

# Linked Multicomponent Robotic Systems: Basic Assessment of Linking Element Dynamical Effect

Borja Fernandez Gauna, Jose M. Lopez-Guede, Manuel Graña, Ekaitz Zulueta

Computational Intelligence Group, Universidad del Pais Vasco  
www.ehu.es/ccwintco

**Abstract.** The Linked Multicomponent Robotic Systems are characterized by the existence of a non-rigid linking element. This linking element can produce many dynamical effects that introduce perturbations of the basic system behavior, different from uncoupled systems. We show through a simulation of a distributed control of a hose transportation system, that even a minimal dynamical feature of the hose (elastic forces opposing stretching) can produce significant behavior perturbations.

## 1 Introduction

Controlling wheeled mobile robots to follow a predefined path is a well-known problem that has been approached in a broad set of ways: smoothed bang-bang controllers [5], PID and adaptive controllers [3], fuzzy controllers [9,6], tracking-error model-based predictive controllers [4], or through dynamic feedback linearization [8]. Some authors have taken the path following problem one step further to that of keeping multi-robot formations along the path [11], but very little literature exists nowadays about the constraints imposed in physically linked multi-robot systems [12,2]. We assume as the cooperative control paradigm the works of [10], although for lack of space we will not be able to detail here our formulation of the distributed controller being simulated.

A Linked Multicomponent Robotic System (L-MCRS)[1] is a collection of robotic units coupled through a passive non-rigid element, such as a hose or a cable. It is our working hypothesis that this passive connection imposes dynamic constraints to the robot dynamics and transmits non-linear dynamical forces among robots. That means that definite effects can be observed that differentiate the L-MCRS from a collection of uncoupled robotic units. We assume as the problem paradigm the transportation of a hose. The basic question is: does the hose introduce any perturbation on the behavior of the system under the command of (distributed) controller? To show that the definitive answer is yes, we introduce the simplest dynamical effect of the hose: it behaves as a spring when it is stretched longer than its nominal length. Simulations show this effect clearly.

This paper is organized as follows: in Section 2 we give the system definition, including the dynamical description and the two performance measures of the individual and overall system behavior, that will quantify the observed effects.

Section 3 gives some hints about the definition of the distributed control, which can not be given in full detail here. Section 4 shows the results of the conducted simulation experiments. Finally, some conclusions are given in Section 5.

## 2 System definition

Consider the problem of an elastic hose fixed to some wheeled mobile robots that are moving following a known path so that there are  $L$  meters long hose segments between every pair of consecutive robots. To avoid the hose interfering the robots' motion, a  $L$  meters Euclidian distance between consecutive pairs of robots is desired. Our simulation will focus on the effects caused by elastic traction forces between robots that appear when the distance between consecutive robot units is larger than  $L$ , and how the whole system behavior is affected.

### 2.1 Definitions and restrictions

The path followed by the robot units will be defined as a function of the travelled distance  $s$ :

$$\mathbf{H}(s) = (h^x(s), h^y(s))$$

The usual path tracking approach will be assumed: each robot will use a reference or virtual robot along the path and its controller will try to minimize the error between its position and the reference. The  $i^{\text{th}}$  robot's bi-dimensional position will be denoted as  $\mathbf{P}_i \equiv [P_i^x \ P_i^y]$ , and the position of its reference will be derived from its position along the path ( $s_i$ ) using the previously defined function:  $\mathbf{H}(s_i)$ .

We can then define  $d(s, L)$  as the function that returns the minimum value greater than  $s$  that fullfills  $|\mathbf{H}(s) - \mathbf{H}(d(s, L))| = L$ . That is, the index along the path for the nearest point ahead that is  $L$  distant from  $\mathbf{H}(s)$ .

For a predefined path to be travelled keeping a fixed distance  $L$  between a pair of robots, a sufficient condition can be set:  $d(s, L)$  must be a continuous monotonically increasing function defined for every value of  $s$ .

For generalization purposes, we further define in a recursive manner:

$$e(s, L, i) \begin{cases} s & i = 0 \\ e(d(s, L), L, i - 1) & \text{else} \end{cases} \quad (1)$$

### 2.2 Dynamical Model

The most basic model of the effect of the linking element is to introduce an elastic traction force due to hose stretching. The robots are assumed to be powerful enough so that any other hose-related forces can be neglected. The elastic force acts when Euclidean distance among robots grows bigger than the nominal linking element size.

