

A brief review of lattice computing

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Abstract—Defining *lattice computing* as the class of algorithms that either apply lattice operators *inf* and *sup* or use lattice theory to produce generalizations or fusions of previous approaches, we find that a host of algorithms for data processing, classification, signal filtering, have been produced over the last decades. We give a fast and brief review, which by no means could be exhaustive; with the aim of showing that this area has been growing during the past decades and to highlight the ones that we think are broad avenues for future research. Although our emphasis is on Artificial Neural Networks and Fuzzy Systems in this review we include Mathematical Morphology as a notorious instance of Lattice Computing.¹

I INTRODUCTION

The main road of definition computational algorithms in many aspects of Computer Science, including Computational Intelligence, works on the algebra of the real numbers, the addition and the multiplication $(R, +, \cdot)$. However there have a parallel line of works working on other algebras, like $(R, \wedge, +)$ or its dual $(R, \vee, +)$, where the role of the addition is taken by the lattice operation *inf* or *sup*, and the multiplication role is taken by the addition. These works have evolved into the application of Lattice Theory as a framework to define new approaches and algorithms that either generalize previous ones [1] or fuse existing approaches [2]. We call this broad class of algorithms and knowledge representation methods Lattice Computing. Of course, the works reviewed didn't refer themselves as belonging to this category.

If we categorize, at a very abstract level, the kind of processes realized in Computational Intelligence as:

- Filtering: maps of objects (i.e. signals) in a high dimensional space into objects of the same space, i.e. $F : R^N \rightarrow R^N$.
- Dimension reduction: maps of objects in a high dimensional space into ones in a lower dimension space, i.e. $F : R^N \rightarrow R^d; d \ll N$.
- Classification: Mapping objects in a (high dimension) space into categories, where the construction can be done in a supervised or unsupervised (clustering) way, i.e. $F : R^N \rightarrow \Omega; \Omega = \{\omega_1, \dots, \omega_c\}$.

There have been instances of Lattice Computing algorithms falling on all these categories. Filtering roughly corresponds to Mathematical Morphology applied to image and signal processing [3-5], which has been quite successful

and with a large body of literature. In fact this success has been the source of inspiration for researchers trying to extend this approach to Computational Intelligence problems and applications. There have been many attempts and proposals of the construction of classification systems based on Lattice Computing ideas. They fall in a no man's land between Artificial Neural Networks and Fuzzy Systems. The results reported by these classifiers sometimes improve conventional classifiers and sometimes fall behind. There are few instances of dimension reduction based on Lattice Computing ideas, may be [6-9] are the only works falling in this category.

A lattice is a partially ordered set (poset) any two of whose elements have a supremum and an infimum. The *inf* and *sup* operations are binary relations that give, respectively, the infimum and the supremum of any pair objects in the set. The classical reference on Lattice Theory is the book [10]. Perhaps the first extensive work that proposed computations with lattice operations is the definition of the Minimax Algebra [11] in the context of operational research problems. The following historical landmark is the introduction of Mathematical Morphology in [3]. Image algebra [12-14] is another early attempt to define lattice computing methods devoted to image processing. The Fuzzy-ART architecture [15, 16] maybe the earliest Lattice Computing learning approach. Recent works [2, 17, 18] identify the algebraic lattice structure as a central concept for a whole family of methods and applications. In short, the Lattice Computing approach has served to bridge the gap between computational paradigms as diverse as fuzzy systems, morphological signal processing and artificial neural networks [19-27].

In section 2 we comment on the problems and the approaches followed to implement learning procedures in Lattice Computing algorithms. In section 3 we comment on some of the approaches that we find more promising and in section 4 we finish with some conclusions.

II LEARNING AND PARAMETER ESTIMATION

Construction by learning is the induction of the system parameters from the available data. The key to the success of the Artificial Neural Networks paradigm is the development of easy to implement, robust (up to some degree) methods to estimate the parameters of the system: backpropagation, competitive learning algorithms, among others. These algorithms were designed as gradient descent procedures for a given energy or cost function. Some Lattice Computing proposals had tried to follow this same pattern [28-31]. For instance, [29] tries to minimize the MSE of the output, [28] tries to maximize an equality index between the output and

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the desired output of the system. The major difficulty in these approaches is the discontinuity of the *min* and *max* functions. These discontinuity has been avoided proposing continuous approximations to them [28, 29, 32] that allow to obtain close expressions that give approximations of the gradient. Some approaches had added complications, such as the need to express the clause structure in a way that allows differentiation in [32]. Besides the construction of classification systems, there have been works [33, 34] tried to define gradient descent algorithms for the estimation of the morphological filter structural element. In fact, [34, 35] proposes a class of mixed morphological/linear systems which are trained with an algorithm analogous to the error backpropagation.

Some authors have tried to apply non-differential learning approaches, based on heuristic reasoning. For instance, there have been attempts to mimic the perceptron rule for lattice based morphological networks [36-38]. Among the heuristic approaches, one of the earliest and most successful is Fuzzy-ART [15, 16] which applies an ad-hoc heuristic category growing learning procedure. The learning procedure produces hyperrectangles covering the data, corresponding to the categories. These hyperrectangles grow monotonically as the learning proceeds, only stopped by the application of the vigilance parameter. Both the resolution of the result and the number of categories found depend on this vigilance parameter. The learning process is highly dependent on the order of presentation of the data [39], and still research is going on the setting of the algorithm parameters.

