Licensing Information Goods

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Abstract

A significant amount of research on patent licensing and the diffusion of knowledge is organized around static frameworks of analysis. Patent holders, however, may face a dynamic problem, namely the intertemporal consistency problem of the durable-goods monopolist that is induced by durability on the demand side. Licensing practices such as exclusive licensing contracts and most favored customer clauses allow patentees to solve or mitigate this dynamic consistency problem. There are situations, however, where these practices are not possible either due to the nature of the patent (the case of information goods) or due to compulsory patenting laws. We study the effects of the intertemporal consistency problem on patent licensing in these situations. Relative to the existing literature, we obtain the following main results: (i) All of the firms that remain in the industry will be using the innovation; (ii) Royalty licensing may be superior to fixed-fee or auction licensing from the licensor’s point of view; (iii) Social welfare and consumer surplus may be lower than when the patent holder can commit not to make additional sales; (iv) Even for non-drastic innovations, the price of the good that is produced may be lower than the competitive price corresponding to the initial situation (before the innovation was discovered).

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1 Introduction

As documented by the Survey of Current Business (1994, 2006), OECD (2004) and other sources, over the last few decades patent licensing has become an increasingly important aspect of the dynamics of innovation. One source of profit for an inventor is through his own working of the patent. The other, of course, is through licensing of the patent. In this case, patent holders may face the intertemporal consistency problem of the durable-goods monopolist that is induced by durability on the demand side. This is a problem that may be solved or mitigated by licensing practices such as exclusive licensing agreements and most favored customer (MFC) clauses. There are situations, however, where these practices are not possible (or profitable) either due to the nature of the patent (the case of information goods) or due to compulsory patenting laws.\footnote{There is legislation that does prevent practices such as exclusive contracts and MFC clauses in patent licensing. For example, in Europe, the Paris Convention for the Protection of Industrial Property (1883), Article 5A.(2), allows governments to force compulsory licensing to prevent various possible abuses from the exercise of the exclusive rights conferred by the patent. In the US, the Clean Air Act (2004), Section 308, provides mandatory licensing of air pollution prevention inventions. The US Public Health and Welfare Code allows compulsory licensing of patents that have been found to affect the public interest, and patents whose unavailability might tend to substantially reduce competition or tend to create a monopoly. I thank the co-editor for leading me to these examples.} We study the effects of the intertemporal consistency problem in patent licensing by comparing two cases: (1) First, there is patent protection and practices such as exclusive licensing agreements and MFC clauses are possible, and (2) such practices are not possible. Hereafter we will refer to these cases as situations in which the patent holder does and does not have commitment ability respectively.

Early work on licensing of cost-reducing innovations can be traced to Arrow (1962), who focused on the question of whether it is more profitable to innovate in a competitive or a monopolistic industry. His analysis was extended by Kamien and Schwartz (1982) to licensing in an oligopolistic industry. Analyses of licensing strategies of a process innovation taking into account the inventor's ability to exploit the interdependence and competition among the potential licensees to his advantage were later introduced independently by Kamien and Tauman (1986) and Katz and Shapiro (1986). A significant amount of research in the area has been organized around their framework of analysis, a setting where a number of important insights into the economics of patent licensing are obtained.\footnote{Kamien and Tauman’s (1986) analysis is limited to the case of a linear demand function for the}
In this paper we note that patent holders may face a dynamic problem, namely the classic intertemporal consistency problem of the durable-goods monopolist that is induced by durability on the demand side. In order to study the implications of this problem the analysis proceeds as follows. Market power on the patentee’s side is represented, under perfect and complete information, by a monopolist (patent holder) that has no financial interest in the potential adopters (firms of the industry).\(^3\) As for general licensing mechanisms, we consider the three one-stage mechanisms most often observed in practice and examined in the literature: auction, fixed-fee and royalty.\(^4\) For each mechanism we initially characterize the basic features of the diffusion process without modeling the underlying competitive interaction among potential licensees. We then explicitly model their competitive interactions and study the impact of the monopolist’s inability to commit to a future schedule of sales (e.g., cases where practices as exclusive licensing agreements or MFC clauses are not possible) has on licensing strategies, monopolist’s profits, market structure, market prices, consumer surplus and social welfare. This part of the analysis follows the classic framework in Kamien and Tauman (1986) by considering a cost reducing innovation in a Cournot model in which firms have constant marginal costs and face a linear demand. Finally, this basic framework is extended to an intertemporal setting in order to study the consequences of the ability or inability of the monopolist to commit to a future schedule of license sales.

\(^3\)When the patentee is part of the industry or has financial interest in the potential adopters of the innovation, he may still have an interest in licensing the patent. In section 4 we will briefly discuss this situation. The issue of the strategic use of licenses or intentional sharing in the market for technology information has been analyzed, for instance, by Gallini (1984), Gallini and Winter (1985), Katz and Shapiro (1985), Rockett (1990), Fershtman and Kamien (1992) and Eswaran (1994).

\(^4\)Empirically, the most common methods of patent licensing are a fixed fee that is independent of the quantity produced with the patented technology, a royalty per-unit of output produced with the patented technology, and a combination of a fixed fee plus a royalty (see, for instance, Taylor and Silberston (1973), Caves, Crokel and Killing (1983), Rostoker (1983) and Macho-Stadler, Martinez-Giralt and Perez-Castrillo (1996)). More recently, Edwards, Murray and Yu (2003) conclude that licenses from universities to biotechnology companies include up-front licensing fees, annual maintenance fees, milestone payments and royalties. Vishwasrao (2007) studies factors which determine that licensing contracts include up-front fees, royalties or a combination of both.
To put the main findings of the analysis into perspective, it is worthwhile to first briefly review some of the existing results in this literature, all of which are obtained in the static framework in Kamien and Tauman (1986) or Kamien, Oren and Tauman (1992) (see Kamien, 1992, pp. 334-335): (a) In general, in a Cournot model with an external patentee, auctioning licenses, that is offering a fixed number of licenses to the highest bidders, yields the patentee a higher profit than offering licenses at a fixed fee or royalty rate to any firm wishing to purchase one; (b) The patentee never licenses more firms than the number for which the Cournot equilibrium price equals the perfectly competitive price with the original inferior technology; (c) If the innovation is non-drastic and the inverse demand function for the good produced by the industry is linear then the number of licensees is at least equal to one half the number of potential licensees, and as a consequence both licensees and non-licensees may remain active in the industry. These results coincide, in the dynamic framework of analysis that will be considered in this paper, with those corresponding to the case in which the patent holder has commitment ability. In other words, they are obtained for the case where there is patent protection and practices such as exclusive licensing agreements and MFC clauses are possible. The impact or consequences of these agreements and clauses, therefore, can be determined by comparing the existing results to those corresponding to the case in which the patent holder does not have commitment ability.

Relative to the existing results, the main contributions that arise in a dynamic setting in which regulations do not allow practices that provide firms with commitment ability can be summarized as follows:

(i) Regardless of the patent licensing mechanism, for linear inverse demand functions, only firms that possess the license (i.e., firms that adopt the innovation by buying a license) will remain in the industry.

(ii) The patentee may prefer the royalty mechanism to the auction and fixed-fee mechanisms.

(iii) The price of the good produced by the firms in the industry may be below the competitive price corresponding to the initial situation, before the innovation was discovered, even for non-drastic innovations.

5The analysis may also be relevant in other settings. For instance, in order to protect their intellectual property, firms are sometimes observed to rely on non-patent mechanisms like secrecy. Unlike patent laws, trade secrets laws do not protect against discovery by reverse engineering. As a result, the owner of the secret might not be interested in exclusive licensing contracts given that these would provide incentives to non-licensees to develop their own R&D, which in turn decreases the value of the license to each adopter.
(iv) In general, social welfare and consumer surplus are greater when the monopo-
lisit can commit than when he cannot. The exception may only occur if the innovation
is large enough to induce firms that do not possess the license to stop producing.

(v) For a wide range of innovations, which include non-large innovations, the
adoption process coincides with the one corresponding to a shorter term of the patent.
So, consumer surplus and social welfare are identical in both cases although the
patentee prefers the longest possible term of the patent.

Finally, it is important to note that, in general, the problem of patent licensing
and the diffusion of innovations and new knowledge has all the characteristics of
the durable good monopolist problem (Coase 1972), plus the additional aspects that
are associated with the specific nature and extent of the competition among the
potential licensees, that is from the fact that the demands for the licenses are generally
interdependent as the value of the license to each potential adopter generally depends
upon the number of other firms that also buy the license. This is the general problem
that this paper addresses where for the case of patent licensing we study the impact
of licensing practices that solve or mitigate the durable good monopolist problem
identified by Coase (1972).

With respect to the literature on durable goods monopolies, it shows how the
power held by a monopolist in the production and sale of a durable good can be
substantial, but is notably less than the power held by a monopolist who produces a
non-durable good. Pioneering work on durable-goods monopolists was done by Coase
(1972), who conjectured that a monopoly seller of an infinitely durable good without
some commitment or restraints to limit future production would saturate the market
with the competitive output “in the twinkling of an eye” (p.143). The extensive
literature generated from this insight has often studied the extent to which Coase’s
conjecture holds in different scenarios as well as the means and business practices
at the disposal of the monopolist to reduce the commitment problem. It has also
analyzed the implications of the ability or inability of commitment to a future schedule
of production on sales and social welfare. Kahn (1986), for example, shows that social
welfare is greater if the monopolist cannot commit to a future schedule of production;
Dudine, Hendel and Lizzeri (2006) find that the opposite may be true when demand
changes over time and the good is storable. Viewed from this perspective, this paper
suggests a new context, namely the sale of licenses, in which social welfare may be

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6See, for instance, Bulow (1986), Butz (1990), Gul, Sonnenschein and Wilson (1986), Horner and

greater when the monopolist can commit than when he cannot (see result v).

In terms of practices that reduce or mitigate the consistency problem, MFCs and exclusive licensing agreements are often present in licensing contracts. This suggests that patent holders seek to solve or mitigate the intertemporal consistency aspect of patent licensing that is the focus of the analysis in this paper, and hence that this problem is empirically relevant.

The rest of the paper is organized as follows. Section 2 presents a general intertemporal model of patent licensing. Section 3 considers the three licensing mechanisms most often studied in the literature: Section 3.1 studies the auction licensing mechanism and provides an explicit model of interaction among licensees in which in equilibrium, always, only licensees will remain in the industry; Section 3.2 is devoted to the analysis of the fixed-fee mechanism, and also presents a comparison with the results obtained under the auction mechanism, and Section 3.3 analyzes the per-unit royalty mechanism. Section 4 discusses potential applications and extensions of the analysis. Section 5 concludes with some final remarks.

