trend. Second, $Tr^* - Tr = 7.29 > \chi^2_{(2)}$, thereby rejecting the null hypothesis of absence of a linear trend. Given that both results are contradictory, the final decision was based on the graphical analysis of the residuals: the model without a linear trend provides some more satisfactory graphic representations in stationarity terms.

25 The graphic representation of the cointegrating residuals is illustrative in this case: a stationary
consequence, we conclude with the existence of only one long run relationship in a model without a linear trend.

\( iv) \) The same result is obtained for young unemployed female workers: the presence of only one long run equilibrium relationship between the series in \( X_t \) is not rejected (\( U_{1m}, E_m, T_m \) and \( Y \)). Under \( Tr^* \), \( H_0 : r = 1 \) and \( H_0 : r = 2 \) are rejected, although the values of the statistics are almost equal to the critical values tabulated. This explains why the final decision was based on \( \lambda_{\text{max}}, \lambda_{\text{max}}^* \) and \( Tr \).\(^{26}\)

\( v) \) When the model includes \( U_{2m} \), we can conclude that there is no equilibrium or long run relationship among the middle aged unemployed female workers and the rest of the series in \( X_t \). The values of the statistics to test for \( H_0 : r = 0 \) allow us to reject this hypothesis in a model without a linear trend. However, these are close to the critical values tabulated. That is, this hypothesis would not be rejected at a significance level only slightly lower than 5%.

\( vi) \) To conclude, we consider the model that includes the unemployment rate of the eldest female workers \( U_{3m} \). The result is similar to the one obtained for the unemployed lower aged female workers: there is no long run relationship between \( U_{3m}, E_m, T_m \) and \( Y \).

A summary of the results is shown in Table IX.

**INSERT TABLE IX AROUND HERE**

The disparity in the results is worthy of note. The use of Engle and Granger’s (1987) approach gives unexpected results. We find no relationship among the unemployment rates and the variables representing the evolution of the population pyramid, regardless of sex and age of unemployed, except when the first differences in \( U_{1h} \) are included.\(^{27}\)

Following Johansen’s procedure, however, the conclusions are modified in some cases. We do not detect long run equilibrium relationships among the population variables and behavior can be seen in those obtained from a model without a linear trend, being the opposed when the trend is not present.

\(^{26}\)There exists, nevertheless, some ambiguity about whether the analysis must be made in a model with or without a linear trend. A visual inspection of the cointegrating residuals would support this last result. The graphs do not seem to be stationary in a model with this deterministic component. The representations are improving remarkably (in stationarity terms) if the trend is not present. The tests (on \( \beta \) and \( \alpha \)) that we are going to implement later have been carried out for both cases, obtaining robust results under both specifications.

\(^{27}\)As for coherence, we make new regressions using Engle and Granger’ approach. The new feature appears in the dependent variables we use. These will be the first differences of the unemployment rate and not their levels. In particular, we analyze two cases: the one including \( \Delta U_{1h} \) and the one including \( \Delta U_{2h} \). The results, with regard to the existence of equilibrium relationships, are similar to the ones obtained with Johansen’s procedure in the first case (\( DFA_{\text{res}} = -4.866 \)) but differ in the second one (\( DFA_{\text{res}} = -3.926 \)). That is, when we use Engle and Granger’s approach for \( \Delta U_{1h} \) and \( \Delta U_{2h} \), only one long run relationship among the demographic variables, the \( GDP \) growth rate and the first differences in the unemployment rates of the youngest male workers (between 16 to 29) is detected, rejecting this relationship if the male workers in the higher age segment are analyzed (from 30 to 44).
the unemployment rates of the middle aged (from 30 to 44) and oldest (45 and over) female workers, but we do detect equilibrium relationships for the remaining sex-age groups.

To sum up, there is at least one cointegrating relationship between the series. This is the case for male workers (regardless the age), and for the youngest female workers. For these groups, the unemployment rate, the relative mean age and relative size of adult population, and the GDP growth rate evolve jointly thus keeping at least one long run equilibrium relationship. There exists, therefore, a common evolution between these series, thus showing that population ageing has significant long run effects upon some specific Spanish unemployment rates.

Zimmermann (1992) and Schmidt (1993) obtain similar results. None of them finds a clear long run relationship between the age distribution of German population and unemployment rates for specific age and sex. Schmidt (1993), who represents the demographic structure through the relative size of the own cohort, can only justify the existence of long run relationships in five out of fourteen sex-age groups analyzed [these corresponding to age segments: 15 to 19, 20 to 24, 25 to 34 and 35 to 44 for female, and 55 to 59 for the workers]. Something similar occurs in Zimmermann (1992). He uses the same classification for population (by sex and for seven age segments, that is, fourteen groups), and only detects such long run relationship for eight cases [15 to 19, 20 to 24, 25 to 34, 55 to 59 and 60 to 64 for female, and 20 to 24, 25 to 34 and 55 to 59 for male workers]. To capture the demographic effect, this author uses the same variables as in this paper.

