Identity, Existence, Cardinality and Direct Reference*

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The aim of this work is to offer one analysis of relations between concepts of identity, existence and cardinality and some very general ‘takes’ on the interconnection of these concepts and direct reference theories. Even though there are several direct reference theories, let us refer to all of them by means of the term “Millian Theory” (MT). As far as this work is concerned, MT is, in a nutshell: “Sentences containing a proper name express singular propositions, namely, those propositions which contain the bearer of the name as constituent.” In Millian terms, let us assert that proper names contribute denotation to propositions expressed by sentences in which the proper name is a constituent part.

It is a well known fact that, currently, a schism has split philosophy of language into those who favor MT and those partisan to the Theory of Indirect Reference, or Fregean Theory (FT). Kripke, in his already classic work Naming and Necessity, proposes a working agenda for MT. He considers FT satisfactory in theory because it helps explain well-known puzzles which act upon existence-sentences and identity-sentences. The problem is that, as far as Kripke is concerned, FT is a false theory, independently of its recognized explanatory

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usefulness. MT must, therefore, replace FT. MT must, nonetheless, be capable of a satisfactory answer to the aforementioned puzzles.

'Singular existential sentences and identity sentences between names need some other analysis.'

Point in fact, said puzzles are natural occurrences of an MT framework. What proposition, for example, does a sentence, having as its constituent a proper name devoid of denotation, express? In two different sentences derived one from another by means of a substitution of co-denotational proper names, are their respective propositions identical? In principle, MT followers may find themselves in a bind: to our first question, prima facie at least, they would respond that ‘it expresses no proposition’ and to the second that ‘they are identical’. Both answers, however, are contrary to our intuition. This is, then, the working agenda which we referred to above. I will try to justify that singular cardinality-sentences, even when they prove, at first glance, to be problematic for MT, do not, in fact, offer any novelty. In other words, problems do not extend beyond those raised by concepts of existence and identity.

Almost paradoxically, G. Frege himself, that spokesman for FT, will guide us in this work. There are two basic reasons for this selection. On the one hand, according to authors

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1 Kripke (1980), 59.
2 Clearly, puzzles do not only act upon existence-sentences and identity-sentences. In reality, MT must confront both the no-denotation problem and the co-denotation problem.
like Mendelsohn or Perry, Frege went from espousing MT to defending FT, the latter becoming what one might describe as Frege’s official theory.

‘Frege has three different accounts to consider. The first is very much like the referentialist account. This is the account delivered by the basic semantic theory of his *Begriffsschrift*…’

FT’s point of departure is his now-classic *Über Sinn und Bedeutung* (1892) (and works from this period); whereas for MT, *Begriffsschrift* (1879) is. *Grundlagen* (1884) hovers in orbital approach around MT. A change takes place, noteworthy precisely because, while it acted upon Fregean analysis of the identity puzzle, it did nothing to affect the analysis of the existence puzzle. The resulting asymmetry merits investigation, if only because it is an interesting exercise that sheds light on the issues that occupy us in this paper. Mendelsohn, for example, categorizes it in the following terms:

‘Just as he had abandoned his early view about identity, Frege should have abandoned the correlative view about existence.’

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3 Perry (2001), 175 (Emphasis is mine). Perry’s referentialist account is our MT (see also Mendelsohn (2005)). If Perry distinguishes between three different accounts, two correspond to MT.

4 Mendelsohn (2005), 114.
'It remains something of a mystery why Frege never changed his view of existence, although we suspect that it was too deeply developed in conjunction with the definition of number for him to give it up.'

This last quote encapsulates, quite vividly at that, our second reason for selecting Frege as guide-author. Interpreting his ideas (especially in Grundlagen) has generated undoubtedly important debate over the problems that occupy us and, specifically, over existing relations between concepts of identity, existence and cardinality. Geach initiated that dialogue, and Perry, Alston-Bennett and Blanchette took up the gauntlet.

1. Cardinality, identity and direct reference

Let us begin by examining existing relations between concepts of cardinality and identity using Peter Geach’s approach.