2 A Dynamic Model of Patent Licensing

The model is essentially an intertemporal extension of the ones presented in static contexts by Katz and Shapiro (1986), Kamien and Tauman (1986), and Kamien, Oren and Tauman (1992). Consider, as in those papers, an oligopolistic industry with \( N \) identical firms that produce a homogeneous non-durable good. Entry into the industry is assumed to be unprofitable, i.e., the cost of entry exceeds the profits an entrant could realize. There is also an upstream monopolist selling a durable input that can be used by producers in the downstream industry. The input can be thought of, for instance, as a license to use an innovation or the right to use an industry standard. We shall emphasize in what follows the innovation interpretation. The terms ‘input’ and ‘license’ will be used indistinguishably. The monopolist maximizes his profits by selling licenses that allow the use of the innovation; that is, he does not use the innovation to compete with the firms in the industry and has no financial interest in them. The oligopolistic firms are engaged in competition, and each firm maximizes its profits from production minus the cost of the license.

As in the papers cited above, a key assumption underlying our analysis is that each downstream firm has use for at most one unit of the input. This restriction limits the applicability of the analysis to markets in which buyers have zero-one demands, such as in the case of new ideas, knowledge or information goods. All of these are
goods for which their physical possession is sufficient for a firm to invent around it costlessly and have the property of irreversibility in trading the good. As a result the monopolist is not interested in leasing these goods for a relative short period of time.\textsuperscript{8} As Muto (1986) indicates, a new process for producing chemical products or a new computer program are examples of information goods.\textsuperscript{9}

In order to study the consequences derived from the ability or inability of the monopolist to commit to a future schedule of license sales, the model must be adapted to an intertemporal framework. This is done with the help of the following assumptions:

A1 - There are two discrete periods of time $t = 1, 2$.
A2 - The innovation can be used, without depreciation, during both periods.
A3 - Markets are complete and the discount factor is $v = \frac{1}{1+r}$, where $r$ is the interest rate.
A4 - All agents have perfect and complete information, i.e.: (a) the potential buyers of the licenses have complete information about the existence and characteristics of the innovation, and (b) the monopolist patentee knows the costs of production and the demand faced by the firms in the industry.
A5 - The potential buyers of the innovation have perfect foresight.
A6 - The demand function for the good produced by the downstream industry is the same in each period.

These assumptions are equivalent to those considered in the durable-goods model of Bulow (1982).

We allow each firm to buy at most one unit of the input in order to prevent anticompetitive hoarding, which may be individually profitable. As assumed in Katz and Shapiro (1986), we do not consider “sleeping licenses,” i.e., a single downstream firm buying multiple licenses for a given innovation because “they would be blatantly anticompetitive” (p. 572). In our dynamic model this assumption implies that in the second period only non-licensees may buy a license. Therefore, the profits of any given firm in the industry (licensees and non-licensees) depend in each period only upon the number of licensees in the industry. As for general licensing mechanisms, we consider three one-stage mechanisms: auction, fixed-fee and per-unit royalty.\textsuperscript{10}

\textsuperscript{8}This is a business practice at the disposal of a monopolist who produces a durable good to solve or mitigate his commitment problem. Coase (1972) considers two additional practices, namely that the monopolist can establish quantity commitments and that he can agree to buy back the good if the price decreases. These practices are similar to those allowed by patent laws such as, for instance, exclusive licensing contracts or MFC clauses.

\textsuperscript{9}For a detailed discussion on information goods see Muto (1986) and other referencees therein.

\textsuperscript{10}An insightful licensing mechanism is analyzed in Jehiel, Moldovanu and Stachetti (1996). This
analysis is modeled as a non-cooperative game that consists of three stages in each of the two periods $t = 1, 2$. In the first stage the monopolist either makes available $K$ licenses subject to the buyers paying at least a minimum bid for each, sets a fixed fee for the licenses, or sets a royalty per-unit of output produced with the license. In the second stage all the firms in the industry are informed of the number of licenses that will be auctioned, the fixed fee or the royalty. At that point they simultaneously and independently decide either how much to bid for a license, or whether or not to buy the innovation at the announced price. In the auction mechanism, licenses are sold to the highest bidders at their bid price, and in the event of a tie licensees are chosen arbitrarily. In the third stage, once the number of licensees is known, all firms, with and without licenses, engage in a competition game and maximize profits.

The solution concept is that of a subgame perfect Nash equilibrium in pure strategies. Therefore, the solution is derived by backward induction from the third stage. The profits of each firm will depend on the number of licenses sold, the type of licensing mechanism, whether or not the firm is a licensee, and if so in which period. The differences in profits between licensees and non-licensees is what generates the demand for licenses. Given this demand, the monopolist will then choose the amount of licenses to be sold, or the price, that maximizes his profits. Furthermore, note also that if the monopolist patentee cannot commit to a future schedule of license sales, the durable-goods monopolist problem of Coase (1972) is present. Each period, the monopolist maximizes the present discounted value of profits starting from that period. Therefore, the monopolist has to find a sequence of sales (or prices) such that: (i) his behavior is optimal given the expectations of the firms in the industry, and (ii) the firms’ expectations are rational given the behavior of the monopolist. In order to calculate such a schedule, the maximization problem has to be resolved recursively by backward induction: first determine the trivial optimal strategy for period $t = 2$ given any strategy in period $t = 1$, and then calculate the best strategy in period one.

The following notation will be used in the formalization of the model:

$K_t$: Number of firms that own a license (licensees or adopters) in period $t$.

$W_h(K_t)$: Gross profits in period $t$ (that is, without subtracting the cost of a li-

mechanism exploits the fact that potential licensees may be asymmetric. In particular, they analyze the optimal licensing mechanism when there are asymmetries among the potential licensees and there is only one unit that may be sold or licensed to a single agent. As a result, the willingness to pay for the license and the profits of non-licensees depend on the precise identity of the licensee. In our framework licensees cannot buy a second license and, hence, all the potential buyers are identical every period. In consequence, their mechanism coincides with the auction mechanism that will be examined in the next section.
cense) of a firm that becomes a licensee in period $h$.\footnote{Notice that even though all the firms are initially identical it may be that $W_1(K_2) \neq W_2(K_2)$ due, for instance, to learning by doing. For simplicity, and without loss of generality, we will consider $W_1(K_2) = W_2(K_2)$. None of the main results in this paper depends on this assumption.} $L_t(K_t)$: Profits in period $t$ of a non-licensee. $P_t$: Amount paid for a license sold in period $t$. $p_t$: Amount paid for a license rented in period $t$.

3 Patent Licensing Mechanisms

In this section we proceed to the resolution of the intertemporal model of patent licensing, first under the auction mechanism (Section 3.1), then under the fixed fee mechanism (Section 3.2), and lastly under the royalty mechanism (Section 3.3).

3.1 Auction Licensing

The optimal sale schedule takes into account the subsequent equilibrium behavior of the potential buyers, that is, the patentee computes his equilibrium intertemporal demand function for licenses and chooses his preferred point on it. The solution is thus derived by working backwards from the third stage:

(a) THIRD STAGE

Firms that own a license have an advantage that allows them, in equilibrium, to behave in a different way than firms that do not own a license. As a result, their gross profits are different. This difference in profits depends on the number of licensees and is what drives the demand for licenses.\footnote{The number of firms that buy the license each period is a natural number. Without loss of generality, we treat it as a continuous variable in this section in order to determine certain relevant properties of the intertemporally consistent schedule of sales that is optimal from the point of view of the monopolist. The term ‘optimal’ will refer to his viewpoint from now on.} Note that the sale of the input may, on occasion, originate a “natural oligopoly.” That is, the input may be such that non-licensees will be unable to remain active in the market. Let $x$ be the minimum number of licensees necessary to induce a natural oligopoly. Then $L_t(K_t) = 0$ for all $K_t$ such that $N > K_t \geq x$.

The following assumptions about payoffs of the firms are quite general in that they encompass a wide range of downstream oligopoly behavior:

B1. The gross profits earned each period by a licensee are greater than those earned by a non-licensee, that is $W_t(K_t) > L_t(K_t)$. 

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The following assumptions about payoffs of the firms are quite general in that they encompass a wide range of downstream oligopoly behavior:

B1. The gross profits earned each period by a licensee are greater than those earned by a non-licensee, that is $W_t(K_t) > L_t(K_t)$.
B2. The per period profit of an active non-licensee decreases with the number of licensees: $L'_t(K_t) < 0$, for all $K_t < x$.

B3. If the gross profits earned each period by a licensee decrease with the number of licensees, then the total gross profits of a “natural oligopoly” decrease with the size of the oligopoly; that is, if $W'_t(K_t) < 0$ then $K_t \cdot W_t(K_t)$ is strictly decreasing in $K_t$ for all $K_t \geq x$.

These assumptions are those considered in the static analysis of Katz and Shapiro (1986).

(b) SECOND STAGE

In the second stage, firms decide independently and simultaneously how much to bid for a license. At this time, each firm takes as given the corresponding bids of the other firms knowing that if its bid is not a winner then another firm will buy the license. Given that licenses are durable goods that do not depreciate over time, firms that acquire the license in the first period can continue using it in the second period without losing performance. In this intertemporal framework future sales have an effect on the licenses’ value to the first licensees, and the number of licensees may in fact increase over time. This aspect is taken into account by potential licensees in determining their willingness to pay for a license. It implies the following:

(i) The difference between the profits of a firm that owns a license only in the second period and its profits without the license determines the maximum amount that a firm is willing to pay for a license in that period. Therefore, since for $K_2 < N$ each firm knows that if its bid does not win another firm will buy the license, the maximum amount that a firm is willing to pay for a license in the second period is:

$$W_2(K_2) - L_2(K_2) \quad \text{if } K_2 < N,$$

$$W_2(N) - L_2(N - 1) \quad \text{if } K_2 = N.$$  

These expressions represent the inverse demand function for licenses in period $t = 2$ for all $K_2 > K_1$. They also represent the inverse rental demand that the monopolist would face if he rented the input by means of the auction mechanism (or equivalently, the demand faced by the monopolist in a static context). In equilibrium, because of the initial homogeneity of the firms in the industry, all the bids must be equal.

