Patent Licensing under Strategic Delegation

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The modern corporation is characterized both by a separation of ownership from management and by managerial incentives that often include strategic elements in addition to the standard incentive elements. Despite the importance of these two features in the agency and corporate-governance literatures, they are absent in the treatment of the firm in the patent-licensing literature. The analysis in this paper shows how, by simply taking into account these two features of the modern corporation, it is possible to offer a new explanation for the use of royalties in licensing agreements.

1. Introduction

A patent serves as an incentive for invention by providing the inventor (patentee) a certain period of time during which he controls the diffusion of the invention, so that he can attempt to realize a profit on his investment in research and development. One source of profit for the inventor is through his own working of the patent. The other, of course, is through licensing of the patent. Early work on licensing of cost-reducing innovations can be traced to Arrow (1962), who focused on the question of whether it is more profitable to innovate in a competitive or a monopolistic industry. His analysis was extended by Kamien and Schwartz (1982) to licensing in an oligopolistic industry, and by Kamien and Tauman (1986) and Katz and Shapiro (1986), who studied licensing strategies for a process innovation by considering, in addition, the inventor’s ability to exploit the interdependence and competition among the potential licensees to his advantage. A significant amount of research in the area has been organized around

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their framework of analysis, which has allowed important insights into the economics of patent licensing.\(^1\)

The literature distinguishes two kinds of patent ownership, both of which arise in practice. The first one is that of an independent lab that has no financial interest in the firms of the industry in which the innovation applies. The second one is that of a patentee that is one of the firms in the industry. According to Kamien and Tauman (1986) and Kamien (1992), when the industry’s aggregate output and product price are determined by the Cournot equilibrium, licensing a nondrastic innovation by means of a royalty per unit of output produced with the patented technology is less profitable for an external patentee than licensing by means of an auction or a fixed fee independent of the quantity produced with the new technology.

Understanding how innovations are marketed is important in a number of dimensions. For instance, extensive analyses by the Organization for Economic Cooperation and Development (1997) and the Survey of Current Business (1998) provide strong empirical support for the relevance of the diffusion of innovations as a fundamental source of economic growth, productivity, jobs, and competitiveness. Clearly, the way innovations are licensed has a crucial effect on the diffusion of innovations, on which firms obtain the licenses, and on the revenues that accrue to inventors.

Empirical evidence reveals that royalties are found in most of the licensing agreements observed in practice [see, for instance, Taylor and Silbertson (1973), Caves et al. (1983), Rostoker (1983), Macho-Stadler et al. (1996), and Jensen and Thursby (2001)]. The presence of royalties in these agreements is justified in the literature by appealing to asymmetric information (Beggs, 1992), risk sharing (Bousquet et al., 1998), moral hazard (Macho-Stadler et al., 1996; Jensen and Thursby, 2001), and product differentiation issues (Muto, 1993), or by assuming that the patentee is one of the firms in the industry issues (Katz and Shapiro, 1985; Wang, 1998; Kamien and Tauman, 2000).

This paper provides a new justification for the superiority of the royalty mechanism over the fixed-fee mechanism within Kamien and Tauman’s (1986) theoretical framework of analysis. The analysis

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1. Kamien and Tauman’s (1986) analysis is limited to the case of a linear demand function for the product to which the innovation applies. Their framework has been extended by further research. Kamien et al. (1992) extend the analysis to the class of product demand functions that are downward sloping, that are differentiable, and such that the total revenue function is strictly concave in the quantity sold. Kamien et al. (1988) consider the licensing of product innovation. Muto (1987) analyzes the case in which the patentee cannot prevent his competitors from reselling licenses. See Kamien (1992) for an excellent review of the literature on patent licensing.
will therefore maintain all the useful properties and features of their analysis and, in addition, deliver an explanation for the use of royalties. More precisely, it will be shown how, contrary to their result, royalty licensing may allow the patentee to obtain greater profits than fixed-fee licensing. In particular, the analysis in this paper draws attention to two important features of the modern corporation that have received no attention in the treatment of the firm in the patent-licensing literature: the actual objective function of the firm and its basic institutional structure of production.

Economists have long debated about the objective function of corporations. The patent-licensing literature has followed traditional economic theory and treated firms as economic agents with the sole objective of profit maximization. However, during the last couple of decades, as economists began to consider seriously the fact that the modern corporation is characterized by a separation of ownership and management, the analysis of the firm’s objective function began to focus on managerial objectives and the owner-manager relationship. This relationship is typically described as a standard principal-agent problem where the manager’s objective depends on the structure of incentives designed by the owner to motivate him. The incentive scheme often implies managerial incentives different than profit maximization, as managerial compensation is usually indexed to profits, sales, relative performance, and other variables. In fact, a significant amount of literature on strategic delegation has argued quite persuasively that the choice of managerial incentives often includes strategic elements, in addition to the standard incentive elements. A principal may indeed benefit by hiring an agent and giving him incentives to maximize something other than the principal’s payoff function when such an action creates strategic advantages (d’Aspremont and Gerard-Varet, 1980). This new angle on principal-agent models has generated a substantial body of theoretical and empirical literature and provides a novel rationale for the existence of managers. The literature has examined the strategic value of publicly observable managerial incentive contracts based, for instance, on sales and profits (not profits alone) and concluded that this class of incentive schemes can be used to change a firm’s strategic position in the market. Vickers (1985), for example, clearly

2. See, for instance, Holmström (1979) on the standard incentives problems, where the principal is concerned only with the action of his agent and where the optimal contract is independent of other contracts. Aggarwal and Samwick (1999) and Joh (1999) provide empirical tests of strategic interactions in executive-compensation data.

3. For instance, an owner’s optimal choice of managerial incentives may depend on rival managers’ choices, [e.g., in duopolistic markets (Fershtman, 1985; Vickers,
states these points: “The fact that delegation can have strategic advantages has a bearing on several issues on the theory of the firm. . . . Indeed the separation [of ownership from control] may be in some cases essential for the credibility of some threats, promises and commitments.” As noted above, these potentially important features have not been taken into consideration in the way the firm is assumed to operate in the patent-licensing literature, even though they represent fundamental building blocks in the agency-theory and corporate-governance literatures.

This paper takes into account the explicit consideration of these features of the firm and analyzes the implications for optimal patent licensing mechanisms within the Kamien-Tauman framework.

The analysis proceeds as follows. Market power on the patentee’s side is represented by a monopolist (patent holder) who sells licenses that allow the use of a process innovation that reduces the costs of production of the firms of a given industry. The monopoly (a research lab) has no financial interest in the potential adopters. These firms compete in quantities (Cournot competition) and may be characterized by a separation of ownership from control. Firm owners decide whether to buy a license and choose the incentive contracts that are offered to managers. The class of managerial incentive schemes examined are those indexed to profits and sales that have been analyzed and reported in the literatures on managerial compensation and contract theory.