Geach, a defender of the doctrine of relative identity (DRI), says that DRI goes hand-in-hand with the doctrine of relative cardinality (DRC). Alston and Bennett arrive more or less at the same conclusion. In any event, these authors do not consider it possible to

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5 Mendelsohn (2005), 118. Emphasis is mine.
7 Terminology borrowed, with slight adjustments, from Perry (1978).
8 Even though, Geach’s purpose, on the one hand, and Alston-Bennett’s, on the other, differ. As Blanchette concludes, in arguing that DRC implies DRI, ‘Alston and Bennett intend not to lend support to Geach, but instead to provide an argument against Frege’. Blanchette (1999), 219.
accept DRC and reject DRI. For them, Frege does this; he accepts DRC but not DRI. So, Frege’s position seems inconsistent.

Let us describe DRI and DRC by means of the following example.

(1) Dr. Jekyll is the same as Mr. Hyde

Following DRI, it makes sense to ask about identity sentence (1): “The same what?” Not only does it make sense, but different answers to the same question are possible. And depending on the answer, (1) is either true or false. That is to say, whether (1) is true or false depends on how we answer our question. In reality, (1) is an incomplete sentence. By completing it, for example, we obtain, (1') and (1''), where (1') is true and (1''), false.⁹

(1') Dr. Jekyll is the same man as Mr. Hyde

(1'') Dr. Jekyll is the same person as Mr. Hyde

In other words, what is most noteworthy about DRI is not only that (1) is incomplete and that identity sentences require a general term (or more concisely, a sortal-term) to be complete, but that the key to relativism essentially is that the identity sentence ‘a is the same F as c’ may be true, while ‘a is the same G as c’ may be false. So, using and slightly expanding terminology minted by D. Wiggins, one must distinguish sortal-dependency of identity from

⁹ Let us, for the sake of argument, suppose so.
sortal-relativity of identity. DRI is the latter. Wiggins, for example, accepts sortal-dependency of identity and rejects DRI.\textsuperscript{10}

Along those same lines, one might describe DRC in the following terms.\textsuperscript{11} If (2) is a cardinality sentence, one might ask, “What kind of two?”

\begin{enumerate}
\item[(2)] Dr. Jekyll and Mr. Hyde are two
\end{enumerate}

Like in the case of identity, the question makes sense and it may be answered in different ways. Depending on those answers, (2) is either true or false. Once more, (2) is an incomplete sentence. By completing it, we obtain, (2\textsuperscript{'}), and (2\textsuperscript{")}, where (2\textsuperscript{")} is true and (2\textsuperscript{'}) is false.

\begin{enumerate}
\item[(2\textsuperscript{'})] Dr. Jekyll and Mr. Hyde are two men
\item[(2\textsuperscript{")}] Dr. Jekyll and Mr. Hyde are two persons
\end{enumerate}

Geach’s interpretation is that Frege defends DRC and non-DRI. From Geach’s (and Alston-Bennett’s) point of view, this is incoherent, since DRC implies DRI. Could this be

\textsuperscript{10} Wiggins (1980). In general terms, I hold that Wiggins’ thesis is right (Arrieta (2001)).

\textsuperscript{11} As we will see, neither Frege nor Geach express DRC in such terms. They do not consider singular cardinality-sentences. However, for our purposes herein, we will do so.
right? My goal here is not to go into the details of this controversy, but to state my position clearly. ¹²

(T1) It is true that the doctrine of relative cardinality (DRC) implies the doctrine of relative identity (DRI). Thus, it is incoherent to defend DRC and non-DRI.

(T2) However, Frege’s doctrine of cardinality (FDC) is not DRC.

(T3) FDC does not imply DRI. Therefore, it is coherent to defend FDC and non-DRI.

Now, let us organize our discussion. First, a brief discussion in favor of these three theses. Second, following Blanchette, let us extend FDC to include singular cardinality-sentences, such as (2), which Frege may not have taken into account. Let us call it The Extended Frege’s Doctrine of Cardinality (EFDC). Finally, some critical comments concerning EFDC.

With respect to (T1), my comments on the examples above should prove convincing enough. If one accepts that (2) can be read as (2') or as (2''), it seems difficult not to accept that (1) is ambiguous or that it can be read in two ways. Clearly, DRC implies DRI.

Now, before moving onto (T2), a few words about our understandings of DRC. Even though we invoke DRC in the context of a singular sentence which makes reference to particulars, relativity of cardinality is usually expressed by referring to groups or collections,

¹² Except for some details that we will see concerning this issue, I think Blanchette (1999) has the ‘last word’.
and not merely to particulars. In any event, we might reformulate our example so as to accommodate this requirement. Suffice it to say that we can suppose a collection exists in which we ‘include’ both Jekyll and Hyde. As for this collection, DRC says that there can be more than one assignment of cardinality to this collection: one (man) or two (person).