(ii) The maximum additional amount that a firm would pay to acquire a license in the first period rather than in the second period is equal to the difference between the profits from using the innovation in both periods and the profits from using it in
the second period only, given the behavior of all other firms. The discounted gross profits of a firm that buys a license in the first period (without substracting the price paid by the license) are $W_1(K_1) + vW_2(K_2)$. When $K_1 < N$ each firm knows that if its bid does not win another firm will buy the license. Therefore, his discounted gross profits by buying the license in the second period would be $L_1(K_1) + vW_2(K_2)$. Hence, we have that the maximum amount that a firm is willing to pay for a license in the first period is:

$$(1 + v)[W_1(N) - L_1(N - 1)]$$ if $K_1 = K_2 = N,$

$W_1(K_1) - L_1(K_1) + v[W_2(N) - L_2(N - 1)]$ if $K_1 < K_2 = N,$

$W_1(K_1) - L_1(K_1) + v[W_2(K_2) - L_2(K_2)]$ if $K_2 < N.$

These expressions represent the inverse demand function for licenses in period $t = 1$.

\(c\) FIRST STAGE

In the first stage the objective of the monopolist is to choose the number of licenses to be auctioned and the minimum bid to be accepted that maximize the present value of total profits.\(^{13}\) As discussed earlier, in each period the monopolist who cannot commit maximizes the present discounted value of his profits starting from that period. Therefore, in order to calculate the optimal intertemporally consistent sequence of sales, the problem has to be solved by backward induction. Given that we allow each firm to buy at most one license, in period 2 the monopolist can sell licenses only to those firms that did not buy them in the first period. As a result, the optimal intertemporally consistent license sale schedule $\{K_1^*, K_2^*\}$ is calculated by solving sequentially the following two problems:

Given $K_1$, find the $K_2(K_1) \leq N$ that solves:

$$\max P_2 \cdot (K_2 - K_1) \quad (1)$$

subject to:

$$P_2 = W_2(N) - L_2(N - 1) \quad \text{if } K_2 = N;$$

$$P_2 = W_2(K_2) - L_2(K_2) \quad \text{if } K_2 < N;$$

\(^{13}\)If the monopolist sells fewer than $N$ licenses then, because of the nature of the competition among the firms, it is not necessary to establish a minimum bid. However, if he auctions $N$ licenses, since each firm knows that it will not be replaced by another one if it chooses to lower its bid, it is necessary to establish a minimum bid. Otherwise the bid will be arbitrarily close to zero. The highest minimum bid that will induce all $N$ firms to purchase a license in $t = 1$ is $(1 + v)[W_1(N) - L_1(N - 1)]$, and to purchase it either in period $t = 1$ or $t = 2$ is $W_2(N) - L_2(N - 1)$. 11
and at \( t = 1 \) find the \( K_1 \) that solves:

\[
\max P_1 K_1 + vP_2 [K_2(K_1) - K_1]
\]  

(2)

subject to:

\[
P_1 = (1 + v)[W_1(N) - L_1(N - 1)] \quad \text{if } K_1 = N
\]
\[
P_1 = W_1(K_1) - L_1(K_1) + vP_2 \quad \text{if } K_1 < N.
\]

From problem (1) we obtain that \( P_2 \) is equal to the rental price the monopolist would charge in the second period if he rented \( K_2 \) licenses in that period (that is, \( P_2 = p_2 \)). Moreover, from problem (2), we get that the price paid in the first period for the input, \( P_1 \), is equal to the present discounted value of the rental prices the monopolist would charge if he rented each period the amount \( K_t, t = 1, 2 \), by means of an auction. That is, \( P_1 = p_1 + v p_2 \) with \( p_t = W_t(K_t) - L_t(K_t) \) if \( K_t < N \) and \( p_t = W_t(N) - L_t(N - 1) \) if \( K_t = N \). As a result, the present discounted value of the total profits of the monopolist is equal to the one he would obtain if he rented \( K_t^* \) licenses in periods \( t = 1 \) and \( t = 2 \):

\[
P_1(K_1^*) \cdot K_1^* + v P_2(K_2^*) \cdot (K_2^* - K_1^*) = p_1(K_1^*) \cdot K_1^* + v p_2(K_2^*) \cdot K_2^*.
\]

At this point it is important to note that the demand faced by the monopolist may not be decreasing. It depends on the nature of the downstream industry. Given that \( P_2 = p_2 \) for all \( K_2 \), we may conclude that if the implicit rental demand were decreasing (increasing) so would be the demand faced by the monopolist in the second period. We may thus distinguish two basic cases:

(i) First, the implicit inverse rental demand for all \( K_t < N - 1 \), that is \( W_t(K_t) - L_t(K_t) \) may be increasing. Note that \( p_t(N) = W_t(N) - L_t(N - 1) \). This is likely to be the case when \( W_t(K_t) - L_t(K_t - 1) \), that is the value of renting a license given that \( K_t - 1 \) firms are doing so, increases with \( K_t \). For example, it is increasing when the per-period gross profit of every licensee, \( W_t(K_t) \), increases with the number of licensees due to network effects (in other words, when there are positive externalities among licensees).\(^{14}\) When the inverse rental demand is increasing for all \( K_t < N - 1 \) it is not difficult to show that if the monopolist does not have commitment ability then the number of licenses auctioned, \( K_2^* \), will be at least as high as when the monopolist

\(^{14}\)See Katz and Shapiro (1986) for a discussion of the circumstances under which the implicit rental demand is increasing for all \( K_t < N - 1 \).
can commit to a future schedule of sales.\textsuperscript{15}

(ii) Second, the implicit rental demand may be decreasing. Many oligopoly models in the literature have the property that the value of renting a license given that \( K_t - 1 \) firms are doing so decreases with \( K_t \). For instance, this is the case in the classic homogeneous good Cournot model with linear demand in which the innovation reduces the constant marginal costs of production of the firms in the industry. In fact, it turns out that most of the relevant effects of the dynamic consistency problem for the patent licensing literature will take place when there are negative externalities among licensees, something which has as a likely consequence that the implicit rental demand will be decreasing. We next analyze the main features of this case in more detail.

### 3.1.1 Decreasing Implicit Rental Demand for the Licenses

The first order condition for the problem in (1) implies that every consistent schedule of sales such that \( K_2 > K_1 \) and \( K_2 \neq \{x, N\} \) satisfies:

\[
p_2(K_2) + p'_2(K_2) (K_2 - K_1) = 0. \tag{3}
\]

Differentiating (3) and taking into account the second order condition that \( 2p_2'(K_2) + p_2''(K_2) (K_2 - K_1) < 0 \),\textsuperscript{16} we obtain that these intertemporally consistent schedules of sales are such that:

\[
\frac{dK_2(K_1)}{dK_1} = \frac{p_2(K_2)}{2p_2'(K_2) + p_2''(K_2) (K_2 - K_1)} > 0. \tag{4}
\]

Therefore, since \( P_1(K_1) = p_1(K_1) + v p_2(K_2(K_1)) \) and \( P_2(K_2) = p_2(K_2) \), we can then conclude that if the implicit rental demand is decreasing, \( p'_1(K_t) < 0 \), then the monopolist who does not have any commitment ability will face a decreasing demand in both periods.

\textsuperscript{15}From problem (1), it is clear that \( K_2(K_1) \geq N - 1 \) for all \( K_1 \geq 0 \) with \( K_2(N - 1) = N \) given that \( p'_2(K_2) > 0 \) for all \( K_t < N - 1 \). As a result, the optimal schedule of sales is such that \( K_2^* = N - 1 \) or \( K_2^* = N \). Moreover, if the optimal schedule of sales is such that \( K_2^* = N \) then the number of licenses sold in the first period will be either \( K_1 = N - 1 \) or \( K_1 = N \). However, if \( K_2^* = N - 1 \), the number of licenses sold in the first period will be \( K_1 = \bar{K} \), where \( \bar{K} \) satisfies \((N - 1 - \bar{K}) \cdot p_2(N - 1) = (N - \bar{K}) \cdot p_2(N)\). Note that if the monopolist had commitment ability, or in a static context, the number of licenses sold would also be \( N - 1 \) or \( N \). More precisely, it would be \( N \) if and only if \( N \cdot p_1(N) \geq (N - 1) \cdot p_1(N - 1) \). Therefore, as \( \bar{K} \cdot p_1(\bar{K}) < (N - 1) \cdot p_1(N - 1) \), \( K_2^* \) will be at least as high as when the monopolist can commit to a future schedule of sales.

\textsuperscript{16}Without loss of generality we assume that it is satisfied with inequality.
Notice that in this case the willingness to pay for the licenses in the first period decreases with the expected total number of licenses to be auctioned. The reason is that, after the first licenses have been sold, future sales will reduce their value to the first period licensees.

We next discuss two specific cases that may arise when the implicit rental demand for the licenses is decreasing:

1. **Natural Oligopoly.** As mentioned earlier, the sale of the input may, on occasion, induce a natural oligopoly. That is, some of the existing firms in the industry (those without a license) may stop producing and even exit the industry. More precisely, if the number of licenses sold is such that \( N > K_t \geq x, \ t = 1, 2 \), then in period \( t \) only licensees will remain active in the industry. Notice that, if possible, inducing a decrease in the number of active firms in the first period may act as a commitment device for the monopolist not to sell additional units in the future. That will be the case if, for example, it is not possible to become an active firm when one has not produced in the first period, perhaps because it is too costly. As a result, in this framework the schedules of sales such that \( K_1 = K_2 \) with \( N > K_t \geq x, \ t = 1, 2 \), are intertemporally consistent. Notice that the existence of this commitment device depends on the size of the industry (in particular on \( x \) being below \( N \)).

2. **Potential Saturation of the Market.** Consider that the demands of the potential licensees for the input are interdependent and \( p_t(K_t = \min\{x, N\}) > 0 \). In this case, the optimal consistent schedule of sales might imply a saturation of the market. In other words, it might be the case that all firms that remain active in the industry will use the input (i.e., \( N \geq K_2 \geq \min\{x, N\} \)).\(^{17}\) In subsection 3.1.2 we will study a framework in which in equilibrium saturation always occurs, as well as the additional implications of intertemporal consistency for licensing intangible property by means of an auction in that framework.

After discussing these two cases, we get back to the general analysis and study in detail some features of the optimal consistent schedule of license sales.