In doing so, the analysis uncovers a heretofore unrecognized relationship between the patent licensing and the modern agency-theory and corporate-governance literatures. As mentioned earlier, the analysis provides a new explanation for the superiority of royalty licensing over fixed-fee licensing from the point of view of the patentee. In this sense, it adds one more explanation to previous explanations considered in the literature by bringing two novel, observable features of firms and markets to bear on the literature on patent licensing. These features may help explain patent licensing practices and hence can contribute to our understanding of the specific transmission processes and the extent of dissemination of new knowledge.

1985; Fershtman and Judd, 1987; Sklivas, 1987], may serve to deter entry (Sen, 1993), or may serve to “create” leadership (Basu, 1995).
4. See Basu (1993) for a review of this literature.
5. The patentee may be thought of as a firm whose output is not associated with that produced in the industry under consideration. When the research lab is part of the industry or has financial interest in the potential adopters of the innovation, it may still have an interest in licensing the patent. The issue of the strategic use of licenses or intentional sharing in the market for technology information has been analyzed, for instance, by Gallini (1984), Gallini and Winter (1985), Katz and Shapiro (1985), Rockett (1990), Fershtman and Kamien (1992), and Eswaran (1994).
The rest of the paper is organized as follows. Section 2 describes the general features of a model of patent licensing under strategic delegation that captures both the features of the classic Kamien-Tauman framework and those of strategic delegation in the agency and corporate-governance literatures. Section 3 is devoted to the specific analysis of the duopoly. This allows us to generate in a transparent way the qualitative nature and main insights of the analysis. In Section 4 the general framework and analysis for the case of \( N \) firms is provided. Section 5 discusses the effects of separation of ownership from control and strategic delegation on the diffusion of the innovation and social welfare. Lastly, Section 6 contains some concluding remarks.

2. A Model of Patent Licensing under Strategic Delegation

Consider, as in Kamien and Tauman (1986) and Kamien (1992), an oligopolistic industry with \( N \) identical firms that produce a homogeneous good. The inverse demand function for this good is of the form

\[
p = a - bQ\quad \text{with}\quad Q = \sum_{i=1}^{N} q_i,
\]

where \( q_i \) represents the quantity produced by firm \( i = 1, \ldots, N \). Entry into the industry is assumed to be unprofitable, i.e., the cost of entry exceeds the profits an entrant could realize. The average cost of production of each firm is independent of the level of production and equal to \( c \), with \( a > c > 0 \). The oligopolistic firms are engaged in quantity competition and may choose to delegate production decisions to managers in order to improve their strategic position in the market. A given research laboratory owns a patent on a process innovation and sells licenses to the downstream firms in the oligopolistic industry. The innovation reduces their marginal cost of production from \( c \) to \( c' = c - \epsilon, \epsilon > 0 \), and is such that \( \frac{a - c}{\epsilon} \geq N \). The marginal cost of selling licenses is zero. Obviously, the value of the license to each firm depends upon the number of rival firms that also buy the license.\(^7\)

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6. In the context of Kamien and Tauman (1986) a necessary condition for the sale of licenses to induce a decrease in the size of the industry is that \( \frac{a - c}{\epsilon} < N \). As will be shown in this paper, however, this is not the case when firm owners delegate production decisions to managers and these do not maximize profits.

7. If there were some initial asymmetries among the \( N \) firms in the industry, the value of a license for a potential licensee would depend on both the number of other licensees and their identities.
The analysis is modeled as a noncooperative game in four stages. In the first stage the monopolist sets either a fixed fee or a royalty for the licenses. In the second stage all firms in the industry are informed of the fixed fee or the royalty, and simultaneously and independently decide whether or not to buy a license at the announced fee or royalty. In the third stage of the game, once the number of license purchasers is known, all firms, with and without licenses, decide on the manager compensation incentives. Given the evidence from the theoretical and empirical literatures on delegation of control and managerial compensation incentives, we consider linear incentive compensation contracts that are a function of profits $\pi_i$ and sales $s_i$, and not a function of profits alone, that firms $i = 1, \ldots, N$, can offer to their managers. The contract is such that each firm $i$ will pay its manager the quantity $A_i + B_i \Phi_i$, where $A_i$ and $B_i$ are constants, $B_i > 0$, and $\Phi_i = \alpha_i \pi_i + (1 - \alpha_i) s_i = (p - \alpha_i c_i) q_i$. Each firm owner chooses the parameter $\alpha_i$ to determine his manager’s incentives. The terms $A_i$ and $B_i$ are chosen by owner $i$ in such way that the manager’s compensation is equal to his opportunity cost (reserve wage). The contract must be legally enforceable, irreversible, and observable. This is the contract typically considered in the literature on the strategic design of management compensation incentives. Finally, in the fourth stage all firms engage in a quantity competition game (Cournot competition) in which each manager determines the level of production that maximizes his compensation.

The solution concept is that of a subgame-perfect Nash equilibrium in pure strategies. Therefore, the solution is derived by backward induction from the fourth stage.

We first examine in Section 3 the case of a duopoly in order to provide clear intuition for the results that will be obtained for $N$ firms. Section 4 is devoted to the general analysis of oligopolies with $N > 2$ firms in the industry.

8. The auction licensing mechanism is not explicitly considered in this paper. As Katz and Shapiro (1986) show, the auction mechanism is at least as good as the fixed-fee mechanism from the viewpoint of the patentee when the demands for the innovation are interdependent. However, as Kamien (1992, p. 342) remarks, “fixed-fee licensing may be a more practical alternative than licensing by means of an auction if the cost of organizing the auction is taken into account.” Auction licensing would be superior to both fixed-fee licensing and royalty licensing in the framework developed in this paper if there were no costs associated with it.

9. Note that the marginal cost of production for a licensee will depend on the licensing mechanism. Therefore, each owner will set his manager’s contract once the structure of the industry (i.e., costs of each firm) is known.

3. Duopoly Case

Given that the patentee may decide to sell only a license, it is clear that in equilibrium firms may potentially have different costs. In a novel analysis, Fershtman and Judd (1987) examine the case of a duopoly in which firms with different costs of production $c_i$ delegate, for strategic reasons, production decisions to managers, and compensation contracts are, as described above, indexed to profits and sales. Their analysis corresponds to the fourth and third stages described earlier. They refer to the equilibrium under strategic delegation as the incentive equilibrium and to the equilibrium under no strategic delegation, where firms strictly maximize profits, as the Cournot equilibrium. These terms will also be used throughout our analysis. Following their analysis, the incentive parameters $a_i$, the production levels of each firm $q_i$, $i, j = 1, 2$, and the gross profits $\pi_i$ (that is, without subtracting the fixed fee paid if a license was bought) are

$$
\begin{align*}
\alpha_i &= 1 - \frac{a + 2c_j - 3c_i}{5c_i}, & i \neq j, \\
q_i &= \frac{2a - 6c_i + 4c_j}{5b}, & i \neq j, \\
\pi_i &= \frac{2(a - 3c_i + 2c_j)^2}{25b}, & i \neq j.
\end{align*}
$$

Directly of the resolution may be found in their paper. We examine next the second and first stages of the game for royalty and fixed-fee licensing.