But did Frege defend DRC? According to (T2), the answer is ‘no’.

Frege’s words concerning cardinality may prove misleading, given that he sometimes seems to favor a kind of relativism of cardinality. Here is one example:

“While looking at one and the same external phenomenon, I can say with equal truth both ‘It is a copse’ and ‘It is five trees,’ or both ‘Here are four companies’ and ‘Here are 500 men.’ ”\textsuperscript{13}

While Frege holds that different cardinality-sentences can be ascribed with equal right to one and the same external phenomenon, such as ‘four companies’ and ‘500 men,’ he goes on to say:

“If I place a pile of playing cards in (someone’s) hands with the words: Find the Number of these, this does not tell him whether I wish to know the number of cards, or of complete packs of cards, or even say of honour cards at

\textsuperscript{13} Frege (1884), 46.
skat. To have given him the pile in his hands is not yet to have given him completely the object he is to investigate; I must add some further word –cards, or packs, or points.”

Furthermore,

“…an object to which I can ascribe different numbers with equal right is not what really has a number.”

With this in mind, let us propose the following scenarios, (F1) and (F2).

(F1) It is indeed possible to ascribe relativity of cardinality to an exterior phenomenon. This is only possible if the exterior phenomenon is expressed, for example, as multitude, set or plurality.

“The terms ‘multitude’, ‘set’, and ‘plurality’ are unsuitable, swing to their vagueness, for use in defining number.”

Nevertheless, this type of ‘entity’ (set, multitude, plurality) proves inadequate for defining a number. That relativity of cardinality is possible seems symptomatic of an

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14 Frege (1884), 22. The emphasis is mine.
15 Frege (1884), 22.
16 Frege (1884), 45.
inadequate ‘entity.’ Any apparent relativity of cardinality merely implies that we have not yet determined our object or domain of investigation. This means we are immersed in vagueness.

(F2) Determining domains of investigation basically consists in providing a (Fregean) concept. At the same time, some number or some absolute cardinality is associated to said domain of investigation. Analogously, the (absolute) identity of objects that fall under the concept (or of those objects belonging to the extension of the concept) is set.

“As soon as we restore possession to the rightful owner, the concept, numbers reveal themselves as no less mutually exclusive in their own sphere than colours are in theirs.”

Therefore, once a concept is set, the cardinality associated with it and the identity of the objects that fall under this concept become absolute. Frege’s only requisite for any concept is that it not be vague, that is, it must be determined for every object whether it falls under a concept or not.

Geach’s take on this is different. Generally speaking, says Geach, once domain of investigation D is specified, we can simultaneously assign to D the cardinality m A-s and n

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17 As we shall see, Frege considers the term ‘set’ a vague expression. Clearly, the use given by Frege to the term ‘set’ does not correspond to the use for ‘set’ understood by Set Theory. When referring to sets from Set Theory, Frege employs the term ‘extension’ (of a concept).
18 Frege (1884), 48. Emphasis is mine.
B-s, when each particular belonging to this domain of investigation is both an A and a B.19

Once a domain of investigation is specified, depending on the relation of (relative) identity for that domain of investigation, we will associate to said domain one cardinality or another. Basically, the problem is that Frege and Geach understand the term 'domain' differently. Frege considers that something which is assigned different numbers ‘is not what really has a number.’ Thus we do not have the object of investigation completely. According to Frege, a concept simultaneously specifies the domain of investigation, its cardinality and the identity of its elements. Nonetheless, the relativist position does not envision such a conceptual approach to specification of the domain of investigation. It is no coincidence that Geach proposed non-standard ways to specify the domain and a non-standard theory concerning proper names. (See his distinction between names-of and names-for.) Furthermore, his proposed quantification theory is also quite non-standard.

As such, the Geach approach revolves around the following theses:

(G1) Identity sentences, in general terms, consist of four components: an identity expression, two proper names and a general term. The basic thesis is that, by not varying the first three components, variations of the fourth can generate, at the same time, variations in the truth-value of a sentence.

(G2) The general terms in (G1) are sortal terms. An equivalence relations is an essential part of a sortal. That equivalence relation determines identity.

19 Blanchette (1999), 217.
(G3) Cardinality is determined, not by a concept, but by an equivalence relation. That is to say, the equivalence relation determines cardinality.