In general, the optimal consistent schedule of sales of the monopolist who cannot commit will differ from that of the monopolist who can commit. Since the marginal

\(^{17}\) Notice that if \( p_t(K_t = \min\{x, N\}) = 0 \), then the maximization problem will imply that \( K_2^* \) is below \( \min\{x, N\} \). Otherwise, given \( K_1 \) the monopolist’s profits in the second period will be zero (the same profits he would obtain if he did not sell any licenses), although total profits will be greater by not selling in that period because \( P_1 \) decreases with \( K_2 \).
cost of licensing is zero, if the monopolist has the ability to commit to a future schedule of license sales, licenses will be auctioned only in the first period and the number of licenses sold will be that of the static case. Denote as $K^m$ the number of licenses auctioned in a context in which the monopolist can commit to a future schedule of license sales. That is, $K^m$ is such that $p_t(K^m) \cdot K^m \geq p_t(K) \cdot K$ for all $K \neq K^m$ and $K \leq N$.\textsuperscript{18} Then we may establish the following proposition:

**PROPOSITION 1.** If the monopolist cannot commit to a future schedule of license sales then:

(i) A necessary condition for the monopolist to saturate the market is that $K_2(K^m) \geq \min \{x, N\}$.

(ii) If the monopolist does not saturate the market then, even though in the first period he will not sell more licenses than if he had commitment ability, the total number of licenses sold will be higher than if he had such ability.

**Proof:** See Appendix.

We now may establish the following corollaries:

**COROLLARY 1.** The total number of licenses sold when the monopolist cannot commit to a future schedule of sales will be greater than (or equal to) the total number sold when he can commit.\textsuperscript{19}

**COROLLARY 2.** If the market is saturated and the input is not important enough to induce a decrease in the size of the industry (i.e., if $K_2^* = N \leq x$), then the optimal schedule of sales for the monopolist is $K_1^* = K^m$, $K_2^* = N$.\textsuperscript{20}

It is important to note that Proposition 1 applies to other contexts. In particular, the results in this proposition are valid even in frameworks in which the value of the good to each potential buyer is independent of the number of buyers, that is even when the demands for the good are independent. Such framework may be more

\textsuperscript{18}Without loss of generality, throughout the analysis we assume that when the patentee is indifferent between two equilibria then $K^m$ is the number of licenses auctioned in the equilibrium associated with the greater social welfare. Obviously, if $K^m = N$ then there is no commitment problem since the monopolist will always auction $N$ licenses in the first period. Note also that assumption B3 implies $K^m \leq x$.

\textsuperscript{19}This corollary follows from Proposition 1, part (ii), and from assumption B3, which implies that $K^m \leq x$, since saturation means that $N \geq K_2^* \geq \min \{x, N\}$.

\textsuperscript{20}This corollary follows from Proposition 1, part (i), since $p_1(K^m) \cdot K^m \geq p_1(K) \cdot K$ for all $K \neq K^m$. 

15
appropriate in the case in which the potential buyers are consumers, e.g., when the
durable good is a product innovation as in the case of a new electrical appliance.\textsuperscript{21}

We have so far examined the main features of both the intertemporal framework of
analysis and the optimal intertemporal consistent schedule of sales. This was possible
without explicitly modeling the underlying competitive interaction among potential
licensees. However, in order to be able to study the effects of intertemporal consist-
tency on market structure, the market price, consumer surplus and social welfare, it
is necessary to model their interaction explicitly. We analyze next the specific frame-
work most often referred to in the literature on patent licensing, one in which because
of the commitment problem there will always be saturation in the market.

3.1.2 Cournot Oligopoly and Linear Demand

Consider the inverse demand function for the good produced by the industry:

\[ P = a - bQ \quad \text{with} \quad Q = \sum_{i=1}^{N} q_i, \]

where \( q_i \) represents the quantity produced by firm \( i = 1, \ldots, N \). The marginal cost of
production of each firm is constant and equal to \( c \), with \( a > c > 0 \). There are no fixed
costs of production. Assume that the monopolist owns a patent on an innovation and
sells licenses to the downstream firms in the oligopolistic industry. The innovation
reduces their marginal cost of production from \( c \) to \( c' = c - \varepsilon \), with \( \varepsilon > 0 \), and the
marginal cost of selling licenses is zero. This linear case is the one analyzed in a
static context by Kamien and Tauman (1986), Kamien (1992) and others. It has
the virtue that it encompasses the main results that can be obtained in more general
frameworks in which the demand faced by the firms in the industry is non-linear (e.g.,
the patentee prefers the auction mechanism to the fixed-fee and royalty mechanisms;
see Kamien, Oren and Tauman (1992)).

As discussed earlier, if possible, the monopolist may find it optimal to induce a
reduction in the number of active firms in the first period, in which case non-licensees

\textsuperscript{21}Consider for example the following case. The monopolist faces an inverse rental service demand
\( p = 100 - q \), \( \forall q \leq N \). The maximum amount that a potential buyer is willing to pay for the good
is independent of the number of buyers and the marginal cost of production for the monopolist is
zero. It can easily be shown that, for instance, if \( v = 1 \) and \( N = 75 \), then \( K_1^* = 40 < K^m = 50 \),
and \( K_2^* = 70 \). However, if \( N = 71 \) then \( K_1^* = K^m = 50 \) and \( K_2^* = N = 71 \). Lastly, if \( N > 75 \) then
\( K_2^* (K^m = 50) < N \) and \( K_1^* = \frac{200}{4+v}, \ K_2^* = \frac{100(6+v)}{8+2v} < N \). Note that when the demands for the good
are independent, rather than interdependent, \( K^m < N \) is independent of \( N \).
will not produce. Assume that there is a cost \( F \geq 0 \) of reentry into the active industry in period 2. We first consider that \( F \) is high enough so that reentry is impossible. In this case we have:

\[
x = \frac{a - c}{\varepsilon}
\]

\[
W_t(K_t) = \begin{cases} 
\frac{e^2(x+N-K_t+1)^2}{b(N+1)^2} & \text{if } x > K_t \\
\frac{e^2(x+1)^2}{b(K_t+1)^2} & \text{if } x \leq K_t
\end{cases}
\]

\[
L_t(K_t) = \begin{cases} 
\frac{e^2(x-K_t)^2}{b(N+1)^2} & \text{if } x > K_t \\
0 & \text{if } x \leq K_t.
\end{cases}
\]

We consider that \( x \) is a natural number, as in Reinganum (1981), Kamien and Tauman (1986) and Muto (1987).\(^{22}\) From (1) and (2), we have that the two problems the monopolist will solve sequentially under an auction mechanism are the following:

Given \( K_1 \), find the \( K_2(K_1) \leq N \) that at \( t = 2 \) solves:

\[
\max P_2 \cdot (K_2 - K_1)
\]

subject to:

\[
P_2 = e - fN \quad \text{if } K_2 = N \leq x \\
P_2 = e' - f'K_2 \quad \text{if } K_2 < \min \{x, N\} \\
P_2 = \frac{e^2(x+1)^2}{b(K_2+1)^2} \quad \text{if } x \leq K_2
\]

and at \( t = 1 \) find the \( K_1 \) that solves:

\[
\max P_1 \cdot K_1 + vP_2 \cdot [K_2(K_1) - K_1]
\]

subject to:

\[
P_1 = (1 + v)(e - fN) \quad \text{if } K_1 = N \leq x \\
P_1 = e' - f'K_1 + vP_2 \quad \text{if } K_1 < \min \{x, N\} \\
P_1 = (1 + v)\frac{e^2(x+1)^2}{b(K_1+1)^2} \quad \text{if } x \leq K_1
\]

where:

\[
e' = \frac{e^2[2x+N+1]}{b(N+1)}, \quad f' = \frac{2e^2}{b(N+1)^2};
\]

\[
e = \frac{e^2N[2x+N+2]}{b(N+1)^2}, \quad f = \frac{2e^2N}{b(N+1)^2}.
\]

\(^{22}\)Sen (2005) shows that if \( x \) is not an integer then the uniform superiority of fee over royalty that occurs in the static context of Kamien and Tauman (1986) no longer holds.
The resolution of these two problems yields the following proposition:

**PROPOSITION 2.** If licenses for a cost-reducing innovation are sold under an auction mechanism, reentry is impossible and the monopolist cannot commit to a future schedule of license sales, then the intertemporally consistent sale schedule that maximizes his profits is such that all firms that remain in the industry will eventually adopt the innovation.

**Proof:** See Appendix.

At this point a natural question that arises is whether Proposition 2 holds true even if reentry was possible or if firms had positive fixed costs of production. The answer lies in the proof of this proposition where it is shown how the optimal consistent sale schedule that induces \( K_2 = \min \{ x, N \} \) generates more profits to the patentee than every intertemporally consistent schedule of sales in which \( K_2 < \min \{ x, N \} \).

We may thus conclude that this proposition holds both if reentry was possible and in the presence of fixed costs of production, since in each case the inverse demand function faced by the monopolist for all \( K_2 \leq \min \{ x, N \} \) is the same as the one with no fixed costs of production.

Once we have examined how the commitment problem induces that saturation always occurs in equilibrium, we can address next the effects of the intertemporal consistency on two relevant additional aspects: (i) the diffusion process of the innovation and the resulting structure of the industry, and (ii) consumer surplus and social welfare. In order to examine these matters we need to compare the case in which the monopolist has commitment ability with the case in which he does not.

(i) **The effects on the diffusion process**

Consider first the case in which the monopolist has the ability to commit to a future schedule of license sales. As mentioned earlier, given that there are no costs of licensing, licenses will be sold only in the first period and the number of licenses to be auctioned will be that of the static case \((K^m)\). More precisely, taking into account the restriction that the number of licenses to be auctioned must be a natural number, the following proposition can be easily proven along the lines of Kamien (1992):

\[ \frac{(q - c + K)^2}{q(N+1)^2} - H = 0, \] which is lower than \( \frac{q}{x} \). Moreover note that if reentry was possible with \( F > 0 \) then \( W_2(K_2) = W_1(K_2) - F \) for all \( K_2 > K_1 \geq x \).
**PROPOSITION 3.** If licenses for a cost-reducing innovation are sold under an auction mechanism and the monopolist can commit to a future schedule of license sales, then the schedule of sales that maximizes his profits is such that:

\[
K_1^* = K_2^* = K^m = \begin{cases} 
N & \text{if } \frac{N^2+N-3}{2} \leq x \\
K^A & \text{if } \frac{N+3}{2} < x < \frac{N^2+N-3}{2} \\
x & \text{if } x \leq \frac{N+3}{2}
\end{cases}
\]

where \(K^A\) is the natural number closest to \(\frac{2x+N+1}{4}\) that is lower than or equal to \(N-1\).