Ahn et al. (2000) do not obtain conclusive results either when they use a specification similar to the one presented here [the cohort size represents the demographic effect and the GDP growth rate works as a variable to control for the business cycle]. They find that the influence of the size of the youth population upon the youth unemployment rate differs depending on the population group considered (from 16 to 19, from 20 to 24 or from 25 to 29) and on the demographic variables included (the own cohort size and/or the cohort size of the younger adjacent cohort). They find positive incidence in some cases and non statistically significant incidence in others.

4.2.3 Tests relative to the cointegrating vectors and the weighting matrix

We are going to analyze some characteristics relative to the cointegrating vectors, \( \beta_j \) \((j = 1, \ldots, r)\), and the weighting matrix, \( \alpha \), for those groups for which we have not rejected the existence of, at least, one equilibrium relationship: namely, male unemployed in the three age segments and youngest female unemployed. Unlike Engle and Granger’s approach, where standard inference procedures are invalid due to non-stationary series in the models, hypothesis tests relative both to the cointegrating vectors, \( \beta \), and to the adjustment coefficient matrix, \( \alpha \), can be implemented with Johansen’s methodology [see Johansen and Juselius (1990, 1992) and Johansen (1991b)] with three aims being pursued.
Firstly, following Johansen and Juselius (1992), we analyze the stationarity in the series \((U_i, E_i, T_i, Y)\) in a multivariate framework and obtain, as expected, non-stationarity in all of them. More precisely, we reject the null hypothesis that cointegrating vectors are composed by only one variable, for each component in \(X_t\) [i.e., we test if each cointegrating vector is such that \(\beta_k^j = 1\) and the remaining \(\beta_j^\ell\) are zero, where \(k \neq \ell\) with \(k = U, E, T, Y\), for \(j = 1, 2, ..., r\)]. We reject, in short, the presence of trivial cointegrating vectors.\(^{28}\)

Secondly, we test whether all series \((U_i, E_i, T_i, Y)\) are part of the cointegrating vector(s) or if some of them should be excluded. In the former case, all the series play the same role in the long run relationships. In the latter case, the long run behavior of the system would not depend on the excluded variable(s). To ascertain the cointegrating relationship between \(U_i, E_i\) and \(T_i\) none of them should be excluded. Although some of these series cannot affect the long run behavior, its short run dynamics could be affected by deviations from this long run equilibrium.

Johansen and Juselius (1990, 1992) propose a test for this purpose. The results, for all the variables in \(X_t\), are presented in Table X. [Remember that the analysis is made only for those models in which we have not rejected the existence of equilibrium relationships: namely, those containing \(\Delta U_1, \Delta U_2, \Delta U_3\) and \(\Delta U_m\).]

| INSERT TABLE X AROUND HERE |

The hypothesis of exclusion of the \(T_i\) variable is not rejected in three out of four models. Therefore, the relative size of the adult population does not seem to influence the long run behavior of the system. Thus, the effect of changes in the demographic structure is captured by the relative mean age of adult population, \(E_i\).

However, in the fourth model, the one that includes \(\Delta U_2\), the incidence of changes in population is shown by both demographic variables, \(E_h\) and \(T_h\). In this case, we reject the exclusion of all the variables in the cointegrating vectors, so that all of them affect the long run behavior of the system.

There are two models (those containing oldest unemployed male workers and youngest unemployed female workers) in which both \(T_i\) and the unemployment rates must be excluded. This implies that the cointegrating vectors are integrated only by the GDP growth rate and by the relative mean age of adult population (i.e., \(Y\) and \(E_i\)). A long run relationship between these variables might have a reasonable justification from an economic point of view to the extent that the population ageing might really affect the GDP.

To sum up, we cannot justify, in general, the existence of a long run equilibrium relationship between the demographic variables (the relative size and the relative mean age of adult population), the variable that represents the economic situation (the growth

\(^{28}\)The results are not shown here for reasons of space but they are available on request to the authors.
rate of GDP) and the unemployment rates, disaggregated by sex and age. The exception is given when we analyze the youngest and the middle aged male workers. In these groups, one can actually find a long run joint evolution between the unemployment rates and the population variables.

Thirdly, a cointegrating vector can be interpreted as an equilibrium relationship from which variables can deviate only temporarily. The elements in the $\alpha$ matrix represent the speed of response of these variables in the short run to transitory deviations from the long run relationships. Thus, it is worth wondering whether all the variables in the system react to such deviations or, on the contrary, this adjustment is produced only by some of them. A weak exogeneity test would give an answer to this question.

We must be aware that the study of short run adjustments of some variables (such as $T_i$ and $E_i$) might not be appropriate here, because the changes in the age composition of population are not expected to be instantaneous. As a result, when we implement the tests, we are implicitly assuming that the remaining variables makes the corresponding short run changes (in this case, there is only one variable left, GDP growth rate).