For the basic motion of the robots, a simple dynamic model can be used:  $\dot{V}_i^y = \frac{F_i^y}{m}$  and  $\dot{V}_i^x = \frac{F_i^x}{m}$ , where  $m$  is the mass of the robots,  $V_i^x$  and  $V_i^y$  represent the components of the the  $i^{\text{th}}$  vehicle's velocity vector,  $F_i^x$  and  $F_i^y$  represent the components of the force vector applied on the  $i^{\text{th}}$  vehicle in the  $x$  and  $y$  axis by its locomotor system. The elastic traction force  $T_i$  between  $i^{\text{th}}$  and  $(i+1)^{\text{th}}$  robots are modelled as a clamped spring neglecting compression forces. Therefore, the linking element has no effect if the Euclidean distance between two robots is less than  $L$ :

$$\begin{aligned} T_i^x &= K \cdot \max(0, |\mathbf{P}_i - \mathbf{P}_{i+1}| - L) \cdot \cos(\beta_i), \\ T_i^y &= K \cdot \max(0, |\mathbf{P}_i - \mathbf{P}_{i+1}| - L) \cdot \sin(\beta_i), \end{aligned}$$

where  $K$  is the spring constant and  $\beta_i$  is the angle of the segment connecting the  $i^{\text{th}}$  and  $(i+1)^{\text{th}}$  robots relative to the  $x$ -axis. Including these terms into our dynamic model, we obtain:

$$\begin{aligned} \dot{V}_i^x &= \frac{F_i^x - T_{i-1}^x + T_i^x}{m}, \\ \dot{V}_i^y &= \frac{F_i^y - T_{i-1}^y + T_i^y}{m}. \end{aligned} \quad (2)$$

The expected position for the  $i^{\text{th}}$  robot can then be calculated as:

$$\begin{aligned} \dot{P}_i^x &= V_i^x, \\ \dot{P}_i^y &= V_i^y. \end{aligned} \quad (3)$$

### 2.3 System performance measures

We need to define a system performance measurement-function so that we can compare the performance of the system with and without the hose dynamics. Two functions have been used to measure individual error:

Mean square Euclidian distance error between robots ( $e_i^{dis}$ ):

$$e_i^{dis} = \frac{\int_0^t (|\mathbf{P}_i - \mathbf{P}_{i+1}| - L)^2}{t}$$

Mean square Euclidian distance error between robots and their desired position ( $e_i^{pos}$ ):

$$e_i^{pos} = \frac{\int_0^t (|\mathbf{P}_i - \mathbf{H}(\mathbf{s}_i)|)^2}{t}$$

Using these two functions, system error has been measured as the sum of the mean square deviations:  $e^{dis} = \sum_{i=0}^{n-2} e_i^{dis}$  and  $e^{pos} = \sum_{i=0}^{n-1} e_i^{ref}$ .

### 3 Consensus-Based Control Approach

In [10] two basic consensus-based methodologies are described to approach distributed multi-vehicle cooperation problems: with and without an optimization objective. In the first case, an objective is desired to be optimally achieved while in the second, only cooperation among the individuals is desired. The first approach has been applied on the reference level, so that the references to be followed by the robots are controlled in a cooperative way. The essence of the methodology can be summarized in four steps: (a) defining the cooperation objective and constraints, (b) defining the coordination variables and coordination functions [7], (c) designing a centralized cooperation scheme and (d) building a consensus-based distributed cooperation scheme.

Two ways to represent the system state are proposed in [10]: as a group-level reference state or as individual local vehicle states. The first implies that individual control decisions can be derived from a group-level set of variables, while the second assumes that each individual acts according to the states of its neighbors. For the purposes of this work, the former can be used, denoting as  $\xi$  the base-position of the robot formation, this is, the desired position of the last robot. Using the previously defined function 1, the desired position for each robot can be expressed as  $e(\xi, L, i)$ , where  $i$  is the zero-based index of a robot in the formation.