Therefore, in this context the sale of licenses may sometimes imply breaking up permanently the initial homogeneity of the firms in the industry given that firms with different marginal costs, \(c\) and \(c'\), may coexist. This is always the case when \(K^m \neq \min\{x, N\}\).

We now examine the case in which the monopolist cannot commit to a future schedule of license sales. In this case the sale of licenses may also sometimes imply breaking up the initial homogeneity of the firms in the industry in the first period (that is, it may be the case that \(K_1 < \min\{x, N\}\)). In other words, while there is an initial symmetry among the firms in the industry, there may be a temporal asymmetry and hence diffusion over time. The diffusion is due to the interplay between the decrease over time in the cost of adoption (price of the license) and the decrease in profits that are derived from the use of the innovation.

To begin with, we consider that reentry into the active industry is not possible.\(^{24}\) In this case, all the schedules of sales in which \(K_1 = K_2 = K\) with \(K \geq x\) are intertemporally consistent. In other words, the fact that reentry is not possible implicitly provides the monopolist with some commitment ability: if he sells in the first period \(K \geq x\) licenses, he will not be able to sell any additional licenses. Notice that since the implicit rental demand for licenses is inelastic for all \(K \geq x\), then the optimal schedule of sales of a monopolist who chooses to decrease the size of the industry in the first period is \(K_1 = K_2 = x\). Moreover, if \(K^m = x\) (that is, if \(x \leq \frac{N+3}{2}\), see Proposition 3), then the monopolist will behave as if he had commitment ability. Obviously, this would also be the case if the monopolist with commitment ability decides to sell a number of licenses equal to \(N\). Hence, a necessary condition for the

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\(^{24}\)Since the gross profits of each licensee when non-licensees do not produce, \(\frac{\alpha^2(x+1)^2}{b(K_t+1)^2}\), are decreasing in \(K_t\) then reentry will not be possible if \(F > \frac{\alpha^2(x+1)^2}{b(K_t+1)^2}\bigg|_{K_t=x+1} = \frac{\alpha^2(x+1)^2}{b(x+2)^2}\).
diffusion to take place over time is that the patentee with commitment ability chooses not to saturate the market.\textsuperscript{25}

The resolution of the two sequential problems described earlier gives the optimal intertemporally consistent schedule of license sales:

**PROPOSITION 4.** If licenses for a cost-reducing innovation are sold under an auction mechanism, reentry is not possible, and the monopolist cannot commit to a future schedule of license sales, then the intertemporally consistent sale schedule that maximizes his profits is such that

\[
\begin{align*}
K_1^* &= K^m, & K_2^* &= N \\
K_1^* &= K^m, & K_2^* &= N \quad \text{or} \quad K_1^* = K_2^* = x > K^m \\
K_1^* &= K_2^* = x = K^m \\
\end{align*}
\]

where

\[
N = \frac{x + 3}{2}.
\]

**Proof:** See Appendix.

Before examining in greater detail the effects of intertemporal consistency on the diffusion process it is worth recalling that the present discounted value of the total profits of the monopolist is equal to what he would obtain if he rented \(K_t^*\) licenses in period \(t\). From Proposition 4 we know that if \(N < x < N\), then \(K_1^* = K^m, K_2^* = N\) or \(K_1^* = K_2^* = x > K^m\). Therefore, in this case the trade-off faced by the monopolist may be interpreted as follows: he can either choose to “sacrifice” some profits in the second period in order to obtain the profits of the monopolist renter in the first period or he may use the exogenous commitment device at his disposal, namely the sale of \(x\) licenses in the first period, and sacrifice some profits in the first period in order to obtain greater profits in the second period.

Given this trade-off, we can have a better understanding of some important results that derive from the commitment problem:

1. The price of the good produced by the firms in the industry may be below the competitive price corresponding to the initial situation, before the innovation was discovered. In other words \(K_2^*\) may be greater than \(x\). This result is in sharp contrast with the one obtained in a static context where \(K^m \leq x\) (see Katz and Shapiro (1986), Kamien, Oren and Tauman (1992) and other references therein).

\textsuperscript{25}Quirmbach (1986) compares the diffusion rates under various market scenarios. One is that of a patent holder who leases units of a new equipment on a period-by-period basis. As he recognizes, this allows him to avoid the durable goods monopoly problem that is studied in this paper. Moreover, assuming zero marginal costs of producing the new equipment his dynamic model collapses to the static model by Katz and Shapiro (1986), whose results for the case of linear demand and constant marginal costs are those contained in Proposition 3.
2. Innovations that do not induce a decrease in the size of the industry when the monopolist can commit to a future schedule of license sales may do so when he cannot (To see this, simply compare Propositions 3 and 4 for \( \frac{N+3}{2} < x < N \)).

3. The speed of diffusion \( (K_1) \) is never slower and the total diffusion level \( (K_2) \) is never smaller when the monopolist does not have the ability to commit to a future schedule of sales than when he does.

4. Given that for all \( K_t < x \) the elasticity of the inverse rental demand function \( p_t(K_t) \) increases with \( N \), then \( K^m \) does not decrease with \( N \). Therefore, from Proposition 4 we have that an increase in \( N \) will induce an increase in the number of licenses sold in the first period (or no change at all if \( K^*_1 = K^*_2 = x \)). Moreover, the greater the number of firms in the oligopolistic industry \( N \), \( \text{ceteris paribus} \), the more likely the innovation will induce a decrease in the size of the industry.

5. Changes in the discount factor \( v \) may have an effect on the diffusion process. If they do, it must be that \( \frac{N+3}{2} < x < N \) and an increase in \( v \) will imply an increase in the rate of adoption and a decrease in the total level of adoption in the industry given that in this case \( K^m < x < N \). The reason is that when the future is important enough \( (v \) is high enough), \( \text{ceteris paribus} \), the monopolist is more likely to induce a decrease in the size of the industry as a commitment device not to overproduce in the future. In other words, an increase in \( v \) makes it more likely that the patentee prefers the schedule \( K_1 = K_2 = x \) to the schedule \( K_1 = K^m, K_2 = N \).

Lastly, consider the case in which reentry into the active industry is possible, for example because \( F = 0 \). All the results previously obtained when reentry is not possible are maintained except those that involve the number of licenses sold in the first period when \( \min \{x, N\} = x \). Let us discuss why this is the case.

First, when the monopolist does not have the ability to commit to a future schedule of sales then the speed of diffusion will be slower than or equal to the one in which he does \( (i.e., \ K^*_1 \leq K^m) \). The reason is that the only commitment mechanism not to flood the market in the second period available to the monopolist is to decrease the number of licenses sold in the first period \( (\text{see condition (4)}) \). Now he could also choose to induce a decrease in the size of the industry in the first period but, as reentry is possible, he would end up selling additional licenses in the second period so he is not interested in doing that.

Second, note that in this context changes in \( v \) may also affect the diffusion process. However, if this is the case then an increase in \( v \) will induce a decrease, rather than an increase, in the speed of diffusion \( K^*_1 \) and, hence, in the total diffusion level of
the innovation $K_2$. The reason is that the implicit rental demand is inelastic for all $K_2 \geq x$. (Recall that $K^*_2 \geq x$.) This in turn implies that an increase in $N$ may induce a decrease (rather than an increase) in the number of licenses sold in the first period.

Next, we examine the effects of the commitment problem on consumer surplus and aggregate social welfare (defined as the sum of consumer surplus, monopolist’s profits and profits of the firms in the industry in present value terms).

(ii) Consumer Surplus and Social Welfare

First, it is clear that production in the industry increases with the adoption level of the innovation. As a result, the price of the good decreases and, since licensing costs are zero, consumer surplus and aggregate welfare also increase with the adoption level. Therefore, from Result 3 in (I) we have that when reentry is not possible both consumer surplus and social welfare are greater (or equal) when the monopolist cannot commit to a future schedule of license sales than when he can. However, when reentry is possible this need not be the case:

**PROPOSITION 5.** When reentry is possible social welfare may be higher if the monopolist has commitment ability than if he has not.

**Proof:** Consider the following example: $F = 0$, $P = 206 - 50Q$, $\varepsilon = 10$, $N = 41$, $c = 16$, and $v = 1$. In this case $x = 19$ and $K^m = 19$. Therefore, if the monopolist has commitment ability then $K^*_1 = K^*_2 = 19$. However, if the monopolist does not have commitment ability then $K^*_1 = 9$, $K^*_2 = 19$. As social welfare and consumer surplus increase each period with the number of licenses sold, it is straightforward to conclude that both will be higher if the monopolist can commit than if he cannot.

Obviously, this result derives from the effect of the commitment problem on the diffusion process, that is from the fact that $K^*_1$ may be lower than $K^m$.\(^{26}\)

Lastly, when the optimal schedule of sales for the monopolist who cannot commit implies a total diffusion of the innovation, which is the case for example for non-large

\(^{26}\)Note that a necessary condition for Proposition 5 to hold is that $x < N$. In this case the implicit rental demand has a kink in $K = x$. See Malug and Solow (1989) for a similar result in a different framework in which the monopolist seller faces an exogenous kinked demand. Moreover, as shown in Saracho (1997), the result in Proposition 5 holds when $N \leq x$ if the marginal cost of licensing is positive. As will be shown below, this proposition holds even if we take into account the fact that the optimal licensing mechanism may depend on whether the monopolist can commit or not.
innovations (i.e., \( N \leq x \)), consumer surplus and social welfare coincide with those in
the case of a shorter term of the patent (one period). The reason is that in the first
period the number of licenses sold is identical in both cases, \( K_1 = K^m \), and in period
2 all firms use the innovation, \( K_2 = N \), given that after the patent expires it goes to
public domain. Obviously, the patentee profits are lower in that case.

Summing up, this subsection has examined the implications of intertemporal con-
sistency for patent licensing when licenses are sold by means of an auction. Relative to
the existing literature we obtain a number of results that are substantively di¤erent.
We next analyze the cases of fixed-fee and royalty licensing.

3.2 Fixed Fee Licensing

This mechanism has received substantial attention in the literature.

3.2.1 General Features

Each period the monopolist sets a fixed price at which the input will be sold. This
price is independent of the production level that will take place using the input.