3.1 Royalty Licensing

If the patentee sets a royalty $h$ per unit of output produced with the new technology, then the marginal cost of production for a firm that buys the license is $c_i = c - \varepsilon + h$. Obviously, if $h > \varepsilon$, no firm will buy the license, and if $h \leq \varepsilon$, both firms will buy the license. In the first stage the patentee will then choose the royalty that maximizes his profits. Taking into account the equations in (1), the optimal royalty per unit of output may be found by solving the following problem:

$$
\max_{h \leq \varepsilon} h \cdot (q_i + q_j) = \max_{h \leq \varepsilon} \frac{4a - 4(c - \varepsilon + h)}{5b}.
$$

Then, since $\frac{a}{\varepsilon^2} > 1$ implies $\frac{a(c+\varepsilon)}{\varepsilon^2} > \varepsilon$, we have that the patentee will set, as in the Cournot equilibrium, a royalty per unit of output $h^* = \varepsilon$. Note that in the incentive equilibrium managers act as
if they were strict profit-maximizing firms whose production costs are lower than the true ones their firms have (the reason is that \( \alpha_i = \alpha_j = \frac{-a + c}{3c} < 1 \)). In other words, firms are more aggressive than in the Cournot equilibrium, and, as a result, total production in the industry increases. Interestingly, this in turn implies that the patentee obtains greater profits from royalty licensing in the incentive equilibrium than in the Cournot equilibrium.\(^{11}\) More precisely, the patentee’s profits in the incentive equilibrium are \( \varepsilon \cdot 4(a - c)/5b \), whereas in the Cournot equilibrium they are \( \varepsilon \cdot 2(a - c)/3b \).

### 3.2 Fixed-Fee Licensing

Given a number of licensees, \( k \), the difference between the profits obtained by an adopting firm (adopter), \( \Pi^a(k) \), and those that it would obtain as a nonadopter (that is, with one less adopter in the industry), \( \Pi^a(k - 1) \), is the maximum amount that an adopter would be willing to pay for a license. That difference represents the inverse demand function for licenses. From the equations in (1), the inverse demand function for licenses is obtained as

\[
\Pi^a(2) - \Pi^a(1) = \frac{6\varepsilon}{25b}(2a - 2c - \varepsilon) \quad \text{if} \quad k = 2,
\]

\[
\Pi^a(1) - \Pi^a(0) = \frac{6\varepsilon}{25b}(2a - 2c + 3\varepsilon) \quad \text{if} \quad k = 1.
\]

Let \( x \equiv \frac{a - c}{c} \). Then it is straightforward to show that the number of licenses sold and the profits the patentee obtains under fixed-fee licensing, \( \Pi^{ff} \), are\(^{12}\)

\[
k = 2 \quad \text{and} \quad \Pi^{ff} = \frac{12\varepsilon}{25b}(2a - 2c - \varepsilon) \quad \text{if} \quad x \geq 2.5,
\]

\[
k = 1 \quad \text{and} \quad \Pi^{ff} = \frac{6\varepsilon}{25b}(2a - 2c + 3\varepsilon) \quad \text{if} \quad 2 \leq x < 2.5.
\]

\(^{11}\) This result also applies to the case in which the patentee is one of the firms in the industry. As a result, the superiority of royalty over fixed-fee and auction licensing shown by Wang (1998) is reinforced when considering the role of strategic delegation. Notice also that under Bertrand competition [see, for instance, Fershtman and Judd (1987) and other references therein] managers behave less aggressively, prices tend to be greater, and thus production in the industry tends to be smaller, than without strategic delegation. Therefore, under the royalty mechanism the patentee will obtain smaller revenues than in the equilibrium without delegation. However, even in this case, it is not difficult to show that royalty licensing may be a superior means of licensing in a differentiated-good Bertrand duopoly when strategic delegation is taken into account. In other words, Muto’s (1993) result is robust to the consideration of strategic delegation.

\(^{12}\) Without loss of generality, throughout the analysis we assume that when the patentee is indifferent between two equilibria, then the one associated with the greater social welfare (measured as the sum of consumer surplus and profits) obtains.
Comparing these results with the ones obtained in the Cournot equilibrium [see Kamien and Tauman (1986) or Kamien (1992)], we may conclude that the diffusion level (the number of adopters) in the incentive equilibrium is lower than or equal to the one corresponding to the Cournot equilibrium. More precisely, in the Cournot equilibrium $k = 2$; however, in the incentive equilibrium $k = 2$ if and only if $x \geq 2.5$. Otherwise only one license will be sold. This result suggests that strategic delegation of output decisions to managers slows down the diffusion of the innovation. The two reasons for this result are: (i) the positive difference between the fixed fees corresponding to $k = 1$ and $k = 2$ is greater in the incentive equilibrium than in the Cournot equilibrium, given that $-\partial^2 \bar{\Pi}_i / \partial c_i \partial c_j$ is greater in the former than in the latter equilibrium, and (ii) the fixed fee corresponding to $k = 2$ may be lower in the incentive equilibrium than in the Cournot equilibrium, given that firms induce their managers to behave aggressively ($\alpha_i < 1$) and a firm that has a cost disadvantage has a lower market share than in the Cournot equilibrium. More precisely, the low-cost firm’s market share is $1 + \varepsilon / (a - c + 2\varepsilon)$ times greater in the incentive equilibrium than in the Cournot equilibrium.

By comparing the patentee’s profits under the royalty and fixed-fee licensing mechanisms, we may conclude:

**Result 1:** Royalty licensing is preferred to fixed-fee licensing from the patentee’s viewpoint if and only if the size of the innovation ($\varepsilon$) is intermediate ($\frac{a - c}{3} < \varepsilon < \frac{a - c}{2.25}$).

**Proof.** We have two possible cases:

A. $x \geq 2.5$. In this case the patentee’s profits under the fixed-fee mechanism, $\Pi^f$, and the royalty mechanism, $\Pi^r$, are

$$
\Pi^f = \frac{12\varepsilon}{25b}(2a - 2c - \varepsilon) \quad \text{and} \quad \Pi^r = \frac{4\varepsilon(a - c)}{5b}.
$$

Given that

$$
\Pi^f - \Pi^r = \frac{4\varepsilon(a - c - 3\varepsilon)}{25b},
$$

then fixed-fee licensing is preferred to royalty licensing if and only if $\frac{a - c}{3} \geq \varepsilon$. 


B. 2 ≤ x < 2.5. In this case, his profits under fixed-fee and royalty licensing are

$$\Pi^g = \frac{6\varepsilon}{25b}(2a - 2c + 3\varepsilon) \quad \text{and} \quad \Pi^r = \frac{4\varepsilon(a - c)}{5b},$$

respectively. Given that

$$\Pi^g - \Pi^r = \frac{2\varepsilon}{25}(-4a + 4c + 9\varepsilon),$$

we have that fixed-fee is preferred to royalty licensing if and only if $\frac{a - c}{2.25} \leq \varepsilon$.