(G4) One must distinguish between restricted and unrestricted quantification.

(G5) One must distinguish between names-of and names-for.\(^\text{20}\)

Both cardinality and identity are incomplete. They require an equivalence relation (implicit to sortal terms that complete cardinality-sentences and identity-sentences). Thus different equivalence relations determine different cardinalities and identities. (G1)-(G5) remove Geach from the Fregean approach. And, as such, from what might be called the standard approach. One must, at least, recognize that Geach, to be coherent with the demands of DRC, unveils a non-standard approach which drags DRC along with it. Against the Geach’s better judgment, this very reliable indicator proves that, with regard to cardinality, Geach and Frege travel along absolutely divergent paths.

So, our conclusion is that Fregean doctrine of cardinality (FDC) has little to do with the doctrine of relative cardinality (DRC). From FDC, any cardinality sentence is a sentence about a concept (or the extension of a concept) and only one assignment of numerical value to said concept (or its extension) is possible.

But what about singular cardinality-sentences like ‘\(a_1, \ldots, a_n\) are \(m\)’? Apparently, Frege does not worry himself about this type of sentences. In any event, he supposes that if

\(^{20}\) For a critical examination of these theses, see Arrieta (1999).
such sentences are expressed, they are merely something vacuous; they are sentences that contribute no information about particulars \( a_1, \ldots, a_n \).

‘It is not easy to imagine how language could have come to invent a word for a property which could not be of the slightest use for adding to the description of any object whatsoever.’\(^{21}\)

For this reason, Frege concludes that such sentences are not, in a real sense, about cardinality. Language does not revel in such vacuities. As a result, they must be reinterpreted.

“It is true that ‘Solon was one’ can actually occur, but not in a way to make it intelligible on its own in isolation. It may, for example, mean ‘Solon was a wise man’, if ‘wise man’ can be supplied from the context. In isolation, however, it seems that ‘one’ cannot be a predicate.’\(^{22}\)

Singular cardinality-sentences merely \textit{appear} to express cardinality, but once they are supplied (with a context), they transform into standard descriptive sentences becoming sentences which are not about cardinality at all.

\(^{21}\) Frege (1884), 29.  
\(^{22}\) Frege (1884), 29.
Notwithstanding, if the perspective described above were indeed Frege’s own, then he would clearly be wrong about singular cardinality-sentences. Cardinality-sentences whose logical form is ‘a₁, ..., aₙ are m’ appear to be completely meaningful and informative, without having to be reinterpreted as standard descriptive sentences in which a property is attributed to a particular.

By accepting this, FDC would need to account for said singular cardinality-sentences, even if Frege himself should take it amiss.

This presents a difficult challenge. It remains unclear to what concept a number m in any ‘a₁, ..., aₙ are m’ sentence is applied. FDC ‘needs’ such a concept. Via Millian Theory (MT), our chances of a successful search for a concept are reduced to the limit because proper names ‘a₁’, ..., ‘aₙ’ only contribute denotation to a proposition expressed by cardinality-sentences. From where shall we extract a concept? One descriptivist, or Fregean, theory of reference may provide a possibility.

Keeping in mind that FDC’s birthing takes place in Grundlagen and presupposing that during this period Frege still dabbled in MT, it is no wonder he opted for avoiding genuine, singular cardinality-sentences. Nonetheless, singular cardinality-sentences generate puzzles analogous to identity sentences: Does the sentence ‘Hesperus and Phosphorus are one’ share the same content as ‘Hesperus is Phosphorus’? Why was Frege interested in the latter and not the former?

In reality, we do not believe that the data permits us to formulate definitive conclusions about Frege’s actual position.

Perry (1978), 31; Alston-Bennett (1984), 560-567; Blanchette (1999), 219-222.
Blanchette extends FDC, while maintaining a certain compatibility with Frege’s ideas and ensuring that the extended doctrine covers cases such as (3).\textsuperscript{25}

\begin{equation}
(3) \ a_1, \ldots, a_n \text{ are } m
\end{equation}

Blanchette holds that the disjunctive concept *identical with* $a_1 \text{ or } \ldots \text{ or with } a_n$ fulfills all the requisites set by Frege. Hence (3) should be considered a cardinality-sentence; which means that (3) should be interpreted as a (non-singular) sentence about our concept *identical with* $a_1 \text{ or } \ldots \text{ or with } a_n$.\textsuperscript{26} Let us call this extended FDC intended for singular cardinality-sentences, transforming them into general cardinality-sentences, EFDC. In any case, EFDC is a narrow bridge between identity and cardinality.