The procedure is as follows. If $\alpha_{kj} = 0$, where $k = U, E, T, Y$ and $j = 1, ..., r$ (where $r$ denotes the maximum number of long run equilibrium relationships or cointegrating vectors), this would be interpreted as the absence of adjustment of the $k$ variable in the presence of transitory deviations from the equilibrium relationships. In other words, $k$ would be a weakly exogenous variable. [See Johansen and Juselius (1990, 1992)]. The values of the test statistics are shown in Table XI.

\textbf{INSERT TABLE XI AROUND HERE}

One does not find homogeneous results for all groups. The systems including the youngest (from 16 to 29) and the middle aged (from 30 to 44) unemployed male workers reject the weak exogeneity of all variables. That is, those unemployment rates, the population variables and the GDP growth rate do get adjusted in the short run to reach the long run equilibrium relationship among them.

Looking at the system that includes the oldest (45 and over) unemployed male workers, only the relative size of adult population is adjusted to achieve this relationship. $T_i$, therefore, does not affect the long run behavior of the system [see Table X], but it does have influence on its short run path.

Finally, we consider the unemployed female workers with ages between 16 and 29. The weak exogeneity hypothesis is not rejected for the relative mean age of adult population and for the GDP growth rate. Such variables, therefore, are not affected by the cointegrating relationships in their short run evolution. Their unemployment rate and the relative size of female adult population will be adjusted to achieve such relationships, in which the other two variables ($E_m$ and $Y$) will take part.
In short, these results do not confirm the starting hypothesis about the weak exogeneity of the demographic variables. Once again, the reduced informative contents may explain this result.

### 4.2.4 Interpretation of the results

Once the characteristics of the cointegrating vectors and the weighting matrix are known, we analyze the structural relationships which can be inferred from their estimations. \( \beta'X_t = 0 \) represents the equilibrium relationships between the variables in \( X_t \), and the elements of \( \alpha \) represent the adjustment of these variables towards the equilibrium. Appendix II shows the estimations of \( \beta \) and \( \alpha \).

There are certain limitations to interpreting the meaning of the estimated long run relationships among the series. When there is more than one relationship, we cannot be sure about which one should be picked up. Moreover, even though the cointegrating vector is unique, we would not have enough information to identify the long run equilibrium relationships. Why? The matrix \( \Pi = \alpha\beta' \) that appears in the estimated error correction model is identifiable, but \( \beta \) and \( \alpha \) matrices are not. Therefore, the cointegrating vectors cannot be interpreted directly as long run equilibria, because one would be ignoring the dynamic relationships between the variables of the short run system.

Nevertheless, it is possible to prove the existence of the cointegrating relationships. Thus, we can confirm the presence of relationships between the demographic trends and the youngest and middle aged unemployed male workers. Moreover, although the interpretation of these relationships by using the coefficients of the normalized equation is not satisfactory, we can infer, at least, whether this is positive or negative. So, we can write \( \beta'X_t = 0 \) as:

\[
\beta_j^U U_{jt}^j + \beta_j^E E_{jt} + \beta_j^T T_{jt} + \beta_j^Y Y_t^j + \beta_0 \frac{y^j}{j} = 0, \quad j = 1, 2, ..., r,
\]

and we can normalize the cointegrating relationships for a specific unemployment rate (the variable of our interest). We only consider the two models in which long run equilibrium relationships have been found, i.e., those including the youngest and middle aged male workers. [Remember that two cointegrating vectors have been found in both cases.]

The equations referring to the youngest unemployed male workers (from 16 to 29) do not provide clear information about the effect that demographic structure can exert on the unemployment rate: the sign associated with the population variable \( E_{ht} \) is positive in one case and negative in other.

When we analyze the middle aged unemployed male workers, the sign associated with \( T_{ht} \) is not clear, although the sign of \( E_{ht} \) is positive in both cointegrating vectors. Thus,

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29 We consider that the error correction model does not include the vector of constants, so that the dimension of \( \beta \) increases in one unit.
there is a positive relationship between the age and the change in the unemployment rate, while the relationship between the evolution of GDP and the unemployment rate is, as expected, negative.

In short, we cannot confirm our initial hypothesis about the existence of a negative relationship between the relative size of adult population and the unemployment rates, disaggregated by sex and age. However, the results show that increases in the relative mean age of adult workers increase the middle aged unemployment rate of male workers. Zimmermann (1992) offers a similar conclusion.

One might be tempted to follow an impulse response analysis to obtain additional information about these relationships between the variables. This procedure may be appropriate in other contexts but not here. Our purpose in this paper is to analyze the population incidence on unemployment rates, disaggregated by sex and age segment: an analysis that considers shocks in the demographic structure is of reduced relevance because the changes in the Spanish population pyramid, as in most countries, are produced very slowly.

5 Conclusions

In this paper, we have analyzed the long run incidence of changes in the age structure of the population upon unemployment rates, disaggregated by age and sex in Spain during the period 1976:3 – 1998:4. The age structure has been approximated by the relative mean age of adult workers and the relative size of the group of adult workers with respect to the group of young workers. Unemployment rates have been defined considering three age segments: the first between 16 and 29 years old; the second between 30 and 44; and the third of 45 and over.