This result is in sharp contrast with the one obtained in the Cournot equilibrium, where, as Kamien and Tauman (1986) show, fixed-fee licensing is always preferred to royalty licensing.\(^{13}\)

There are other relevant conclusions that derive from comparing the results of the incentive equilibrium and from the Cournot equilibrium.

Efficiency, for instance, is a relevant concern in models of strategic delegation. For example, Fershtman and Judd (1987, Corollary 1) find that in a duopoly the incentive equilibrium in the quantity game generates a more efficient allocation than the usual Cournot equilibrium. The two reasons for this result are: (a) price gets closer to marginal cost (industry production is higher), and (b) in an asymmetric duopoly production rises relatively more at the low-cost firm.

However, when we take into account the facts that strategic delegation may have an effect on the optimal licensing method and on the number of licenses sold, this result may no longer hold true. This may actually be the case because, as we know from our previous analysis, strategic delegation may cause the number of licenses sold to decrease and thus may induce a decrease in the number of firms that

\(^{13}\) If the patentee were to sell the licenses by means of the auction mechanism considered by Katz and Shapiro (1986) and Kamien and Tauman (1986) and there were no costs associated with it, then the number of licenses sold and the profits $\Pi^{au}$ the patentee obtains would be

$$\begin{align*}
k = 2 \quad \text{and} \quad \Pi^{au} &= 2[\Pi^a(2) - \Pi^a(1)] = \frac{12\varepsilon}{25b}[2a - 2c - \varepsilon] \quad \text{if} \quad x \geq 5.5, \\
k = 1 \quad \text{and} \quad \Pi^{au} &= [\Pi^a(1) - \Pi^a(1)] = \frac{2\varepsilon}{5b}[2a - 2c + \varepsilon] \quad \text{if} \quad 2 \leq x < 5.5.
\end{align*}$$

As a result, the auction mechanism would be preferred to royalty licensing. Note also that the diffusion level would be at least as high under fixed-fee licensing as under auction licensing. It is not difficult to show that these results also apply for the case of oligopolistic industries.
produce with a lower marginal cost $c - \varepsilon$. Moreover, we have shown that under strategic delegation the patentee may prefer the royalty licensing mechanism. In this case, the royalty per unit of output is $\varepsilon$, and hence the total production in the industry is just equal to the one that would take place without the innovation. With no strategic delegation, however, total production in the industry is greater than without the innovation, given that licenses are sold at a fixed fee.

By comparing social welfare in both scenarios the following result is obtained:

**Result 2:** Social welfare will be greater in the incentive equilibrium than in the Cournot equilibrium if and only if both the number of licenses sold and the licensing method used are the same in either equilibrium.

**Proof.** As shown earlier, in the Cournot equilibrium fixed-fee licensing is preferred to royalty licensing and the number of licenses sold is equal to 2. Therefore, total production is $Q = 2\varepsilon(x + 1)/3b$, and social welfare is $W = 4\varepsilon^2(x+1)^2/9b$. In the incentive equilibrium total production and social welfare are

\[
Q = \frac{4(a - c + \varepsilon)}{5b}, \quad W = \frac{12\varepsilon^2(x + 1)^2}{25b} \quad \text{if } 3 \leq x,
\]

\[
Q = \frac{4(a - c)}{5b}, \quad W = \frac{4\varepsilon^2x(3x + 5)}{25b} \quad \text{if } 2.25 < x < 3,
\]

\[
Q = \frac{2(2a - 2c + \varepsilon)}{5b}, \quad W = \frac{4\varepsilon^2(3x^2 + 3x + 7)}{25b} \quad \text{if } 2 \leq x \leq 2.25.
\]

Hence, social welfare is greater in the incentive equilibrium if and only if $x \geq 3$.

Note that this result implies that social welfare may be lower in the incentive equilibrium. This would be the case if (i) the patentee prefers royalty licensing to fixed-fee licensing ($2.25 < x < 3$), or if (ii) the number of licenses sold in the incentive equilibrium is smaller than the number sold in the Cournot equilibrium ($2 \leq x \leq 2.25$). Notice that in case (ii), even though total production is higher in the incentive equilibrium, part of that production is produced without the innovation, that is, at the higher marginal cost $c$, whereas in the Cournot equilibrium all units are produced at the lower marginal cost $c - \varepsilon$.

Clearly, the patentee’s incentive to innovate depends on the expected profits to be earned. It is thus important to determine whether the patentee obtains greater profits in the incentive equilibrium or in the Cournot equilibrium. Profits may be greater under
strategic delegation than under no strategic delegation, but this is not necessarily the case. The following result may be established:

**Result 3:** The patentee’s profits are greater in the incentive equilibrium than in the Cournot equilibrium if and only if the size of the innovation is low enough ($\varepsilon < \frac{a}{6.75}$).

*Proof.* In the Cournot equilibrium the patentee’s profits are $8\varepsilon^2 x / 9b$. As shown earlier, in the incentive equilibrium the patentee’s profits are

\[
\frac{12\varepsilon^2}{25b} (2x - 1) \quad \text{if} \quad 3 \leq x,
\]
\[
\frac{4\varepsilon^2 x}{5b} \quad \text{if} \quad 2.25 < x < 3,
\]
\[
\frac{6\varepsilon^2}{25b} (2x + 3) \quad \text{if} \quad 2 \leq x \leq 2.25.
\]

Therefore, the patentee’s profits in the incentive equilibrium are greater than in the Cournot equilibrium if and only if $x > 6.75$. 

Consequently, in this context, strategic delegation does not imply greater profits for the patentee. This result obtains even though the number of licenses sold and the licensing method used are the same in both equilibria (i.e., $x \geq 3$). This is because strategic delegation has two effects on the firm’s profits that operate in opposite directions on the fixed fee being set. First, in a symmetric duopoly each firm obtains greater profits in the Cournot equilibrium than in the incentive equilibrium, given that total production in the industry is lower. Second, in an asymmetric duopoly the high-cost firm (nonadopter) obtains greater profits in the Cournot equilibrium given that it has a greater market share and total production is lower than in the incentive equilibrium.

Having examined in detail the effects of strategic delegation on optimal patent licensing mechanisms in the duopoly framework, we proceed next to analyze the general case of an oligopoly with $N$ firms. One of the main results that will be shown is that royalty, even in this case, may be superior to fixed-fee licensing from the point of view of the patentee.

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14. This amount is twice the difference between the profits obtained by each firm when they both use the innovation and the profits of a firm without the innovation when its competitor uses the innovation.
4. Oligopolistic Industries

Next we analyze the fixed-fee licensing method, then we analyze the royalty licensing mechanism, and lastly we compare the two methods.

4.1 Fixed-Fee Licensing

We solve the problem by backward induction to get the subgame-perfect equilibrium. Details of the calculations are relegated to Appendix A.