Due to lack of space we can not describe in detail the cooperative control implemented in the simulation. It has been defined so as to minimize an objective function  $J_{obj} = \sum_{i=0}^{n-1} J_{cf,i}$ , defined in terms of local objective functions at each robot  $J_{cf,i} = \int_0^t (s_i - e(\xi, L, i))^2 dt$ , subject to the constraint  $J_{const} = \sum_{i=0}^{n-2} |[H(s_i) - H(e(\xi, L, i))] - [H(s_{i+1}) - H(e(\xi, L, i + 1))]|$ . The minimization leads to a distributed control where each local control rule takes the form of a PID controller.

### 4 Simulation experimental results

Our goal is to measure the individual performance impact on linked MCRSs performance in comparison to non-linked MCRSs and, for that purpose, each of the simulated robots was assigned a maximum force output  $F_i^{max}$  and its output force vector was then clamped so that the individual performance could be individually affected. An experiment was conducted with 5 robots travelling along the path represented in figure 1 keeping a fixed separation of  $L = 0.2m$  between every consecutive pairs. The system was first simulated including the hose dynamic model ( $K^s = 40Nm$ ) and then without it ( $K^s = 0Nm$ ), so the performance impact due to the physical link could be quantified. In the simulation experiment the last robot was made the weakest.  $F_0^{max} = 2.5N$  and  $F_1^{max} = F_2^{max} = F_3^{max} = F_4^{max} = 5N$ . The results are presented in table 1 and figure 2.

Table 1 shows that the poor individual performance of the last robot ( $i = 0$ ) doesn't affect the individual performance of the remaining robot units when no elastic force due to the physical link is present ( $K^s = 0$ ), and  $e_i^{dis}$  and  $e_i^{pos}$  remains constant ( $e_i^{dis} = 0.000$  and  $e_i^{pos} \simeq 0.002$  for all  $i = 1, \dots, 4$ ).

When there is some elastic force due to hose stretching ( $K^s = 40$ ),  $e_0^{dis}$  drops to zero as could be expected because the model used avoids separations between robots bigger than  $L$ . As the last robot moves slower than the rest and the dynamic model tries to keep distance between robots under  $L$ , the rest of the robots are forced to go slower so that the maximum segment length is respected, and that makes them unable to follow their references without the physic link. The system error  $e^{pos}$  grows from 0.0111 to 0.0216, which implies a 95% error growth due to the hose dynamics.

Figures 2a and 2c represent the Reference-Position error ( $|P_i - P_{i+1}|$ ) for  $K^s = 0$  and  $K^s = 40$  respectively. The traction effect can be clearly seen if both figures are compared: while the former shows that the error for the last does not influence the rest, the latter shows how error between references and robots is spread. Nearest neighbors show the poorest individual performance. The Position-Reference distance oscillates as the references go faster at the curves to compensate the change in the growth of the euclidian distance between robots, and oscillations get closer to zero as time goes on, due to the Integrative component of the Proportional-Integrative controller used. Observing figs. 2b and 2d, one can easily see how the hose propagates trough the whole system the local perturbations of the behavior of robot units.

## 5 Conclusions

The basic question addressed in this paper is: there is any definite effect due to the existence of a flexible linking element between robot units? To give an answer we have introduced one of the simplest models: an elastic hose that introduces an elastic traction force when stretched longer than its nominal value. Otherwise the hose does not have any effect. Through a simulation of a collection of robots following a given path we observe that even this simplified model of the hose dynamics can introduce significant perturbations in the system's behavior. It is our conclusion that further work must be devoted to this kind of systems to fully explore their behaviors, and to prepare the grounds for its exploitation in some applications and complex environments.

## References

1. R.J. Duro, M. Graña, and J. de Lope. On the potential contributions of hybrid intelligent approaches to multicomponen robotic system development. *Information Sciences*, in pres.
2. Zelmar Echegoyen-Ferreira. *Contributions to Visual Servoing for Legged and Linked Multicomponent Robots*. PhD thesis, UPV/EHU, 2009.
3. L. Huang. Speed control of differentially driven wheeled mobile robots: Model-based adaptive approach. *Journal of Robotic Systems*, 22(6):323–332, 2005.