The framework of analysis when the licenses are sold at a fixed fee is similar to
that in the previous section. The important difference with respect to the auction
mechanism is the way the demand function is derived. Each buyer at period \( t \) knows
that if he deviated by not buying there would be one less licensee for sure in that
period. Therefore, a buyer in period 1 must recognize that his deviation might a¤ect
the behavior of the monopolist in the future (that is, his choice of a fixed fee at \( t = 2 \)).
Given this feature of the fixed-fee licensing mechanism we may conclude that:

i. The di¤erence between the profits from being a licensee and from being a non-
licensee in the second period, given the behavior of all other ﬁrms, determines the
maximum amount that a ﬁrm is willing to pay for a license in that period. This
amount is:

\[
W_2(K_2) - L_2(K_2 - 1) \text{ for all } K_2 \leq N.
\]

ii. The discounted gross proﬁts of a ﬁrm that buys a license in the ﬁrst period
(without substracting the price paid by the license) are

\[
W_1(K_1) + vW_2(K_{2ff}(K_1)).
\]

However, if he deviated there would be one less licensee in period 1 (i.e., \( K_1 - 1 \)
l licenses) and, as a result, the total number of licenses sold would be \( K_{2ff}(K_1 - 1) \).

\[27\] From now on we will use the superindex \( i = ff \) to denote the case in which the input is sold at
a fixed fee and \( i = A \) the case in which it is sold by means of an auction.
Hence, we have that the maximum amount a firm is willing to pay for a license in the first period is:

\[
W_1(K_1) - L_1(K_1 - 1) + v(W_2(K_2^{ff}(K_1)) - L_2(K_2^{ff}(K_1) - 1)), \text{ for all } K_1 \leq N.
\]

Given the demand function, the monopolist chooses the intertemporally consistent sequence of prices that maximizes his profits. Notice that, on the one hand, the analysis of intertemporal consistency is not relevant when the value of renting a license given that \(K_t - 1\) firms are doing so, \(W_t(K_t) - L_t(K_t - 1)\), increases with \(K_t\). In this case it is straightforward to conclude that the monopolist will behave as if he had commitment ability and will saturate the market by selling the input to all \(N\) firms in the first period. On the other hand the analysis of intertemporal consistency is important when \(W_t(K_t) - L_t(K_t - 1)\) is decreasing. In this case the monopolist may or may not choose to saturate the market. However, saturation always occurs in the Cournot model analyzed in subsection 3.1.2.

Let \(K^{mf}\) denote the number of licenses sold under the fixed-fee mechanism in a static context. More precisely, \(K^{mf}\) is the integer no greater than \(\min \{N, x\}\) that is closest to \(\frac{2x+N+2}{4}\). Using a strategy similar to the one used in the auction case it can be proven that:

**Proposition 6.** If licenses for a cost-reducing innovation are sold for a fixed fee, reentry is not possible, and the monopolist cannot commit to a future schedule of license sales, then the intertemporally consistent diffusion schedule that maximizes his profits is such that:

\[
\begin{align*}
K_*^1 &= K^{mf}, K_*^2 = N, & \text{if } N \leq x \\
K_*^1 &= K^{mf}, K_*^2 = N \text{ or } K_*^1 = K_*^2 = x > K^{mf}, & \text{if } \frac{N+4}{2} < x < N \\
K_*^1 &= K_*^2 = x = K^{mf}, & \text{if } x \leq \frac{N+4}{2}.
\end{align*}
\]

In consequence, all the results derived in subsection 3.1.2 in the analysis of the effects of intertemporal consistency on patent licensing under the auction mechanism are identical to the ones that can be derived in the analysis of fixed-fee licensing.

We next compare the implications of the two mechanisms analyzed so far. Their different effects on market price, consumer surplus and social welfare are studied in the context of the Cournot model formalized earlier.
3.2.2 Comparing the Auction and Fixed-Fee Mechanisms

We begin by comparing the profits of the monopolist under both mechanisms. The comparison is immediate when the value of renting a license given that \((K_t - 1)\) competitors are doing so increases with the number of licensees. In this case, as mentioned above, under the fixed-fee mechanism the monopolist will sell \(N\) licenses in the first period. Given that the monopolist obtains the same profits by auctioning \(N\) licenses with a minimum bid equal to \((1 + v) (W_1 (N) - L_1 (N - 1))\) than by selling licenses at a fixed fee, it is clear that his profits cannot be lower under the auction method.

The comparison, however, is not trivial when \(W_t (K_t) - L_t (K_t - 1)\) decreases with \(K_t\). For one, it may be that the implicit rental demand under the fixed-fee mechanism is at least as elastic as under the auction mechanism. This is certainly the case in the Cournot framework analyzed earlier where, as a result, \(K^m \leq K^{mf}\). More generally, whenever this property holds the following proposition can be proven:

**PROPOSITION 7.** If the implicit rental demand under the fixed-fee mechanism is at least as elastic as under the auction mechanism, then the monopolist without commitment ability will prefer to auction the licenses rather than sell them for a fixed fee.

**Proof:** See Appendix.

The result in Proposition 7 has already been obtained, in a static context, in the literature on patent licensing in presence of complete and perfect information.\(^{28}\)

Other relevant comparisons involve the different effects that these mechanisms have on market price, consumer surplus and social welfare. Although the patentee may prefer the auction to the fixed-fee method, consumers may prefer the opposite. For instance, consider the case of the Cournot model analyzed in subsection 3.1.2. In that case, Proposition 7 applies so the patentee prefers the auction mechanism to the fixed-fee method. Furthermore, as mentioned earlier, consumer surplus increases with the number of licensees. Given \(N\), this number depends on the licensing mechanism, on the magnitude of the innovation, and on whether reentry is possible or not. It is not difficult to show that although the speed of diffusion is at least as high under the fixed-fee method as under the auction mechanism, the number of licenses sold may

\(^{28}\)Jensen (1992) provides a context in which the patentee may prefer the fixed-fee mechanism to the auction mechanism. He shows that this will be the case when the following two conditions are met: (i) there is uncertainty about whether or not the innovation will succeed, and (ii) the life of the patent is shorter than the life of the innovation.
be greater under the auction mechanism than under the fixed-fee method.²⁹

This means that contrary to the case in which the monopolist can commit, the price of the good produced in the industry may be higher under the fixed-fee mechanism than under the auction mechanism. In principle it would seem possible that in this case the auction mechanism generates greater total consumer surplus and social welfare than the fixed-fee method. However, this scenario can never occur because of the effects induced by the discount factor \( v \).

### 3.3 Royalty Licensing

As the previous two licensing methods, this mechanism occupies an important role in the literature. Moreover, it is the one most often observed in practice.

Consider that the monopolist charges each buyer a uniform per-unit of production royalty \( h \).³⁰ Assume that the firms in the industry have constant marginal costs \( c \) and that the monopolist owns a patent on a cost-reducing innovation that reduces the marginal cost of production in a quantity \( \varepsilon \). Then the marginal cost of production for a firm that buys the license is \( c' = c - \varepsilon + h \). In the first stage the patentee will then choose the royalty that maximizes his profits. After \( h \) is announced firms decide independently and simultaneously whether to pay it or continue producing without the innovation.

Obviously, if \( h > \varepsilon \), no firm will buy the license since its marginal cost would then be higher with the innovation than without it, that is \( c' > c \).³¹ However, if the patentee sets a price \( h \leq \varepsilon \), then all firms in the industry will choose to buy the license. Therefore, the patentee solves the following problem:

\[
\max_h (1 + v) \cdot h \cdot Q (c')
\]

²⁹From Propositions 4 and 6 it is immediate to conclude that, since \( K^m \leq K^{mf} \), if \( \min \{ x, N \} = N \) then \( K^A_1 \leq K^{mf}_1 \), even though \( K^{mf}_2 = K^{mf}_2 = N \). When reentry into the industry is not possible then \( K^A_1 \leq K^{mf}_1 \) and \( K^{mf}_2 \leq K^A_2 \). As an example in which \( K^{mf}_2 < K^A_2 \) consider \( P = 206 - 50Q, c = 16, \varepsilon = 10, N = 21 \) and \( v = 0.8 \). In this case, \( K^A_1 = 15, K^{A}_2 = 21 \) and \( K^{mf}_1 = K^{mf}_2 = 19 \). However, if \( F = 0 \) and \( \min \{ x, N \} = x \), then \( K^A_1 \leq K^{mf}_1 \) for \( t = 1, 2 \). As an example in which \( K^A_1 < K^{mf}_1 \) for \( t = 1, 2 \), consider \( P = 206 - 50Q, c = 26, \varepsilon = 10, N = 26 \) and \( v = 0.8 \). In this case, \( K^A_1 = 9, K^A_2 = 19 \) and \( K^{mf}_1 = K^{mf}_2 = 26 \).

³⁰Some authors point out that there exists an important obstacle to licensing via a per-unit royalty since it is not often possible to monitor licensees’ output levels (see Katz and Shapiro (1986)).

³¹In a static context, if the licensee is one of the firms of the industry there are situations in which the optimal royalty is greater than \( \varepsilon \) (see, for example, Fauli-Oller and Sandonis (2002) and Filippini (2005)). This is because when both firms are operating in the same market, the licensor is able to use the licensing agreement as a facilitating device.
subject to

\[ h \leq \varepsilon. \]

Clearly all firms in the industry will pay a royalty equal to the optimal linear royalty of the static context each period. This means that the implications of the royalty mechanism on the monopolist’s profits, the structure of the industry, the price of the good produced in the industry and social welfare do not depend on the extent of the monopolist’s commitment ability. However, as shown in the previous sections, the commitment problem induced by durability has important effects, for example, on the profits that the monopolist obtains when selling the input by means of auction and fixed-fee mechanisms. More precisely, these profits are never greater when he cannot commit than when he can. The reason is that he has the ability to reduce the capital value of the outstanding stock of licenses (via new sales) and, in general, no way of guaranteeing that this power will not be used. As a result, the differences in profits between the auction and royalty mechanisms on the one hand, and between the fixed-fee and royalty mechanisms on the other, will be lower because of the commitment problem. Assuming that the underlying competitive interaction among potential licensees is characterized by Cournot competition, we may conclude that the royalty mechanism may even be preferred by the monopolist to the two other mechanisms. This result is in sharp contrast to what happens under certainty in a static context where the auction mechanism is always preferred to the fixed-fee and royalty mechanisms by the patentee (see Kamien, Oren and Tauman, 1992). We can thus establish the following proposition:

**PROPOSITION 8.** The royalty mechanism may be preferred by the patentee to the auction and fixed-fee licensing mechanisms because of the commitment problem induced by the durability of the licenses if and only if (i) the innovation is important enough to create a natural oligopoly (i.e., \( \min\{x, N\} = x \)) and (ii) reentry is possible.