There are two feature worthy of mention concerning the number of licenses sold under the fixed-fee licensing mechanism:

1. If \( x > N(N - 1) \), then the sale of licenses cannot induce a reduction in the size of the industry. The inverse demand function for licenses is equal to the difference between the profits obtained by an adopter and those that it would obtain as a nonadopter. Moreover, the patentee will set the fixed fee equal to

\[
\frac{\varepsilon^2 N [1 + N^2 + N^4 + 2(N^2 + 1 - N)(x - k*N)]}{b(1 + N^2)^2},
\]

and the number of licenses sold will be

\[
k^* = \min \left\{ \frac{1 + N + N^2}{4N} + \frac{x}{2N}, N \right\}.
\]

2. If \( x \leq N(N - 1) \), then the sale of licenses may induce a decrease in the size of the industry. The number of licenses sold will be no greater than \( m + 1 \), where \( m \) is the integer that belongs to \([x/N, x/N+1]\). If the monopolist sells a number of licenses that is at least as large as \( m + 1 \), then his profits will be \( k^2\varepsilon^2(1+x)^2/b(1+k^2)^2 \) if \( k^2 > x \), and \( \varepsilon^2 x/b \) if \( k^2 \leq x \). Given that \( k^2\varepsilon^2(1+x)^2/b(1+k^2)^2 \) decreases with increasing \( k \), that \( \varepsilon^2 x/b \) is independent of \( k \), and that \( \varepsilon^2 x/b \) is greater than \( k^2\varepsilon^2(1+x)^2/b(1+k^2)^2 \) when \( k^2 > x \), then the patentee will not be interested in selling more than \( m + 1 \) licenses.\(^{16}\)

15. More precisely, because the number of licenses sold must be a natural number, it will be the integer closest to \( k^* \). None of the results in this paper depend on the consideration of natural numbers.

16. Note that since \( \varepsilon^2 x/b \) is independent of \( k \), we may have multiple equilibria. If this is the case, we select the one with the smallest number of licenses being sold. This choice has no effect on any of the results obtained in the rest of the paper. In fact, all these equilibria are identical from the point of view of the agents and imply the same social welfare. For instance, consider the case in which \( N = 10 \) and \( 49 < x \leq 50 \). In this case, the patentee is indifferent between selling 6 and 7 licenses. In either case, his profits will be \( \varepsilon^2 x/b \), the profits of the firms net of the cost of the license will be zero, and production in the industry will be \( \varepsilon x/b \).
Then, since \( m < \frac{x + N}{N} \), the number of licenses sold will be no greater than
\[
\frac{x + 2N}{N}.
\]

The number of licenses sold and the fixed fee set by the patentee can also be computed analytically. However, they do not add any interesting additional results or insights to the ones already presented in this section.

4.2 Royalty Licensing

As has been shown for \( N = 2 \) in Section 3, if the patentee sets a price \( h \leq \varepsilon \) per unit of good produced with the new technology, he will sell a license to each firm in the industry. Therefore, there will be \( N \) firms in the market, each with a marginal cost of production \( c - \varepsilon + h \). Given that \( c_i = c - \varepsilon + h \), \( i = 1, \ldots, N \), from equation (A2) in Appendix A we get
\[
\alpha_i = \frac{(1 - N)a + N(N + 1)(c - \varepsilon + h)}{(1 + N^2)(c - \varepsilon + h)}, \quad i = 1, \ldots, N.
\]

Note that \( \alpha_i < 1 \), and that it increases with \( N \) (managers are more aggressive in more concentrated industries). By replacing \( \alpha_i \) in equations (A1) in Appendix A we obtain that the patentee will set the royalty per unit of output that solves the following problem:
\[
\max_{h \leq \varepsilon} h \frac{N^2[a - (c - \varepsilon + h)]}{b(1 + N^2)}.
\]

Given that we are considering nondrastic innovations in the sense of Arrow \((x > 1)\), we obtain \( h^* = \varepsilon \). Hence, the patentee’s profits under this method will be
\[
\Pi^y = \frac{\varepsilon N^2(a - c)}{b(1 + N^2)} = \frac{\varepsilon^2 N^2 x}{b(1 + N^2)}.
\]

Notice that when firms are assumed to maximize profits (as in Kamien and Tauman, 1986), the total production in the industry under royalty licensing, \( \varepsilon N x / b(N + 1) \), is lower than in the case in which the manager’s compensation depends not only on profits but also on sales, \( \varepsilon N^2 x / b(N^2 + 1) \). Given that the royalty being set is the same in either case, it is clear that under the royalty method the patentee’s profits are greater when the manager’s compensation is indexed not only to profits. This result is straightforward if we take into account that, as has been already shown, under Cournot competition owners
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distort managerial incentives away from strict profit maximization so as to make the manager more aggressive in the product market. Moreover, these incentives to induce more aggressive behavior decrease as the number of firms in the market increases. In fact, as $N$ tends to infinity (perfect competition), firms cannot afford to do anything other than be profit maximizers (Fershtman and Judd, 1987). Consequently, the difference in the patentee’s profits in the cases of strategic delegation and no strategic delegation under royalty licensing decreases with the number of firms in the industry. More precisely, the patentee’s profits in the incentive equilibrium are

$$\Pi_{sd}^r = \frac{\epsilon^2 N^2 x}{b(1 + N^2)},$$

whereas in the Cournot equilibrium they are

$$\Pi_{nsd}^r = \frac{N x \epsilon^2}{b(N + 1)}.$$

Therefore,

$$\Pi_{sd}^r - \Pi_{nsd}^r = \frac{\epsilon^2 N x (N - 1)}{b(N + 1)(1 + N^2)},$$

which decreases with increasing $N$.

4.3 Royalty Licensing versus Fixed-Fee Licensing

In this subsection we compare the two licensing methods. The previous analyses have shown how, whenever managerial incentives include strategic elements in addition to the standard incentive elements, the patentee’s revenues from royalty licensing increase, and how this licensing mechanism may even be preferred over fixed-fee licensing for $N = 2$. However, as competition increases and concentration decreases, firms are less interested in inducing their managers to act aggressively. This suggests that fixed-fee licensing may be preferred to royalty licensing in less concentrated industries. Next, we analyze the extent to which royalty licensing may be superior to fixed-fee licensing given our assumption on the magnitude of the innovation (i.e., $x \geq N$). The following result can be established:

**Result 4:** If $x \geq N$, then patent licensing by means of a royalty may be superior to licensing by means of a fixed fee only if before the innovation takes place the number of firms in the industry is small enough ($N < 5$).
Appendix B shows how fixed-fee licensing is superior to royalty licensing for \( N \geq 5 \).

Hence, in our framework of analysis it is possible for royalty licensing to be superior to fixed-fee licensing only in concentrated industries. In particular, under the assumptions of the analysis this will be the case for \( N = 3 \) when \( 3.06 < x < 3.33 \) and when \( 7 < x < 8.75 \), and for \( N = 4 \) when \( 8.125 < x < 8.34 \). In these cases, consumer surplus and firms’ profits are identical to those corresponding to the pre-innovation situation, since the real marginal cost of production for each firm is \( c \).

More importantly, this result suggests that by taking account of some of the \textit{de facto} fundamental features of the internal organization of the modern corporation we may provide a new explanation for the use of royalties in the licensing agreements. In the last section it is briefly discussed how this result becomes notably stronger in a dynamic framework.