We have obtained four main results. First, the estimations obtained in the empirical analysis do not provide a clear scheme about the relationships between the evolution of the age structure of population and the unemployment rates. Second, in a first approximation one can detect the existence of, at least, one long run equilibrium relationship between the unemployment rates, the relative mean age of adult population, the relative size of adult population and the GDP growth rate in all cases, except for middle aged and oldest female workers. Third, a more thorough analysis of the significance degree of the variables in such equilibrium relationships suggests that unemployment rates for the eldest male workers and the youngest female workers can be excluded from their corresponding error correction models. The same happens with the relative size of adult population in all cases except for middle aged male workers. As a result of the tests, therefore, one can only justify the existence of long run relationships between the demographic variables and the unemployment rates of the youngest and middle aged male workers. One cannot
argue, however, joint evolution for population variables and the unemployment rates of the youngest female workers and of the oldest male workers. Fourth, the short run dynamics of the unemployment rates of the youngest and middle aged male workers and that of the youngest female workers are affected by transitory deviations from these long run relationships.

Zimmermann (1992) and Schmidt (1993)’s papers on the German case are no more conclusive. Their estimations show that there exists a long run relationship between the population variables and the specific unemployment rates for only a few groups. Something similar occurs in Ahn et al. (2000) when they study the incidence of Spanish youth population on the unemployment rates of young workers.

One cannot confirm, therefore, that changes in the age structure of Spanish population play an important role in the unemployment rates of all age segments. Only the youngest and middle aged unemployed male workers seem to be affected by changes in the demographic composition of the labor force.

The effect of population variables on the number of unemployed workers may depend on activity rates: low activity rates may cause low incidence of population variables on unemployment rates. This reasoning may be valid to argue the absence of response of the unemployment rates for 30-plus female workers and 44-plus male workers: the activity rates were 28.5% and 19.3%, respectively, in 1976:3, and 63.1% and 18.8% in 1998:4. In the same periods, the activity rates for the oldest male workers were 67.5% and 44.9%. They are all very low compared to those of middle aged male workers: 97.5% and 95.2%.

All results must be interpreted cautiously. The reduced number of observations and, in consequence, the reduced informative contents of the sample limit the interpretation of the estimations in a way.

An extension of the model that allows for adjustments in wages and for a more important role in the different educational levels could open another line of research.
Appendix I: Graphs of the Series

Figure 1: Male Workers

- **UNEMPLOYMENT RATE**
  - **men16-29**
  - **men30-44**
  - **men45+**

- **AGE (adults/youths)**

- **SIZE OF POPULATION (adults/youths)**

- **GDP GROWTH RATE**

- **DIF (UNEMPLOYMENT RATE)**
  - **men16-29**
  - **men30-44**
  - **men45+**

- **DIF[AGE (adults/youths)]**

- **DIF[SIZE OF POPULATION (adults/youths)]**

- **DIF[GDP GROWTH RATE]**
Figure 2: Female Workers
Figure 3: Aggregate Unemployment Rate
Appendix II: Maximum likelihood estimations

The maximum likelihood estimations of the cointegrating vectors, $\beta$, and the weighting matrix, $\alpha$, are shown below.

- Unemployed male workers aged between 16 and 29: $\beta = \begin{bmatrix} 566.02 & -32.59 \\ -52.04 & -54.76 \\ 8.49 & -3.53 \\ 2.91 & 0.45 \\ 86.83 & 108.18 \end{bmatrix}$, $\alpha = \begin{bmatrix} -2.84e-03 & 1.46e-03 \\ 1.05e-03 & -5.43e-05 \\ 5.57e-04 & 1.77e-03 \\ -0.06 & -0.05 \end{bmatrix}$, $\beta^* = \begin{bmatrix} 576.31 & 58.62 \\ -53.16 & 52.75 \\ 2.96 & -0.32 \\ 95.90 & -101.67 \end{bmatrix}$.

- Unemployed male workers aged between 30 and 44: $\beta = \begin{bmatrix} -259.27 & -731.47 \\ 51.57 & 36.90 \\ 9.93 & -24.80 \\ -1.04 & -1.52 \\ -106.15 & -47.90 \end{bmatrix}$, $\alpha = \begin{bmatrix} 5.51e-04 & 1.27e-03 \\ 8.99e-04 & -6.61e-04 \\ -1.77e-03 & 3.41e-04 \\ 0.06 & 0.06 \end{bmatrix}$.

- Unemployed male workers aged over 44: $\beta = \begin{bmatrix} -2.47 \\ -98.84 \\ 4.36 \\ 1.81 \\ 184.52 \end{bmatrix}$, $\alpha = \begin{bmatrix} -4.25e-04 \\ 3.54e-04 \\ 1.86e-03 \\ 0.04 \end{bmatrix}$.