If (3) may be interpreted as ‘there are $m$ objects falling under the concept *identical with* $a_1 \text{ or } \ldots \text{ or with } a_n$’, then ‘Hesperus and Phosphorus are one’, for example, might be interpreted as ‘there is one object falling under the concept *identical with Hesperus or with Phosphorus*’. But, from the perspective of the logical form, is there any difference between ‘Hesperus is the same as Phosphorus’ and ‘there is an object falling under the concept *identical with Hesperus or identical with Phosphorus*’? One might claim that the latter is general, while the former is singular. I am convinced that such is not the case. I believe the two sentences are singular. Now, if we say, ‘Hesperus, Phosphorus and Neptune are two,’ by

\textsuperscript{25} The names which we plug into (3) may refer to objects which have little in common. Indeed, they need not possess a referent. Thus, one could say, for example: ‘Bush, Vulcan and New York are three’

\textsuperscript{26} Blanchette (1999), 221-2.
means of EFDC, we are affirming ‘there are two objects falling under the concept identical with Hesperus or identical with Phosphorus or identical with Neptune.’ Is this a general sentence? I do not believe so. We are, instead, faced with a Boolean combination of singular identity-sentences. No quantification is required.

As such, Blanchette proposes a solution to the problem of singular cardinality-sentences compatible with FDC: singular cardinality-sentences are general cardinality-sentences. Such a solution, however, reduces singular cardinality-sentences to a Boolean combination of identity sentences.

This may justify Frege’s different treatment of singular and general cardinality-sentences and his omission by not extending FDC. Yes, general cardinality-sentences are sentences of a higher order (according to Fregean interpretation of quantifiers). Singular sentences are not (including Boolean combinations of such sentences). Singular cardinality-sentences are, in a real sense, singular.

In general, one might suppose (provide one adopts the MT framework) that there are specific puzzles for singular cardinality-sentences. Do ‘a and a are one’ and ‘a and b are one’ express different propositions? And what about, ‘a, b and c are two’ and ‘a, a and c are two’? As far as MT is concerned (and supposing that ‘a=b’ is true), we must respond ‘no’. Nonetheless our intuition points us in the opposite direction. With this last point in mind, puzzles which act upon different types of singular cardinality-sentences, however, are merely manifestations of puzzles which act upon any identity sentence which reduces them. So, the supposed new puzzle is a rehashing of MT’s same ‘old’ problem.
2. Cardinality, existence and direct reference

Frege’s ‘migration’ from MT to FT clearly takes place in Über Sinn und Bedeutung. This definitive change affects his analysis of identity-sentences. Both facts are closely linked. Such is not the case with singular existence-sentences (and nonexistence-sentences). It is difficult to determine his evolution with regard to this question, given that Frege, to a certain extent, seems to ignore it. At any rate, let us delimit what may well be a Fregean conception using any and all data available to us.

General existence-sentences are closely linked to general cardinality-sentences. In both cases something is affirmed about some concept, which is to say that they are (from a Fregean perspective) sentences of a higher order. In general nonexistence-sentences, the cardinality zero is assigned to a concept; in general existence-sentences, any cardinality, except zero, is assigned to the concept in question.

“In this respect existence is analogous to number. Affirmation of existence is in fact nothing but denial of number nought.”

27 There are those of the opinion that Frege does not change substantially in his analysis of identity. See Thau-Caplan (2001).
28 Frege (1884), 53.
While there is no mention of singular existence-sentences in *Grundlagen*, Frege, in a paper from 1884 published posthumously, offers us the following insights on singular existence-sentences:

“Neither in ‘A is identical with itself’ nor in ‘A exists’ does one learn anything new about A. Neither statement can be denied.”