**Proof:** See Appendix.

The empirical evidence reveals that royalties are found in most of the licensing agreements observed in practice. These agreements are justified in the literature by appealing, for example, to the possible roles of uncertainty (see Jensen and Thursby (2001) and other references therein), product differentiation (Muto, 1993), the separation of ownership from management (Saracho, 2002), or the degree of competitive behavior in the product market (Saracho, 2005). Our results show that the time-consistency problem implied by durability readily induces the superiority of this method.
Lastly, consumer surplus under the royalty mechanism may be identical to that corresponding to the pre-innovation stage given that \( h^* \) may be equal to \( \varepsilon \). This is the case for non-drastic innovations in the sense of Arrow in the Cournot model with linear demand. This result is contrary to what happens under the auction licensing method where both consumer surplus and social welfare always increase. Therefore, since the monopolist with commitment ability prefers the auction method to the royalty mechanism it is clear that consumer surplus and even social welfare may be higher when the monopolist can commit than when he cannot. The example suggested in the proof of Proposition 8 keeps this property.

4 Discussion and Extensions

The analysis in the previous sections has considered a dynamic framework to study the diffusion of innovations. The framework is a related version of the standard two-period model of Bulow (1982) whose main virtue is its tractability and transparency in generating the qualitative results of this paper in a clear way. It is, however, worthwhile to discuss in some detail the implications of various potential extensions of the analysis:

1. The Assumption of Two Discrete Periods of Time. The extension of the analysis to more than two periods would not modify the essence of the main results obtained here. For instance, three of the main contributions would be maintained: (i) regardless of the patent licensing mechanism, only firms that possess the license will remain in the industry; (ii) the price of the good produced by the firms in the industry may be below the perfectly competitive price corresponding to the initial situation, before the innovation was discovered; (iii) the royalty mechanism may be superior from the monopolist’s point of view to the auction and fixed fee mechanisms.

If the model were developed in continuous time and reentry in the industry were costless, the diffusion process under the fixed-fee and auction mechanisms would coincide with the socially optimal one. That is, there would be a total diffusion of the innovation from the initial date. In other words, Coase’s conjecture would hold and, unless some commitment or restraints to limit future production were adopted, the monopolist would forfeit part of his monopoly power and would produce the competitive output “in the twinkling of an eye” (Coase, 1972, p. 143).\(^{32}\) However, if

\(^{32}\)Obviously, even in the absence of commitment ability, he still obtains positive profits by using the interdependence of demands of potential licensees in the industry to his advantage.
it was unfeasible for firms to reenter into the active industry and the innovation was important enough to create a natural oligopoly, then the size of the industry would be reduced from the initial date. Consequently, both licensing mechanisms would be identical from the point of view of consumers, the industry and social welfare.

2. Patent Ownership. In the analysis of patent licensing, it is important to distinguish two kinds of patent ownership, both of which arise in practice. The first is that of an independent lab that has no financial interest in the firms of the industry, the one considered in this paper. The second is that in which the innovation is owned by one or more of the firms in the industry. In this case, a patentee will choose the number of licenses to be sold and the licensing mechanism that maximize the present value of the revenues he will obtain from the sale of licenses plus the profits he will earn in the downstream market. Previous research has shown in a static context that royalties can generate greater profits that either the auction or the fixed-fee mechanisms when the patent holder is itself one of the firms of the industry (see Wang (1998) and Kamien and Tauman (2002)). This result would clearly be reinforced in the dynamic framework considered in this paper. Moreover, it is not difficult to show that for this type of patent ownership, a second result is that regardless of the licensing mechanism, the market is always saturated in the specific Cournot model considered in this paper. Hence, it is straightforward to conclude that these two results would also apply if our independent lab could enter the industry after getting the patent.

3. Bertrand Competition. In the intertemporal framework studied in this paper patent licensing under Bertrand competition by means of auction, fixed-fee and royalty mechanisms would generate the same implications as those in the static case already analyzed in the literature (for example, in Kamien and Tauman (1986) and Kamien, Oren and Tauman (1992)). The reason is that under Bertrand competition the monopolist does not have any commitment problem even in the auction and fixed-fee licensing mechanisms. This is because in a homogeneous-good context price competition among firms with constant marginal costs yields zero profits to each firm, unless there is only one firm with the lowest marginal cost which, in addition, would be the only active firm in the industry. Therefore, in our dynamic framework

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33 Proofs are available upon request.
34 Sen and Tauman (2007) consider optimal combinations of up-front fees and per-unit royalties under the two kinds of patent ownership in the static context analyzed by Kamien and Tauman (1986, 2002). They show that under this mechanism either all \(N\) or almost all \((N-1)\) firms will use the innovation.
the patentee will sell only one license under either the fixed-fee or the auction mechanism, as in the static case. As a result, the patentee will capture all the profits generated by the single licensee in the industry.

5 Concluding Remarks

As a recent OECD report concludes, “the ability of firms to translate innovations into new products and processes and the timely and widespread diffusion of technologies are of critical importance. ... However, it is less the invention of new products and processes and their initial commercial exploitation than their timely and widespread diffusion and use which generate major economy-wide benefits” (OECD, 2004).

This paper draws attention to the dynamic aspects of the transmission of knowledge when market power on the side of the seller of knowledge is represented by a monopolist (patent holder) who cannot commit to a future schedule of license prices and sales either because of the nature of the patent (the case of information goods) or because of compulsory patenting laws. This is a problem that the static framework analyzed in the literature is not suited to address and hence that has not been previously studied in the patent licensing literature. Further, extending the analysis to a dynamic context allows us to study the effects of patent licensing practices such as exclusive licensing contracts and guarantees such as MFC clauses that allow patentee to solve or mitigate the dynamic consistency problem. We find that the analysis generates new insights into the speed and pattern of innovation and novel implications for patent licensing mechanisms as well as for the interactions between market structure and the evolution of the market and industries.

The model developed in this paper does not sacrifice much generality and allows us to simply and compactly formalize basic ideas about the dynamics of innovation. It also represents a tractable framework in which it is possible to derive the empirical implications that are associated with the monopolist’s intertemporal consistency problem, as well as to assess policies designed to enhance technological progress and social welfare. The analysis also yields implications for the study of the socially optimal degree of patent protection and for understanding the extent of the trade-off between promoting innovative effort and securing competitive outcomes.

With respect to future extensions, the same time-consistency issues that are present with buyers that have zero-one demands are also relevant in cases in which
potential buyers have demands for multiple units of the input. The role of uncertainty about the profitability of the innovation and the role of asymmetries among the potential adopters are also aspects that merit future research.
REFERENCES


Clean Air Act (2004), U.S., Environmental Protection Agency.


APPENDIX

This appendix collects the proofs of the propositions in the text.

Proof of Proposition 1:

(i) Consider $K_2(K^m) < \min \{x, N\}$. In order to saturate the market the monopolist must sell in the first period a number of licenses $K^0$ such that $K_2(K^0) \geq \min \{x, N\} > K_2(K^m)$. Therefore, $p_2(K_2(K^0)) < p_2(K_2(K^m))$. Given that $\{K_1 = K^m, K_2 = K_2(K^m)\}$ is a consistent schedule of sales—that is, $p_2(K_2(K^m)) \cdot [K_2(K^m) - K^m] \geq p_2(K_2(K^0)) \cdot [K_2 - K^m]$ for all $K_2 \leq N$—we have $p_2(K_2(K^m)) \cdot K_2(K^m) > p_2(K_2(K^0)) \cdot K_2(K^0)$. Therefore, since $p_1(K^m) \cdot K^m \geq p_1(K^0) \cdot K^0$, it is straightforward to conclude that the monopolist will not saturate the market since

$$p_1(K^m) \cdot K^m + vp_2(K_2(K^m)) \cdot K_2(K^m) > p_1(K^0) \cdot K^0 + vp_2(K_2(K^0)) \cdot K_2(K^0).$$

(ii) Let $\{K_1^* = K^0, K_2^* = K_2(K^0) < \min \{x, N\}\}$ be the optimal consistent schedule of sales with $K^m < K^0$. From condition (4) we have that $K_2(K^m) < K_2(K^0)$. Proceeding as in part (i), we have that $p_1(K^m) \cdot K^m + vp_2(K_2(K^m)) \cdot K_2(K^m) > p_1(K^0) \cdot K^0 + vp_2(K_2(K^0)) \cdot K_2(K^0)$, which is a contradiction. Therefore, $K_1^* \leq K^m$. In addition, from condition (4), we have $K_2^* = K_2(K^*) > K^m$ since $K_2(0) = K^m$. ■

Proof of Proposition 2: We need to show that $K_2^*$ is such that $N \geq K_2^* \geq \min \{x, N\}$. The proof is in two stages. In stage 1, it is proven that $K_2^* \geq \min \{x, N - 1\}$. As a result, if $x \leq N - 1$ there will be saturation. In stage 2 we prove that if $N - 1 < x$, then the optimal sale schedule is $K_1^* = K^m$, $K_2^* = N$ and, as a result, there will also be saturation.