- Unemployed female workers aged between 16 and 29: $\beta = \begin{bmatrix} -8.57 \\ 75.93 \\ 4.12 \\ -1.88 \\ -141.66 \end{bmatrix}$, $\alpha = \begin{bmatrix} 3.19e-03 \\ -4.63e-04 \\ -1.34e-03 \\ -5.46e-03 \end{bmatrix}$.

Note: $\beta^*$ denotes $\beta$ once $T_{ht}$ variable has been eliminated ($T_{ht}$ has been eliminated as the test of exclusion for this variable indicates [see Table X]).
### TABLE I: ADF, PP, KPSS TESTS

**Series in levels (T=90)**

<table>
<thead>
<tr>
<th>Series</th>
<th>$ADF$</th>
<th>$PP$</th>
<th>$KPSS$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$U^h_1$</td>
<td>-2.6282</td>
<td>-1.7028</td>
<td>0.1650*</td>
</tr>
<tr>
<td>$U^h_2$</td>
<td>-2.1775</td>
<td>-1.4778</td>
<td>0.1432</td>
</tr>
<tr>
<td>$U^h_3$</td>
<td>-1.7039</td>
<td>-0.9348</td>
<td>0.3884*</td>
</tr>
<tr>
<td>$E_h$</td>
<td>-1.3284</td>
<td>-1.1022</td>
<td>0.3435*</td>
</tr>
<tr>
<td>$T_h$</td>
<td>1.7672</td>
<td>1.5030</td>
<td>0.3072*</td>
</tr>
<tr>
<td>$U^m_1$</td>
<td>-2.4616</td>
<td>-1.2627</td>
<td>0.3670*</td>
</tr>
<tr>
<td>$U^m_2$</td>
<td>0.6177</td>
<td>0.6284</td>
<td>0.9006*</td>
</tr>
<tr>
<td>$U^m_3$</td>
<td>-2.2768</td>
<td>-2.3162</td>
<td>0.5693*</td>
</tr>
<tr>
<td>$E_m$</td>
<td>-0.6917</td>
<td>-0.7036</td>
<td>2.0079*</td>
</tr>
<tr>
<td>$T_m$</td>
<td>0.4473</td>
<td>-0.2357</td>
<td>0.2986*</td>
</tr>
<tr>
<td>$Y$</td>
<td>-2.6217</td>
<td>-2.2848</td>
<td>0.1212</td>
</tr>
</tbody>
</table>

Note: "*" represents rejecting $H_0$ at the 5% level. $T$ denotes sample size. The critical value at 5% level for $ADF$ and $PP$ tests is $-3.45$ for $T = 100$ [Fuller (1976), p. 373, Table VI]. Critical value for $KPSS$ test is 0.136 [KPSS (1992), p. 166].

### TABLE II: ADF, PP, KPSS TESTS

**Series in first differences (T=90)**

<table>
<thead>
<tr>
<th>Series</th>
<th>$ADF$</th>
<th>$PP$</th>
<th>$KPSS$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Delta U^h_1$</td>
<td>-2.9555</td>
<td>-6.0285*</td>
<td>0.1031</td>
</tr>
<tr>
<td>$\Delta U^h_2$</td>
<td>-2.5797</td>
<td>-7.3910*</td>
<td>0.0832</td>
</tr>
<tr>
<td>$\Delta U^h_3$</td>
<td>-4.0442*</td>
<td>-7.6374*</td>
<td>0.1433</td>
</tr>
<tr>
<td>$\Delta E_h$</td>
<td>-5.6032*</td>
<td>-11.4843*</td>
<td>0.0377</td>
</tr>
<tr>
<td>$\Delta T_h$</td>
<td>-4.6328*</td>
<td>-8.0637*</td>
<td>0.1291</td>
</tr>
<tr>
<td>$\Delta U^m_1$</td>
<td>-2.7114</td>
<td>-7.6656*</td>
<td>0.0988</td>
</tr>
<tr>
<td>$\Delta U^m_2$</td>
<td>-7.6499*</td>
<td>-7.7837*</td>
<td>0.1224</td>
</tr>
<tr>
<td>$\Delta U^m_3$</td>
<td>-9.7168*</td>
<td>-9.8868*</td>
<td>0.1233</td>
</tr>
<tr>
<td>$\Delta E_m$</td>
<td>-10.8066*</td>
<td>-10.9957*</td>
<td>0.0274</td>
</tr>
<tr>
<td>$\Delta T_m$</td>
<td>-5.4719*</td>
<td>-8.8257*</td>
<td>0.0877</td>
</tr>
<tr>
<td>$\Delta Y$</td>
<td>-5.6241*</td>
<td>-4.0323*</td>
<td>0.1003</td>
</tr>
</tbody>
</table>

Note: "*" represents rejecting $H_0$ at the 5% level. $T$ denotes sample size. The critical value at 5% level for $ADF$ and $PP$ tests is $-3.45$ for $T = 100$ [Fuller (1976), p. 373, Table VI]. Critical value for $KPSS$ test is 0.136 [KPSS (1992), p. 166].
### TABLE III: Perron (1990) Test