We have already referred to the vacuous nature of singular cardinality-sentences: they are not informative. This non-informative feature is linked to self-identity-sentences and existence-sentences in the quote above. We simply learn nothing about object A from singular self-identity sentences, existence-sentences and cardinality-sentences. Faced with the apparent vacuity of such sentences, Frege implements different strategies for each. We have seen how singular cardinality-sentences are, in reality, as Frege calls them, camouflaged descriptive sentences (or per Blanchette, general cardinality-sentences). As for singular existence-sentences and identity-sentences, during his MT period, Frege proposes a meta-linguistic reading. In the case of identity, clearly this is so; while for existence, his defense in favor of such an interpretation is less forceful. Be it as it may, he writes:

29 Frege (1884a), 62
30 As we shall see, even though there are some explicit references to such a meta-linguistic reading for singular existence-sentences, Frege simply did not pay the same amount of attention to identity-sentences and existence-sentences. As for meta-linguistic reading of identity a sizable bibliography is available, see Arrieta (2005).
“If ‘Sache exists’ is supposed to mean ‘The word 'Sache' is not an empty sound, but designates something’…”

It is noteworthy that Frege, referring to self-identity and existence-sentences, says: ‘neither statement can be denied.’ It would not be farfetched for us, according to Frege, to say we are before two logical principles. In other words, any statement formulated as ‘A exists’ is something like a logical truth.

Further on, in the same text, Frege proposes an example, substituting the value ‘this table’ for ‘A’ (i.e., ‘this table exists’, ‘this table is identical with itself’). Thus, if ‘this table’ were a Russellian genuine proper name (or, at least, an approximation), in effect ‘this table exists’ may not be denied, and in so doing, our Fregean affirmation takes on a clearly Russellian nature. This example is a real attention-getter because, intuitively speaking, there seem to be ‘A exists’ sentences that may be denied. ‘Vulcan exists’ is just such a sentence. Our example is even more surprising because, though Russell restricted the use of the term ‘proper name,’ Frege did just the opposite by extending it to totally counter-intuitive extremes. Thus, Frege might have proposed many such (non-Russellian) examples, which would show that the supposed logical principle is problematic. However, he upholds this principle (or logical truth), in varying forms, all throughout his work.

31 Frege (1884a), 60
Frege, in the same text, clearly contrasts between singular existence-sentences and general existence-sentences, based on the non-evident, or non-vacuous, nature of general sentences. This contrast between both types of existence-sentences is a recurring theme throughout Frege's work; one must add, however, that said contrast is made manifest in various ways.

In his works dated, 1892 and 1903 respectively, (placing us somewhere within the timeframe of Sinn und Bedeutung) we find the following statements:

"The sentence ‘there is Julius Caesar’ is neither true nor false but senseless; the sentence ‘there is a man whose name is Julius Caesar’ has a sense, but here again we have a concept, as the indefinite article shows. We get the same thing in the sentence ‘there is only a Vienna’…; ‘Vienna’ is here a concept-word, like ‘metropolis’."\(^{32}\)

“Linguistically-speaking, not only is it unacceptable to say ‘there is Africa’ or ‘there is Charlemagne,’ but it does not even make sense. We can say, however, ‘there is something that is called Africa’."\(^{33}\)

Now, Frege’s view seems radical. In his MT period, he referred to the evident and vacuous nature of singular existence-sentences, but he does a meta-linguistic reading of

\(^{32}\) Frege (1892a), 50.
\(^{33}\) Frege (1903), 276.
them. Now, however, he simply denies that singular existence-sentences make sense. It is true, though, that subsequently he perhaps does attribute some sense to them, transforming them into general existence-sentences. The only fitting solution is to attribute to sentences such as ‘A exists’ the sense ‘(∃x) Px’, where ‘Px’ signifies ‘x’s name is 'A’.

Though these new facts may prove insufficient evidence to convict Frege of holding a clear conception about singular existence-sentences, the following conclusions may fit:

A. From his first period, we have some statements favoring a meta-linguistic reading of singular existence-sentences. From his second period, we have no evidence leading us down this path. The strategy implemented seems to have been to transform singular existence-sentences into general existence-sentences. This approach is something analogous to the one proposed for singular cardinality-sentences by Blanchette (in Frege’s name). According to Blanchette, sentences such as ‘Hesperus, Phosphorus and Neptune are two’ may have a Fregean interpretation meaning ‘there are two objects that fall under the concept identical with Hesperus or identical with Phosphorus or identical with Phosphorus’. One might propose an alternative form, in keeping with the Blanchette proposal, which substitutes the concept ‘identical with Hesperus or identical with Phosphorus or identical with Phosphorus’ with the concept ‘is named 'Hesperus' or is named 'Phosphorus' or is named 'Phosphorus’’. And alternately, in keeping with Blanchette, one may interpret ‘A exists’ as ‘(∃x) x=A’. In any event, we are talking about eliminating the singular nature of singular existence-sentences.
If one takes this data into account, one may refer to a definitive change in Frege concerning singular existence-sentences. However, this does not seem to be related to his change concerning identity-sentences. The issues raised over singular existence-sentences (and singular cardinality-sentences) may be resolved by eliminating such sentences altogether and transforming them into general sentences. I have already indicated that, in terms of cardinality-sentences, this may be the wrong approach.\(^{34}\) As for singular existence-sentences, simply eliminating them is also the wrong way, but for other reasons. In any case, Frege simply did not go into the details of singular cardinality-sentences and existence-sentences. One must keep in mind that by 1892, Frege the defender of FT might have dealt with the puzzle of existence in the same way that he dealt with the puzzle of identity, but nonetheless Frege seems to ignore any problematic aspects concerning existence-sentences.