Another approach to build up systems is that of the Morphological Associative Memories [40-45]. It is very easy and does not involve gradient descent algorithms. It consists on the morphological (lattice) analogy to the construction of linear associative memories and Hopfield networks. Most of the theoretical work on their behavior have been devoted to explain their behavior and to understand the shape and properties of their fixed point subspaces [22, 44, 46-49]. The related dendritic morphological neural network [38, 50, 51] has a learning algorithm based on the incremental refining of the covering of the sets in the classes to be discriminated.

III REVIEW OF SOME APPROACHES

The goal of this review is to give a flavour of some of the avenues of work and research in this broad area which we find most involving and promising of future results. It can not be exhaustive, as some fields like Mathematical Morphology have a large number of works.

A Mathematical Morphology

In image and signal processing a big class of tasks to be solved are related to noise removal and/or to segmentation. The basic morphological operators are the erosion and dilation operators, while the basic morphological filters are opening and closing. Sophisticated morphological procedures like texture segmentation, granulometries, watershed segmentation, etc. are compositions of these filters. One of the venues of research is the definition of

appropriate lattice ordering of high dimension spaces, such as color spaces and other vector spaces found in images, so that filters can be generalized to them. The other venue of interest from our point of view is that of defining new morphological operators and filters [2, 27, 52, 53] and the approaches to estimate appropriate structural elements [4, 5, 34, 35, 54, 55].

Lattice Theory was already used to formalize morphological operators [27]. The use of Lattice Theory in [2, 53] to generalize the erosion and dilation operators and the rigorous construction of adjoint operator pairs to systematically construct generalized opening and closing operators is a breakthrough that paves the way for new research areas. The embedding of fuzzy intersection and union norms into the morphological framework gives new fuzzy morphological openings and closings with enhancing noise removal and edge detection properties.

In the mean time, there have been proposals for adaptive morphological operators sometimes mixed with linear operators, using gradient descent algorithms for structural element estimation [34, 35, 55], and they were applied to image restoration and character recognition.

Another example of the fusion of ideas are the Morphological Shared-weight networks [33] were proposed for target recognition in images. There a morphological hit-or-miss transform is implemented through a shared-weight network and its structural elements are adaptively estimated to obtain an optimal target recognition.

B Morphological Associative Memories

The works on Image Algebra [12-14] were the prelude to the proposal of morphological neural networks, in the form of morphological perceptron [36-38] and of associative memories [40, 42, 56]. They were proposed for the storage of binary and gray patterns, with the aim of recovering the original clean image from noisy copies, which is an image restoration process. Also grayscale morphological associative memories are used in [57] to pre-process the data prior to classification with a simple Nearest Neighbour approach

The good properties of Morphological Autoassociative Memories (later renamed Lattice Autoassociative Memories) were hindered by their sensitivity to specific kinds of noise (erosive or dilative noise), so that a big deal of effort was addressed to obtain robust versions [22, 41, 43-45, 48, 49, 57-61]. These efforts produced a new kind of memories in the frontier between associative morphological memories and fuzzy systems [22, 23, 25, 26].

Our own works on spectral unmixing of hyperspectral images [9, 62] have led to the use of the convex coordinates produced by unmixing process as features for classification purposes [6-8]. The sample data points are expressed relative to the so-called endmembers in hyperspectral image processing. The idea of endmember is that they represent instances of pure elements, so that all the other elements in the image correspond to mixtures of these pure elements. Endmembers are vertices of a polygonal convex set covering data cloud. This is a new kind of feature extraction that we think deserves further research.

It happens that Autoassociative Morphological Memories [40, 42, 56, 63] have specific noise sensitivities that allow the detection of endmembers in the data from hyperspectral images [9, 62]. Working ways to enhance the robustness of Associative Morphological Memories against all kinds of noise [58, 60, 61] led to the definition of “morphological independence” and latter to “lattice independence” which turns out to be related to affine independence [49], a condition to be fulfilled by sets of endmembers. So we proposed an algorithm exploiting the Autoassociative Morphological Memories sensitivities to extract the endmembers from the data sample [9]. It is possible also to obtain these endmembers through the construction of the Autoassociative Morphological Memories [49].

The dendritic neuron [38, 51, 59] is a further development of the Morphological Neural Networks where the biological model is a dendrite and the learning procedure is an incremental procedure that is shown to converge, with some similarities to the Fuzzy-ART learning algorithm.

C Fuzzy Lattice Neurocomputing

The Fuzzy Lattice Neurocomputing paradigm [1, 17, 64, 65] arises from the generalization of Fuzzy-ART [15, 16]. The generalization comes from the use of a general measure of inclusion and definition of the vigilance parameter in terms of the diagonal of the generalized object. The general data structure that allows the modelling of several classical structures is called Fuzzy Interval Number [66, 67]. It allows the manipulation of rather different data objects in a common lattice framework. These ideas have matured to propose lattice computing as a framework for new inference systems [68] and for new versions of well known algorithms, such as the grSOM (granular SOM) [21], the grARMA (granular ARMA) [20], the unification of SOM and ART algorithms [69]. The authors also propose FLNMAP as the generalization of the supervised Fuzzy ARTMAP [70].

The works on Fuzzy Lattice Neurocomputing [1, 64-66, 68, 71-75] had a lot of applications in classification and prediction: [76-78] bone drilling for epidural anaesthesia, [79] text classification, [75, 80] air quality monitoring. The FIN structure is a promising way to produce new generalized algorithms able to deal with heterogeneous data structures, different from the conventional Euclidean spaces.

IV CONCLUSIONS

The aim of this paper was to give a glimpse of the computational field composed of algorithms that employ in any way *sup* and *inf* operators and can be, therefore, put into the framework of lattice theory. The focus on lattices reveals the existence of a profound parallelism between areas as divergent as Fuzzy Systems, Mathematical Morphology, Min-max Algebra and Artificial Neural Networks.

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