**Innovational outlier model**

<table>
<thead>
<tr>
<th>Series</th>
<th>$\tau(\alpha^* = 1)$</th>
<th>$p$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Delta U^h_1$</td>
<td>-2.9575</td>
<td>5</td>
</tr>
<tr>
<td>$\Delta U^h_2$</td>
<td>-2.4878</td>
<td>5</td>
</tr>
<tr>
<td>$\Delta U^m_1$</td>
<td>-2.5125</td>
<td>5</td>
</tr>
</tbody>
</table>

*Note: $\tau(\alpha^* = 1)$ denotes the test statistic under the null of unit root in presence of structural change and $p$ the lag truncation parameter (we have followed Hall (1990), Campbell and Perron (1991), Perron (1990) and Perron and Vogelsang (1992) to obtain it). Critical values for 5% significance level: -3.38 for $T = 100$ and $T_B/T = 0.5$, where $T$ denotes sample size.*

### TABLE IV: Perron (1990) Test

**Additive outlier model**

<table>
<thead>
<tr>
<th>Series</th>
<th>$\tau(\alpha^o = 1)$</th>
<th>$p$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Delta U^h_1$</td>
<td>-3.0491</td>
<td>5</td>
</tr>
<tr>
<td>$\Delta U^h_2$</td>
<td>-2.4792</td>
<td>5</td>
</tr>
<tr>
<td>$\Delta U^m_1$</td>
<td>-3.0735</td>
<td>5</td>
</tr>
</tbody>
</table>

*Note: $\tau(\alpha^o = 1)$ denotes the test statistic under the null of unit root in presence of structural change and $p$ the lag truncation parameter (we have followed Hall (1990), Campbell and Perron (1991), Perron (1990) and Perron and Vogelsang (1992) to obtain it). Critical values for 5% significance level: -3.38 for $T = 100$ and $T_B/T = 0.5$, where $T$ denotes sample size.*

### TABLE V: Cointegration Test (Male Workers)

#### Engle and Granger’s Approach

<table>
<thead>
<tr>
<th>Variable to be explained</th>
<th>$U^h_1$</th>
<th>$U^h_2$</th>
<th>$U^h_3$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$cte$</td>
<td>-1.562</td>
<td>0.350</td>
<td>0.009</td>
</tr>
<tr>
<td></td>
<td>(-3.53)</td>
<td>(2.10)</td>
<td>(0.07)</td>
</tr>
<tr>
<td>$E_h$</td>
<td>1.450</td>
<td>0.117</td>
<td>0.248</td>
</tr>
<tr>
<td></td>
<td>(6.12)</td>
<td>(1.30)</td>
<td>(3.53)</td>
</tr>
<tr>
<td>$T_h$</td>
<td>-1.212</td>
<td>-0.574</td>
<td>-0.495</td>
</tr>
<tr>
<td></td>
<td>(-10.58)</td>
<td>(-13.29)</td>
<td>(-14.59)</td>
</tr>
<tr>
<td>$Y$</td>
<td>-0.009</td>
<td>-0.004</td>
<td>-0.002</td>
</tr>
<tr>
<td></td>
<td>(-2.82)</td>
<td>(-2.88)</td>
<td>(-1.91)</td>
</tr>
<tr>
<td>$ADF_{res}$</td>
<td>-2.767</td>
<td>-2.852</td>
<td>-2.546</td>
</tr>
</tbody>
</table>

### Table VI: Cointegration Test (Female Workers)
<table>
<thead>
<tr>
<th>Variable to be explained</th>
<th>$U_1^m$</th>
<th>$U_2^m$</th>
<th>$U_3^m$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$cte$</td>
<td>$-1.496$</td>
<td>$2.509$</td>
<td>$2.263$</td>
</tr>
<tr>
<td></td>
<td>$(−3.91)$</td>
<td>$(14.74)$</td>
<td>$(22.45)$</td>
</tr>
<tr>
<td>$E_m$</td>
<td>$1.713$</td>
<td>$-0.622$</td>
<td>$-0.783$</td>
</tr>
<tr>
<td></td>
<td>$(8.30)$</td>
<td>$(−6.78)$</td>
<td>$(−14.41)$</td>
</tr>
<tr>
<td>$T_m$</td>
<td>$-1.649$</td>
<td>$-1.267$</td>
<td>$-0.726$</td>
</tr>
<tr>
<td></td>
<td>$(−21.16)$</td>
<td>$(−36.55)$</td>
<td>$(−35.78)$</td>
</tr>
<tr>
<td>$Y$</td>
<td>$-0.002$</td>
<td>$-0.0004$</td>
<td>$-0.0004$</td>
</tr>
<tr>
<td></td>
<td>$(−0.74)$</td>
<td>$(−0.32)$</td>
<td>$(−0.58)$</td>
</tr>
<tr>
<td>$ADF_{res}$</td>
<td>$-1.601$</td>
<td>$-3.821$</td>
<td>$-4.064$</td>
</tr>
</tbody>
</table>

Note: Critical values to test the null of non-cointegration: $-4.22$ for $T=100$ and 5% significance level. [See Engle and Yoo (1987).] $ADF_{res}$ is the statistic to test the null hypothesis of a unit root in the residuals.