B. Frege’s attitude toward existence can be explained in that way: he holds that the principle “there is no room in logical language for names with no reference” clearly has some bearing on our discussion. This principle and what he perceives as a close relation between the existential quantifier and the existence predicate lead Frege to suggest an eliminativistic approach toward singular existence-sentences. This, as far as I am concerned, is another error. Really, the close relation between general existential and cardinality

\(^{34}\) This is contrary to Blanchette. As I have already stated, there is another solution compatible with Frege’s notions: reducing singular cardinality-sentences to Boolean identity-sentences, the singular nature of the sentence is maintained.
sentences may be expressed as: they are sentences about the number of objects that fall under a given concept. Nonetheless, singular existence-sentences have an alternate sense which differentiates them from cardinality-sentences. Obviously, by accepting the principle that every name is referential, we must conclude that singular existence-sentences have no function. Issue must be taken with this principle which seems false, with respect to natural language at least. Besides, whether a name is referential or not may not be decided \textit{a priori}. For this very reason, the existence predicate makes complete sense, and singular existence-sentences are informative. We must distinguish between the existence predicate and the existential quantifier.

3. Conclusions

Frege was surprised by the vacuous nature of self-identity sentences, singular cardinality-sentences and singular existence-sentences. This vacuity translates to different aspects of these sentences, meaning: either they are obvious, or they do not offer information on the denoted object. With regard to this issue, his approach, throughout most of his works, is not uniform. Besides, it is divergent from his treatment of other singular sentences. Only in \textit{Über Sinn und Bedeutung} Frege deals with identity sentences and other singular ‘descriptive’ sentences by employing one sole semantic device. Singular existence-sentences and singular cardinality-sentences, however, have been jointly excluded.
This work has examined some reasons justifying our discrepancies with Frege’s position and those of other authors who have interpreted his work. On the one hand, singular cardinality-sentences, from our perspective, are really much closer to identity-sentences than to general cardinality-sentences and general existence-sentences. On the other hand, singular existence-sentences (notwithstanding some of Frege’s own statements) do make sense, and they are not reducible to general cardinality-sentences or general existence-sentences. The task of singular existence-sentences is closely related to the possibility that we may employ proper names that have no denotation without a priori knowledge of this fact. Frege does not consider such a possibility simply because logical language does not contemplate proper names devoid of denotation. For logical language, therefore, either singular existence-sentences have to be ignored, or they merely sustain some alternate logical structure than the one they seem to exhibit. However, it seems that Frege preferred to ignore them.

Finally, singular cardinality-sentences do not heap new problems unto MT’s to-do list, refuting Alston-Bennett; nor do they proffer definitive arguments against MT. Its grocery list of problems simply remains unchanged.

References

35 What truth value sentences such as ‘Vulcan is Vulcan’ and ‘Vulcan and Hesperus are two’ have is controversial. Distinct alternatives are possible within the framework of distinguishing between the existential property and the existential quantifier. McGinn (2000) considers the former a true sentence. Sainsbury (2005) believes the same sentence is false. Both authors agree that language permits proper names devoid of denotation. The same can be said of the latter sentence, in which case it may be supposed that those same authors would respond in the same way. As far as McGinn is concerned, Vulcan clearly satisfies ‘= Vulcan’, even though ‘Vulcan does not exist’ is truth. Self-identity is therefore a more universal property than existence. Sainsbury’s take on this is different. Whichever the case, this paper will not delve into these very interesting divergences.

36 The same may be said about other types of singular terms.