5. Discussion and Extensions

The analysis in the previous section has identified some potentially important reasons based on the fundamental structure of the firm for royalty licensing to be more profitable than fixed-fee licensing. Hence, it provides a new explanation for the use of royalties in many actual licensing contracts. This main result also obtains if we consider the possibility of licensing by means of two-part tariff contracts that involve a combination of a fixed fee (ff) and a linear royalty (h) in production. For instance, consider the case of a duopoly. In the incentive equilibrium the patentee sells two licenses \((k = 2)\) and the optimal contract is such that

\[
h = 0, \quad ff = \frac{6e^2(2x - 1)}{25b} \quad \text{if} \quad x \geq 11,
\]

\[
h = \frac{e(11 - x)}{16}, \quad ff = \frac{3e^2(x + 5)(31x - 5)}{3200b} \quad \text{if} \quad 2 \leq x < 11.
\]

17. The problem that the patentee solves for \( k = 2 \) is

\[
\max_h \frac{4(a - c + e - h)}{5b} + \frac{4[(a - c + e - h)^2 - (a - c - 2e + 2h)^2]}{25b}
\]

subject to \( 0 \leq h \leq e \). The first term represents the revenues obtained from the royalty component \((h \text{ times industry production})\), and the second term the revenues obtained from the fixed-fee component \((that \text{ is, twice the difference between the profits of an adopting firm when both firms use the innovation and their marginal cost is } c - e + h \text{ and the profits of a non-adopting firm whose marginal cost is } c \text{ while its rival’s marginal cost is } c - e + h\)\). For \( k = 1 \) the contract implies \( h = 0 \).
It is not difficult to show that in the Kamien-Tauman (1986) framework the patentee also sells \( k = 2 \) licenses but the optimal contract is
\[
h = 0, \quad ff = \frac{4\epsilon^2 x}{9b} \quad \text{if} \quad x \geq 3, \\
h = \frac{\epsilon(3 - x)}{6}, \quad ff = \frac{2\epsilon^2 x(x + 3)}{27b} \quad \text{if} \quad 2 \leq x < 3.
\]

The range of values in which a combination of a royalty and fixed fee is optimal is greater in the incentive equilibrium. In consequence, not surprisingly, even when we consider two-part tariff contracts (royalty and fixed fee), royalty licensing plays a more important role when separation of ownership from production decisions and strategic incentives are taken into account.

Other relevant aspects of the analysis are concerned with the effects that separation of ownership from management and strategic managerial incentives have on the diffusion process of the innovation and on social welfare. With regard to the diffusion process, it has already been shown in the case of duopoly that strategic delegation will generally induce a decrease in the diffusion level. This result also holds true for any oligopolistic industry. As Kamien and Tauman (1986) show, in the Cournot equilibrium the number of licenses sold is
\[
k_{nd} = \min \left\{ \frac{x}{2} + \frac{N}{4} + \frac{1}{2}, N \right\}.
\]

From the results obtained in Section 4.1 we know that under strategic delegation and fixed-fee licensing the number of licenses sold is
\[
k_{d1} = \min \left\{ \frac{1 + N + N^2}{4N} + \frac{x}{2N}, N \right\} \quad \text{if} \quad x \geq N(N - 1), \\
k_{d2} < \frac{x + 2N}{N} \quad \text{if} \quad x < N(N - 1).
\]

It is then straightforward to show that \( k_{nd} \geq k_{ds} \) for \( s = 1, 2 \).  

18. As mentioned earlier, we may have multiple equilibria that are indifferent from the point of view of all agents. In all these equilibria the diffusion level \( k \) is such that \( x \geq k^2 \). Therefore, in all of them \( k_{nd} > k \).
the duopoly case. However, in addition, for $x > N$ and $N > 2$, it is possible that nonadopting firms will exit the industry. Notice also that under strategic delegation royalty licensing may be superior to fixed-fee licensing (in which case all firms will use the innovation). Moreover, when royalty is superior to fixed-fee licensing but there is no strategic delegation, the patentee will also sell licenses to all firms in the industry but using instead the fixed-fee licensing mechanism ($k^{nd} = N \forall N \leq 4$).

The matter, in principle, is slightly more complicated from the viewpoint of consumer surplus and social welfare. First, both increase (or remain unchanged) with the number of firms that produce with a marginal cost $c - \epsilon$. Therefore, strategic delegation would, in principle, appear to reduce them both. However, note that for a given number of adopters under fixed-fee licensing, strategic delegation also induces greater levels of production. By comparing the two equilibria we may conclude that total production is greater in the Cournot equilibrium than in the incentive equilibrium only when $N = 3$ and $3.06 < x < 3.33$ (see Appendix C). Obviously, in these cases social welfare will be greater in the Cournot equilibrium than in the incentive equilibrium, given that in both equilibria all firms use the innovation. This result may also obtain when the patentee uses the fixed-fee mechanism in both equilibria. In general, a necessary condition for social welfare to be greater in the Cournot equilibrium than in the incentive equilibrium is that the sale of licenses does not induce a decrease in the number of producing firms in the industry. This is because if the number of producing firms decreases, then (i) the price gets closer to the marginal cost $c - \epsilon$ in the incentive equilibrium (and industry production is higher), and (ii) all producing firms use the innovation in the incentive equilibrium (their marginal cost is $c - \epsilon$), whereas in the Cournot equilibrium this is the case only if $k^{nd} = N$.

6. Concluding Remarks

The analysis in this paper suggests that firms’ strategic incentives may play a fundamental role in generating the licensing practices observed in practice, and thus in explaining the forms and patterns of the diffusion of innovations. In consequence, the analysis indicates that it may be important to take account of some of the fundamental features that characterize the modern corporation, especially its separation of

19. Consider for example $N = 3$ and $x = 3.04$. In this case, social welfare is $7.65e^2/b$ in the Cournot equilibrium and $7.632e^2/b$ in the incentive equilibrium.
ownership from control and managerial incentives that may not be indexed only to profits.

Understanding the way innovations are marketed and the form of the licensing agreements observed in practice is important. An extensive recent report by the Organization for Economic Cooperation and Development (1997) concludes that “the ability of firms to translate innovations into new products and processes and the timely and widespread diffusion of technologies are of critical importance. However, it is less the invention of new products and processes and their initial commercial exploitation than their timely and widespread diffusion and use which generate major economy-wide benefits.”

The report also remarks that the nature of the problem of the transmission and adoption of new knowledge requires a careful analysis of the temporal aspects. It is also important to note that, because a patent provides its holder the monopolist’s right to sell licenses that allow the use of new knowledge or an innovation during a certain period of time, the patent holder faces the typical intertemporal consistency problem of the durable-goods monopolist that is induced by durability on the demand side. If the static analysis in the literature is extended to a dynamic framework by explicitly considering this intertemporal consistency problem as in Saracho (1997), then the implications of the analysis in this paper become notably stronger. The reason is that in such an intertemporal framework the time-consistency problem faced by the monopolist decreases the benefits that he may obtain under the auction and fixed-fee licensing mechanisms but does not affect those that may be obtained by means of royalty licensing.