### TABLE VII: Cointegration Tests (male workers)

**Johansen’s approach**

#### Presence of a Linear Trend

<table>
<thead>
<tr>
<th>$U_1^h$ ($p^*=6$)</th>
<th>$U_2^h$ ($p^*=6$)</th>
<th>$U_3^h$ ($p^*=6$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$H_0$</td>
<td>$Tr$</td>
<td>$Tr$</td>
</tr>
<tr>
<td>$r = 0$</td>
<td>$77.56^*$</td>
<td>$73.72^*$</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>$35.19^*$</td>
<td>$28.96^*$</td>
</tr>
<tr>
<td>$r \leq 1$</td>
<td>$42.37^*$</td>
<td>$40.86^*$</td>
</tr>
<tr>
<td></td>
<td>$21.52^*$</td>
<td>$23.71^*$</td>
</tr>
<tr>
<td>$r \leq 2$</td>
<td>$20.84^*$</td>
<td>$17.15^*$</td>
</tr>
<tr>
<td></td>
<td>$10.97$</td>
<td>$11.07$</td>
</tr>
<tr>
<td>$r \leq 3$</td>
<td>$7.77^*$</td>
<td>$6.17^*$</td>
</tr>
</tbody>
</table>

#### Absence of a Linear Trend

<table>
<thead>
<tr>
<th>$U_1^h$ ($p^*=6$)</th>
<th>$U_2^h$ ($p^*=6$)</th>
<th>$U_3^h$ ($p^*=6$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$H_0^*$</td>
<td>$Tr^*$</td>
<td>$Tr^*$</td>
</tr>
<tr>
<td>$r = 0$</td>
<td>$77.84^*$</td>
<td>$75.84^*$</td>
</tr>
<tr>
<td></td>
<td>$29.66^*$</td>
<td>$31.76^*$</td>
</tr>
<tr>
<td>$r \leq 1$</td>
<td>$48.18^*$</td>
<td>$44.07^*$</td>
</tr>
<tr>
<td></td>
<td>$22.81^*$</td>
<td>$21.24$</td>
</tr>
<tr>
<td>$r \leq 2$</td>
<td>$25.36^*$</td>
<td>$22.83^*$</td>
</tr>
<tr>
<td></td>
<td>$15.56$</td>
<td>$16.32$</td>
</tr>
<tr>
<td>$r \leq 3$</td>
<td>$8.18$</td>
<td>$7.27$</td>
</tr>
</tbody>
</table>

#### Cointegration Tests (male workers)

**Presence of a Linear Trend**

<table>
<thead>
<tr>
<th>$\Delta U_1^h$ ($p^*=6$)</th>
<th>$\Delta U_2^h$ ($p^*=5$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$H_0$</td>
<td>$Tr$</td>
</tr>
<tr>
<td>$r = 0$</td>
<td>$83.95^*$</td>
</tr>
<tr>
<td></td>
<td>$47.15^*$</td>
</tr>
<tr>
<td>$r \leq 1$</td>
<td>$36.81^*$</td>
</tr>
<tr>
<td></td>
<td>$23.28^*$</td>
</tr>
<tr>
<td>$r \leq 2$</td>
<td>$13.52$</td>
</tr>
<tr>
<td></td>
<td>$9.43$</td>
</tr>
<tr>
<td>$r \leq 3$</td>
<td>$4.09$</td>
</tr>
</tbody>
</table>

**Absence of a Linear Trend**
\[
\begin{array}{cccccc}
\Delta U_1^h (p^* = 6) & \Delta U_2^h (p^* = 5) \\
H_0^* & Tr^* & \lambda_{max}^* & Tr^* & \lambda_{max}^* \\
\hline
r = 0 & 94.69^* & 47.39^* & 83.26^* & 40.00^* \\
r \leq 1 & 47.29^* & 26.49^* & 43.26^* & 25.07^* \\
r \leq 2 & 20.81^* & 15.49 & 18.19 & 12.07 \\
r \leq 3 & 5.31 & 5.31 & 6.12 & 6.12 \\
\end{array}
\]

Note: In the first row both the unemployment rate (levels or first differences) and the number of lags, \(p^* = p - 1\), included in the corresponding ECM are shown. "\(^*\)" denotes the rejection of \(H_0\) at the 5% significance level. \(Tr\) and \(\lambda_{max}\) (\(Tr^*\) and \(\lambda_{max}^*\)) are the trace and the maximal eigenvalue statistics in a model with deterministic linear trend (without deterministic linear trend). \(r\) denotes the number of cointegrating vectors.