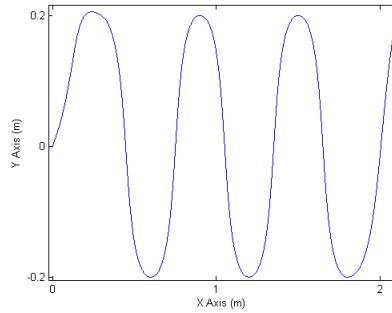
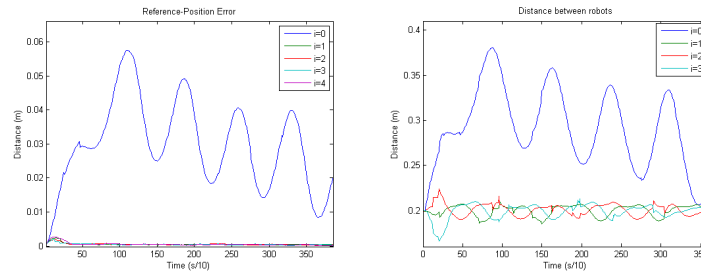
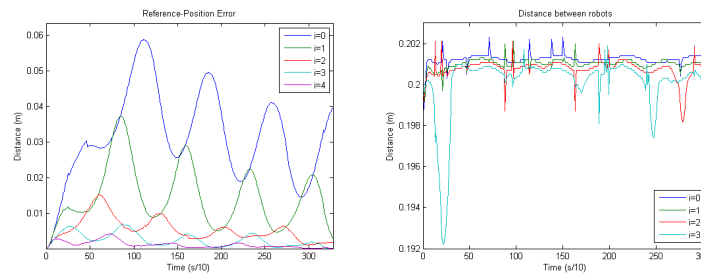


Fig. 1: Nominal path followed by the robot units



(a) Reference position error with- (b) Error on the distance between  
out linking element elastic force robots with no linking element elastic force  $K = 0$



(c) Position error when there is (d) Error in the distance between  
linking element elastic force in- robots when there is a linking element elastic force  $K = 40N$

Fig. 2: Errors in the system with and without linking element elastic force.

Table 1: Simulation experiment results

		Local robot performances					Global	
		$i$	0	1	2	3	4	System performance
$K = 0$	$e_i^{dis}$	0.0102	0.0000	0.0000	0.0000			$e^{dis}$ 0.0104
	$e_i^{pos}$	0.0103	0.0002	0.0002	0.0001	0.0002		$e^{pos}$ 0.0111
$K = 40$	$e_i^{dis}$	0.0000	0.0000	0.0000	0.0000			$e^{dis}$ 0.0000
	$e_i^{pos}$	0.0100	0.0069	0.0025	0.0015	0.0007		$e^{pos}$ 0.0216

4. Gregor Klancar and Igor Skrjanc. Tracking-error model-based predictive control for mobile robots in real time. *Robotics and Autonomous Systems*, 55:460–469, 2007.
5. K. C. Koh and H. S. Cho. A smooth path tracking algorithm for wheeled mobile robots with dynamic constraints. *Journal of Intelligent and Robotic Systems*, 24:367–385, 1999.
6. Ning Liu. Intelligent path following method for nonholonomic robot using fuzzy control. In *Second International Conference on Intelligent Networks and Intelligent Systems*, 2009.
7. Timothy W. McLain and Randal W. Beard. Coordination variables, coordination functions, and cooperative timing missions. *AIAA Journal of Guidance, Control, & Dynamics*, 28(1):150–161, 2005.
8. Giuseppe Oriolo, Alessandro De Luca, and Marilena Vendittelli. Wmr control via dynamic feedback linearization: Design, implementation, and experimental validation. *IEEE Trans. Control Systems Technology*, 10(6):835–852, November 2002.
9. Francesco M. Raimondi and Maurizio Melluso. A new fuzzy robust dynamic controller for autonomous vehicles with nonholonomic constraints. *Robotics and Autonomous Systems*, 52:115–131, 2005.
10. Wei Ren and Randal W. Beard. *Distributed Consensus in Multi-Vehicle Cooperative Control: Theory and Applications*. Springer Verlag, 2007.
11. Liu Shi-Cai, Tan Da-Long, and Liu Guang-Jun. Formation control of mobile robots with active obstacle avoidance. *Acta Automatica Sinica*, 33(5):529–535, 2007.
12. Iván Villaverde De La Nava. *On Computational Intelligence Tools for Vision Based Navigation of Mobile Robots*. PhD thesis, UPV/EHU, 2009.