**APPENDIX A**

We solve by backward induction for the subgame-perfect equilibrium in an oligopolistic industry with \( N \) firms under the fixed-fee licensing method.

**A.1 Fourth Stage**

The manager is assumed to maximize his compensation \( A_i + B_i \Phi_i \), where \( A_i \) and \( B_i \) are constants, \( B_i > 0 \), and \( \Phi_i = (p - \alpha_i c_i)q_i \). That is, he solves the following problem:

\[
\max_{q_i} (a - bQ - \alpha_i c_i)q_i
\]
subject to $q_i \geq 0$, $i = 1, \ldots, N$. Assuming interior solutions, we get

$$q_i = \frac{a - N \alpha_i c_i + \sum_{j \neq i} \alpha_j c_j}{b(N + 1)}, \quad i = 1, \ldots, N.$$  \hfill (A1)

### A.2 Third Stage

All firms simultaneously set their own $\alpha_i$ in order to maximize their profits. Therefore, they solve simultaneously the following problem:

$$\max_{\alpha_i} (a - bQ - c_i)q_i,$$

subject to (A1). From the first-order conditions

$$a - N \alpha_i c_i + \sum_{j \neq i} \alpha_j c_j - Na - N \sum_{j=1}^N \alpha_j c_j + (N + 1)Nc_i = 0, \quad i = 1, \ldots, N,$$

we obtain

$$\alpha_i = \frac{(1 - N)a + N(1 - N) \sum_{j \neq i} c_j + N(2 - N + N^2)c_i}{(1 + N^2)c_i}. \hfill (A2)$$

Let $k$ denote the number of adopters. Taking into account the fact that the innovation reduces production costs by an amount $\varepsilon$ (and that this reduction may induce nonadopters not to produce), then profits, gross of the price of the license, and the incentive parameter of each firm are as follows:

A. If $Nk < x$, then

$$\Pi_i = \begin{cases} 
\frac{\varepsilon^2 N(x + N^2 + 1 - Nk)^2}{b(1 + N^2)^2} & \text{for } i = 1, \ldots, k, \\
\frac{\varepsilon^2 N(x - Nk)^2}{b(1 + N^2)^2} & \text{for } i = k + 1, \ldots, N
\end{cases}$$
and

\[
\alpha_i = \begin{cases} 
\frac{a(1 - N) + Nc(1 + N) - N\varepsilon(N^2 + 1 - Nk + k)}{(c - \varepsilon)(1 + N^2)} < 1 & \text{for } i = 1, \ldots, k, \\
\frac{a(1 - N) + Nc(1 + N) - N(1 - N)k\varepsilon}{c(1 + N^2)} < 1 & \text{for } i = k + 1, \ldots, N.
\end{cases}
\]

B. If \( Nk \geq x \) and \( k^2 < x \), then

\[
\Pi_i = \begin{cases} 
\frac{\varepsilon^2x}{bk} & \text{for } i = 1, \ldots, k, \\
0 & \text{for } i = k + 1, \ldots, N
\end{cases}
\]

and

\[
\alpha_i = \begin{cases} 
-a + c(1+k)/k(c - \varepsilon) < 1 & \text{for } i = 1, \ldots, k, \\
1 & \text{for } i = k + 1, \ldots, N.
\end{cases}
\]

C. If \( Nk \geq x \) and \( k^2 \geq x \), then

\[
\Pi_i = \begin{cases} 
\frac{\varepsilon^2k(x + 1)^2}{b(1 + k^2)^2} & \text{for } i = 1, \ldots, k, \\
0 & \text{for } i = k + 1, \ldots, N
\end{cases}
\]

and

\[
\alpha_i = \begin{cases} 
\frac{a(1 - k) + k(1 + k)(c - \varepsilon)}{(c - \varepsilon)(1 + k^2)} < 1 & \text{for } i = 1, \ldots, k, \\
1 & \text{for } i = k + 1, \ldots, N.
\end{cases}
\]

A.3 Second Stage

From the results obtained in the third stage we have that, given a number \( k \) of adopters, the difference between the profits obtained by
an adopting firm (adopter) and those that it would obtain as a non-adopter (that is, with one less adopter in the industry) are

\[
\frac{k\varepsilon^2(1 + x)^2}{b(1 + k^2)^2} \quad \text{if} \quad (k - 1)N \geq x \quad \text{with} \quad k^2 \geq x,
\]

\[
\frac{\varepsilon^2 x}{bk} \quad \text{if} \quad (k - 1)N \geq x \quad \text{with} \quad k^2 < x,
\]

\[
\frac{k\varepsilon^2(1 + x)^2}{b(1 + k^2)^2} - \frac{N\varepsilon^2 [x - (k - 1)N]^2}{b(1 + N^2)^2} \quad \text{if} \quad kN \geq x > (k - 1)N \quad \text{with} \quad k^2 \geq x,
\]

\[
\frac{\varepsilon^2 x}{bk} - \frac{N\varepsilon^2 [x - (k - 1)N]^2}{b(1 + N^2)^2} \quad \text{if} \quad kN \geq x > (k - 1)N \quad \text{with} \quad k^2 < x,
\]

\[
\frac{N\varepsilon^2 [1 + N^2 + N^4 + 2(N^2 + 1 - N)(x - kN)]}{b(1 + N^2)^2} \quad \text{if} \quad x > kN.
\]

A.4 First Stage

The patent holder sets a price for the licenses. The price, given the demand function, will be the one that maximizes the revenue from license sales.

Appendix B

Proof of Result 3. Three situations can be distinguished:

(A) \( x \geq N(N - 1) \). From the analysis in Section 4.1 we know that the number of licenses sold under fixed-fee licensing \( k^*(x) \) is

\[
\min \left\{ \frac{1 + N + N^2}{4N} + \frac{x}{2N}, N \right\}.
\]

20. Note that this difference corresponds to the inverse demand function for licenses only if it is less than the difference between the profits that a nonadopter would obtain by adopting the innovation and those that it obtains as a nonadopter. As an example in which this does not occur, consider an industry with six firms and an innovation such that \( 18 < x \leq 24 \). In this case, if \( k = 4 \) then \( \Pi^a(k = 4) - \Pi^a(3) < \Pi^a(k = 5) - \Pi^a(4) \). More precisely,

\[
\frac{\varepsilon^2 x}{4b} - \frac{6\varepsilon^2(x - 18)^2}{37^2 b} < \frac{5\varepsilon^2(x + 1)^2}{26^2 b} \quad \forall x \in (18, 24].
\]
The patentee’s profits are then equal to

\[ k^*(x)N e^2 \frac{1 + N^2 + N^4 + 2(N^2 + 1 - N)[x - k^*(x)N]}{b(1 + N^2)^2} \]