Stage 1. Assume that $\{K_1^* = K^0, K_2^* = K_2(K^0) < \min \{x, N - 1\}\}$ is the optimal schedule of sales. From the two sequential problems solved by the monopolist we get that the first order conditions are respectively:

$$e' - 2f'K_2 + f'K_1 = 0,$$
$$e' - 2f'K_1 + v[e' - 2f'K_2(K_1)]dK_2/dK_1 = 0.$$

Hence we obtain:

$$K^0 = \frac{2e'}{(4 + v)f'},$$
$$K_2(K^0) = \frac{(6 + v)e'}{2(4 + v)f'} = \frac{(6 + v)(2x + N + 1)}{4(4 + v)} > \min \{x, N - 1\},$$

which is a contradiction. Therefore, given that for all $K_1$ such that $K_2(K_1) < \min \{x, N - 1\}$ the monopolist’s profits function $K_1 \cdot p_1(K_1) + v \cdot K_2(K_1) \cdot p_2(K_2(K_1))$ is strictly concave, we may conclude that $K_2^* \geq \min \{x, N - 1\}$.
Stage 2. Let \( N - 1 < x \). From stage 1 we know that it must be either \( K_2^* = N - 1 \) or \( K_2^* = N \). Let \( K^I \) denote the level of sales in the first period such that in the second period the monopolist is indifferent between selling \([N - K^I]\) licenses or selling \([(N - 1) - K^I]\) licenses.\(^{35}\) By definition, \( K^I \) is such that \( p_2(N) \cdot (N - K^I) = p_2(N - 1) \cdot (N - 1 - K^I) \). Therefore, taking into account (5) we have:

\[
K^I = \frac{N^2 - 2x + N - 3}{2x + 3}.
\]

Given that \( N - 1 < x \), we get \( K^I < K^m = \frac{e'}{2f'} = \frac{x}{2} + \frac{N}{4} + \frac{1}{4} \). So \( K_2(K^m) = N \). As a result, on the one hand, the optimal consistent schedule of sales that implies a total diffusion of the innovation is \( K_1 = K^m \), \( K_2 = N \). On the other hand, the optimal consistent schedule of sales that implies a diffusion of the innovation equal to \( N - 1 \) is \( K_1 = K^I \), \( K_2 = N - 1 \). Given that \( K^I < \frac{N - 1}{2} < K^m \), the monopolist’s profits associated with the plan \( \{K^I, N - 1\} \) are lower than those associated with the (intertemporally inconsistent) plan \( K_1 = \frac{N - 1}{2}, K_2 = N - 1 \). As a result, a necessary condition for the monopolist not to saturate the market is:

\[
p_1 \left( \frac{N - 1}{2} \right) \cdot \frac{N - 1}{2} + vp_2(N - 1) \cdot (N - 1) > p_1(K^m) \cdot K^m + vp_2(N) \cdot N,
\]

which, taking into account (5) and (6), implies:

\[
v \left[ N^2 - 2x + N - 3 \right] \geq xN^2 - N^3 + 2N^2 - x + N - 2.
\]

This is not possible, since \( v \leq 1 \) and \( x \geq N \). Therefore, \( K_1^* = K^m \), \( K_2^* = N \).\(\blacksquare\)

**Proof of Proposition 4:** We have three cases:

1. \( x \leq \frac{N+3}{2} \). Since \( K_1 = K_2 = x \) is a consistent schedule of sales and \( K^m = x \), then \( K_1^* = K_2^* = x \).

2. \( \min\{x, N\} = N \). From Proposition 2 and Corollary 2 we have that \( K_1^* = K^m \), \( K_2^* = N \).

3. \( \frac{N+3}{2} < x < N \). First, from Proposition 2 we know that the optimal schedule of sales is such that \( K_2^* \geq x \). Moreover, from the maximization problem of the second period,

\[
\max_{K_2} P_2(K_2 - K_1)
\]

\(^{35}\)Note that if \( K^I \leq 0 \), then \( K^m = N \). Therefore, the monopolist will act as if he had commitment ability.
subject to

\[ p_2 = e' - f'K_2 \quad \text{if } K_2 < x \]
\[ p_2 = \frac{e^2(x+1)^2}{b(K_2+1)^2} \quad \text{if } x \leq K_2, \]

we get that the intertemporally consistent schedules of sales in which the monopolist sells licenses both periods and \( K_2 \geq x \) are such that

\[
\begin{align*}
K_2 &= x & \text{if } \frac{2x-N-1}{2} \leq K_1 \leq \frac{x-1}{2} \\
K_2 &= 2K_1 + 1 & \text{if } \frac{x-1}{2} \leq K_1 < \frac{N-1}{2} \\
K_2 &= N & \text{if } \frac{N-1}{2} \leq K_1.
\end{align*}
\]  

(7)

As \( p_1(K_1) \cdot K_1 < p_1(x) \cdot x \) for all \( K_1 < \frac{N-1}{2} \) and \( p_2(K_2) \cdot K_2 < p_2(x) \cdot x \) for all \( K_2 > x \), then from (7) we can conclude that the intertemporally consistent schedule of sales \( K_1 = K_2 = x \) is preferred by the monopolist to every intertemporally consistent schedule of sales in which \( x \leq K_2 < N \). As a result, given that \( K^m > \frac{N-1}{2} \), from (7) we have that the optimal schedule of sales must be either \( K_1 = K^m, K_2 = N \) or \( K_1 = K_2 = x \), whichever implies highest profits for the monopolist.\(^{36}\)

**Proof of Proposition 7:**

Let \( \{K^*_1 = K^{ff}, K^*_2 = K^{ff}_2(K^{ff})\} \) denote the optimal consistent schedule of sales under the fixed-fee mechanism. We then have two cases:

a. \( K^{ff} < K^{ff}_2(K^{ff}) \). Given that for each \( K_1 \) the demand faced by the monopolist in the second period under the fixed-fee mechanism is at least as elastic as under the auction mechanism, we have that \( K^A_2(K^{ff}) \leq K^{ff}_2(K^{ff}) \). Given that under the auction mechanism the schedule of sales \( K_1 = K^{ff}, K_2 = K^{ff}_2(K^{ff}) \) is intertemporally consistent and that \( p^A_1(K_1) \geq p^{ff}_1(K_1) \), we have that \( p^A_2(K^{ff}_2(K^{ff})) \cdot K^A_2(K^{ff}) \geq p^{ff}_2(K^{ff}_2(K^{ff})) \cdot K^{ff}_2(K^{ff}) \). As a result, we may conclude that:

\[ p^A_1(K^{ff}) \cdot K^{ff} + up^A_2(K^{ff}_2(K^{ff})) \cdot K^A_2(K^{ff}) \geq p^{ff}_1(K^{ff}) \cdot K^{ff} + up^{ff}_2(K^{ff}_2(K^{ff})) \cdot K^{ff}_2(K^{ff}). \]

Since the left-hand side of the inequality is less than or equal to the monopolist’s profits under the auction mechanism, we may conclude that the profits under

\(^{36}\)The next two examples show how either one can be optimal. Let \( P = 206 - 50Q, c = 16, \varepsilon = 10 \) and \( N = 21 \). **Example 1:** Consider \( v = 1 \). In this case, \( K_1 = K_2 = x = 19 \), implies monopolists profits \( \Pi = 76 \). On the other hand, \( K_1 = K^m = 15, K_2 = N = 21 \) implies \( \Pi = 75.62 \). Therefore, \( K^*_1 = K^*_2 = x \). **Example 2:** Consider \( v = 0.5 \). In this case, \( K_1 = K_2 = x = 19 \), implies \( \Pi = 57 \). On the other hand, \( K_1 = K^m = 15, K_2 = N = 21 \) implies \( \Pi = 58.26 \). Therefore, \( K^*_1 = K^m, K^*_2 = N. \)
the auction mechanism are greater than (or equal to) the ones under the fixed-fee mechanism.\textsuperscript{37}

b. $K_2^{ff}(K^{ff}) = K^{ff}$. In this case we must have either $K^{ff} = N$ or $K^{ff} \geq x$. If $K^{ff} = N$, the monopolist’s profits will never be lower under the auction mechanism than under the fixed-fee mechanism given that he obtains the same profits by auctioning $N$ licenses with a minimum bid equal to $(1 + v)(W_1(N) - L_1(N - 1))$. If $K^{ff} \geq x$, the monopolist’s profits under the auction mechanism cannot be lower than under the fixed-fee mechanism since by auctioning $K^{ff}$ licenses the monopolist will capture all the profits in the industry. In other words, given that $L_t(K^{ff}) = 0$ each firm pays in equilibrium a bid equal to its profits. $\blacksquare$

\textbf{Proof of Proposition 8:}

We first show that the patentee may prefer the royalty mechanism to the auction and fixed-fee mechanisms. Assume that firms compete à la Cournot and consider $F = 0$, $P = 206 - 50Q$, $\varepsilon = 10$, $N = 41$, $c = 16$ and $v = 1$. Then under the auction mechanism $K_1^* = 9$, $K_2^* = 19$ and the profits of the patentee (II) are 64.57.\textsuperscript{38} Under the royalty mechanism all firms in the industry become licensees. As a result, total industry production is $Q = \frac{N(a-c-e-h)}{b(N+1)}$. It is then straightforward to show that $h^* = 10$, $Q = 3.71$ and $\Pi = 74.2$. So the profits of the patentee will be greater under the royalty mechanism than under the auction and fixed-fee mechanisms.

Now we show why conditions (i) and (ii) are necessary for the patentee to prefer the royalty method to the auction and fixed-fee mechanisms. From Kamien, Oren and Tauman (1992) we know that for a wide range of demand functions for the good produced by the Cournot oligopoly, including the linear case, the monopolist’s profits when auctioning a number of licenses equal to $\min\{x, N\}$ are higher than those he would obtain by selling the licenses by means of a royalty. Moreover, we know that in our dynamic context, the profits of the patentee with the schedule of sales $K_1 = K_2 = \min\{x, N\}$ are equal to $(1 + v)$ times the profits of the patentee in the static context when the number of licenses sold is $\min\{x, N\}$. Therefore given that, as shown above, under the royalty mechanism the profits of the patentee are equal to $(1 + v)$ times the profits of the patentee under that mechanism in the static context, we may conclude that: (1) If $\min\{x, N\} = N$ then the profits of the patentee will

\textsuperscript{37}Notice that since $p_i^{ff}(K_i)$ is decreasing then, proceeding as in the case of the auction mechanism and taking into account the fact that the number of licenses must be an integer no greater than $N$, we have that $K_2^{ff}(K_1 - 1) \leq K_2^{ff}(K_1)$. As a result, assumption B2 implies $P_1^{ff}(K^{ff}) \leq p_1^{ff}(K^{ff}) + v p_2^{ff}(K_2^{ff}(K^{ff}))$. Therefore, the monopolist’s profits under the fixed-fee mechanism are less than or equal to $p_1^{ff}(K^{ff}) \cdot K^{ff} + v p_2^{ff}(K_2^{ff}(K^{ff})) \cdot K_2^{ff}(K^{ff})$.

\textsuperscript{38}Note that, as mentioned earlier, in this context the monopolist’s profits under the fixed-fee mechanism are never greater than under the auction mechanism.
be greater auctioning $N$ licenses than selling the licenses under the royalty mechanism, and (2) if $\min \{x, N\} = x$ and reentry is impossible, then the schedule of sales $K_1 = K_2 = x$ is intertemporally consistent and the profits of the patentee will be greater auctioning in the first period $x$ licenses than selling the licenses using the royalty mechanism. ♦