**TABLE VIII: Cointegration Tests (female workers)**

Johansen’s Approach

<table>
<thead>
<tr>
<th>Presence of a Linear Trend</th>
<th>(U_1^m (p^* = 6))</th>
<th>(U_2^m (p^* = 3))</th>
<th>(U_3^m (p^* = 4))</th>
</tr>
</thead>
<tbody>
<tr>
<td>(H_0)</td>
<td>(Tr)</td>
<td>(\lambda_{max})</td>
<td>(Tr)</td>
</tr>
<tr>
<td>(r = 0)</td>
<td>60.32^*</td>
<td>34.83^*</td>
<td>43.15</td>
</tr>
<tr>
<td>(r \leq 1)</td>
<td>25.49</td>
<td>15.39</td>
<td>25.03</td>
</tr>
<tr>
<td>(r \leq 2)</td>
<td>10.10</td>
<td>8.14</td>
<td>11.31</td>
</tr>
<tr>
<td>(r \leq 3)</td>
<td>1.96</td>
<td>1.96</td>
<td>3.44</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Absence of a Linear Trend</th>
<th>(U_1^m (p^* = 6))</th>
<th>(U_2^m (p^* = 3))</th>
<th>(U_3^m (p^* = 4))</th>
</tr>
</thead>
<tbody>
<tr>
<td>(H_0^*)</td>
<td>(Tr^*)</td>
<td>(\lambda_{max}^*)</td>
<td>(Tr^*)</td>
</tr>
<tr>
<td>(r = 0)</td>
<td>74.90^*</td>
<td>38.50^*</td>
<td>57.31^*</td>
</tr>
<tr>
<td>(r \leq 1)</td>
<td>36.39^*</td>
<td>15.69</td>
<td>29.02</td>
</tr>
<tr>
<td>(r \leq 2)</td>
<td>20.70^*</td>
<td>12.33</td>
<td>14.97</td>
</tr>
<tr>
<td>(r \leq 3)</td>
<td>8.37</td>
<td>8.37</td>
<td>5.25</td>
</tr>
</tbody>
</table>

Note: In the first row both the unemployment rate (levels or first differences) and the number of lags, \(p^* = p - 1\), included in the corresponding ECM are shown. "\(^*\)" denotes the rejection of \(H_0\) at the 5% significance level. \(Tr\) and \(\lambda_{max}\) (\(Tr^*\) and \(\lambda_{max}^*\)) are the trace and the maximal eigenvalue statistics in a model with deterministic linear trend (without deterministic linear trend). \(r\) denotes the number of cointegrating vectors.

**TABLE IX: Cointegrating Relationships**

<table>
<thead>
<tr>
<th>age</th>
<th>Engle and Granger</th>
<th>Johansen</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>male workers</strong></td>
<td>16 – 29(^*)</td>
<td>yes</td>
</tr>
<tr>
<td></td>
<td>30 – 44(^*)</td>
<td>no</td>
</tr>
<tr>
<td></td>
<td>(\geq 45)</td>
<td>no</td>
</tr>
<tr>
<td><strong>female workers</strong></td>
<td>16 – 29</td>
<td>no</td>
</tr>
<tr>
<td></td>
<td>30 – 44</td>
<td>no</td>
</tr>
<tr>
<td></td>
<td>(\geq 45)</td>
<td>no</td>
</tr>
</tbody>
</table>
Note: \( r \) is the number of cointegrating vectors. (*) denotes a model that includes the unemployment rate in first differences.

<table>
<thead>
<tr>
<th>TABLE X: “Exclusion” Test</th>
</tr>
</thead>
<tbody>
<tr>
<td>age ( D^h ) ( E_i ) ( T_i ) ( Y )</td>
</tr>
<tr>
<td>male workers</td>
</tr>
<tr>
<td>16 – 29</td>
</tr>
<tr>
<td>30 – 44</td>
</tr>
<tr>
<td>( \geq 45 )</td>
</tr>
<tr>
<td>female workers</td>
</tr>
<tr>
<td>16 – 29</td>
</tr>
</tbody>
</table>

Note: \( D^h_j = U^h_j \) if \( U^h_j \) is I(1), and \( D^h_j = \Delta U^h_1 \) if \( U^h_j \) is I(2). (*) shows the rejection of \( H_0 \) of exclusion of the variable at the 5% significance level.

<table>
<thead>
<tr>
<th>TABLE XI: Weak Exogeneity Test</th>
</tr>
</thead>
<tbody>
<tr>
<td>age ( D^h ) ( E_i ) ( T_i ) ( Y )</td>
</tr>
<tr>
<td>male workers</td>
</tr>
<tr>
<td>16 – 29</td>
</tr>
<tr>
<td>30 – 44</td>
</tr>
<tr>
<td>( \geq 45 )</td>
</tr>
<tr>
<td>female workers</td>
</tr>
<tr>
<td>16 – 29</td>
</tr>
</tbody>
</table>

Note: \( D^h_j = U^h_j \) if \( U^h_j \) is I(1), and \( D^h_j = \Delta U^h_1 \) if \( U^h_j \) is I(2). (*) shows the rejection of \( H_0 : \alpha_{k_j} = 0 \) at the 5% significance level.

References


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