His profits under royalty licensing are \( N^2 e^2 x / b(1 + N^2) \). Therefore, since \( x \geq N(N - 1) \) the monopolist’s profits under fixed-fee licensing are at least

\[
k^*(x = N(N - 1))N e^2 \\
\times \frac{1 + N^2 + N^4 + 2(N^2 + 1 - N)[x - k^*(x = N(N - 1))N]}{b(1 + N^2)^2}
\]

with \( k^*(x = N(N - 1)) = (3N^2 - N + 1)/4N \). Then, a necessary condition for royalty licensing to be superior to fixed-fee licensing from the patentee’s viewpoint is that

\[
A(x) = 4x(N^4 - 4N^3 + 3N^2 - 2N + 1) - 3N^6 + 13N^5 - 14N^4 \\
+ 13N^3 - 2N^2 + N + 1 < 0.
\]

Note that \( A(x) \) is an increasing function in \( x \) for all \( N \geq 4 \). Given that \( x \geq N(N - 1) \), we have \( A(x) \geq A(N(N - 1)) = N^6 - 7N^5 + 14N^4 - 7N^3 + 10N^2 - 3N + 1 > 0 \) for all \( N \geq 4 \). Therefore, fixed-fee licensing is preferred to royalty licensing \( \forall N \geq 4 \) when \( x \geq N(N - 1) \).

(B) Let \( n < N \) be an integer such that \( nN \geq x > (n - 1)N \) and \( n^2 \geq x \).\(^{21} \) It is clear that under the fixed-fee mechanism the patentee may choose either to reduce the size of the industry by selling \( N - 1 \) licenses or not to reduce it by selling fewer than \( N - 1 \) licenses. Therefore, the patentee’s profits under the fixed-fee mechanism are at least those corresponding to the profits of the patentee selling \( N - 2 \) licenses:

\[ N e^2[1 + N^2 + N^4 + 2(N^2 - N + 1)(x - kN)]k \bigg|_{k=N-2} \]

We thus have that a necessary condition for the patentee’s profits to be greater under royalty licensing than under fixed-fee licensing is that

\[
g(x) = \frac{e^2N(N^3x - 6N^2x + 5Nx - 4N^5 - 8N^4 - 17N^3 + 14N^2 - 7N - 2)}{b(1 + N^2)^2} < 0.
\]

\( ^{21} \) Note that these two conditions imply \( n = N - 1 \).
Given that \( g(x) \) is an increasing function in \( x \) for all \( N > 5 \) and \( x > N(N - 2) \), we may then conclude that
\[
g(x, N) > g(N(N - 2), N) = \frac{\varepsilon^2N(N - 2)}{b(1 + N^2)^2} > 0.
\]
As a result, in this case, fixed-fee licensing will be superior to royalty licensing if \( N > 5 \). In addition, it may be shown that if \( N = 5 \) and \( 16 \geq x > 15 \) then the patentee will sell four licenses when \( x \geq 15.32 \) and three licenses when \( x < 15.32 \), using the fixed-fee mechanism. Therefore, fixed-fee licensing is preferred to royalty licensing \( \forall N \geq 5 \) when \( nN \geq x > (n - 1)N \) and \( n^2 \geq x \).

(C) Let \( n < N \) be an integer such that \( nN \geq x > (n - 1)N \) and \( n^2 < x \). In this case the patentee may or may not choose to induce a reduction in the size of the industry. If he chooses to do so by selling \( k = n \) licenses, his profits will be
\[
\left. \frac{\varepsilon^2x}{b} - \frac{kN\varepsilon^2[x - (k - 1)N]^2}{b(1 + N^2)^2} \right|_{k=n}.
\]
Then, a necessary condition for the patentee’s profits to be greater under royalty licensing than under fixed-fee licensing is
\[
F(x) = -nNx^2 + N^2(1 + 2n^2 - 2n)x + x - N^3n^3 + 2n^2N^3 - nN^3 < 0.
\]
Note that \( F(x) \) is strictly concave and equal to zero in
\[
x_i = \frac{1 + N^2 + 2n^2N^2 - 2nN^2 \pm \sqrt{(1 + N^2)(1 + N^2 - 4nN^2 + 4n^2N^2)}}{2nN},
\]
\( i = 1, 2 \).

Let \( x_1 > x_2 \). Since \( x_2 < N(n-1), Nn < x_1 \), and \( F(Nn) > 0 \), we can conclude that \( F(x) > 0 \) for all \( x \) such that \( nN \geq x > N(n-1) \). Therefore, fixed-fee licensing is superior to licensing by means of royalty.

**Appendix C**

With no strategic delegation, licenses are sold using the fixed-fee mechanism. Therefore, total production in the industry is
\[
\frac{N(a - c) + k\varepsilon}{b(N + 1)},
\]
with \( k \leq N \). That is, total production is at most \( N(a - c + \varepsilon)/b(N + 1) \).
If there is strategic delegation of management we have four possible cases:

A. Fixed-fee licensing.
   
   A1. Reduction in industry size with \( k^2 \geq x > N \). Total production in the industry is given by
   \[
   \frac{k^2(a - c + \varepsilon)}{b(k^2 + 1)},
   \]
   where \( k < N \) is the number of adopters. Note that \( N(a - c + \varepsilon)/b(N + 1) > k^2(a - c + \varepsilon)/b(k^2 + 1) \) implies \( k^2 < N \), which cannot be the case.
   
   A2. Reduction in industry size with \( k^2 < x \). Total production is
   \[
   \frac{a - c}{b},
   \]
   and \( x \geq N \) implies that \( \frac{a - c}{b} \geq \frac{N(a - c + \varepsilon)}{b(N + 1)} \). We may then conclude that total production under strategic delegation is no lower than under no strategic delegation.
   
   A3. No reduction in industry size. Total production in the industry in this case is
   \[
   \frac{N^2(a - c) + k\varepsilon N}{b(N^2 + 1)}.
   \]
   Therefore, \( N(a - c + \varepsilon)/b(N + 1) > [N^2(a - c) + k\varepsilon N]/b(N^2 + 1) \) implies \( h(x) = Nk + Nx + k - N^2 - x - 1 < 0 \). Since \( h'(x) > 0, x \geq N \), and \( h(x) \) is nonnegative for \( x = N \), we can conclude that total production in the industry is greater if firm owners choose strategic delegation of production.

B. Royalty licensing. Total production in the industry in this case is
   \[
   \frac{N^2(a - c)}{b(N^2 + 1)}.
   \]

   Hence, \( N^2(a - c)/b(N^2 + 1) < N(a - c + \varepsilon)/b(N + 1) \) implies \( Nx - N^2 - x - 1 < 0 \). We know that in this framework licensing by means of a royalty is preferred to fixed-fee licensing if \( N = 3 \) for \( 3.06 < x < 3.33 \) and for \( 7 < x < 8.75 \), and if \( N = 4 \) for \( 8.125 < x < 8.34 \). Therefore, since for \( N = 3 \) we have \( k^{\text{nd}} = 3 \), we may conclude that total production in the industry is greater if firm owners choose strategic delegation of production than if they don’t, except if \( N = 3 \) and \( 3.06 < x < 3.33 \).
References


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