

An empirical study of using Rotation Forest to improve regressors

Ana I. González Acuña

by Zhang, C-X., Zhang, J-S., Wang, G-W. in *Applied Mathematics and Computation* (2008)

27/01/2012

Outline

- 1 Introduction
- 2 Rotation Forest regressor ensemble method
- 3 Experimental studies
- 4 Conclusions

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Introduction

- Ensemble or committee machines: a collection of base predictors.
- Construction:
Base learning algorithm over different distributions of the training data
+ combination of the predictions from each ensemble member.
- Techniques for generating ensemble machine:
 - Bagging (bootstrap aggregation)
 - Boosting
- Base learning algorithms:
 - Neural networks
 - Decision trees

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Rotation Forest to improve regressors

└ Introduction

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1. Bagging and Boosting are generally much more accurate than their constituent members when the base learning algorithm is taken to be an instable algorithm. Here an instable learning algorithm refers to that small permutations in its training data or in construction can lead to large changes in the constructed predictor.
2. A classifier is accurate if is better than random guessing. Two predictors are diverse if they make different errors. Intuitively, an ensemble will perform better than the base predictors if the errors in the base predictors are uncorrelated and tend to cancel each other out. Our predictors are all obviously accurate, but are they diverse? To test this we can measure the correlation between the errors made by the different predictors. If they are uncorrelated, then it is likely that we can construct an ensemble with improved performance.

Boosting and Bagging

- (=) Both combine the outputs from different predictors
- (\neq) Permutation of training data:
 - Bagging takes different bootstrap samples from the original training set and trains a predictor on each sample to build its constituent members, which can be generated in parallel.
 - Boosting is a sequential algorithm, initially, a base predictor is constructed by applying the base learning algorithm to the training data set with equal weights assigned to each training instance. In the subsequent iterations, the training data with weights updated according to the performance of the previously built base predictors are provided as the input of the base learning algorithm.

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Rotation Forest to improve regressors

Introduction

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Boosting and Bagging

- (\neq) Combination of base predictors:
 - Bagging the final decision is constructed as combining the predictions of each base predictor with equal weights
 - Boosting the final decision is formed by a weighted voting scheme: the weight of each base predictor is determined by its performance on the training set used to build it.

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1. The boosting algorithm was originally developed for solving binary classification problems and Freund and Schapire extended it to a multi-class case, which they called Adaboost.M1 and Adaboost.M2.

Ensemble methods in regression

- Adaboost.R: reduces regression problems to the corresponding classification ones.
- Random Forest
- Adaboost.R2, Adaboost.RT
- ...

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Rotation Forest to improve regressors

└ Introduction

└ Ensemble methods for solving regression problems

└ Ensemble methods in regression

Ensemble methods in regression

- Adaboost.R: reduces regression problems to the corresponding classification ones.
- Random Forest
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- ...

1. there is not a single boosting algorithm that has been found to be a clear winner in solving regression problems.

Rotation forest for regression

Proposition: Rotation Forest for regression.

- Benchmarks regression data sets.
- Comparison with: Bagging, Random Forest and Adaboost.R2, and a single regression tree.
- Study of the sensitivity of Rotation Forest to the choice of parameters

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Rotation Forest to improve regressors

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Results

- Pruning has some bad effect on the performance of all the considered methods.
- Rotation Forest:
 - Number of attributes in each subset : some influence
 - Ensemble size (not too small): trivial
- Adaboost.R2 generally outperforms Rotation Forest and both of them are better than Random Forest and a single tree. There is not a clear winner between Bagging and Rotation Forest.

Notation

- Training set of N labeled instances: $\mathcal{L} = \{(\mathbf{x}_i, y_i)\}_{i=1}^N = [X \ Y]$
- Each instance (\mathbf{x}_i, y_i) , $\mathbf{x} \in \mathbb{R}^n$ and $y \in \mathbb{R}$
- Regressors in the ensemble machine : C_1, C_2, \dots, C_T
- Number of base regressors: T
- Attribute set : $F = (X_1, X_2, \dots, X_n)^T$
- Number of subsets that the attribute set F should be split into: K

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Rotation Forest to improve regressors
└─ Rotation Forest regressor ensemble method
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Notation

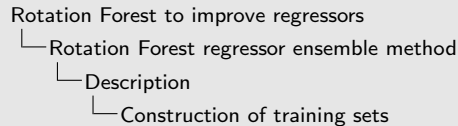
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1. each instance is described by n input attributes and an output attribute

Construction of training sets

1. Randomly split F into K subsets $F_{i,j}$ ($j = 1, \dots, K$).
2. For $j = 1, 2, \dots, K$
 - (a) Select the columns of X that correspond to the attributes in $F_{i,j}$ to compose a new matrix $X_{i,j}$.
 - (b) Draw a bootstrap sample $X'_{i,j}$ (with sample size smaller than that of $X_{i,j}$) from $X_{i,j}$.
 - (c) Apply PCA on $X'_{i,j}$ to obtain a matrix $D_{i,j}$ whose k th column consists of the coefficients of the k th principal component.
3. EndFor
4. Arrange the matrices $D_{i,j}$ ($j = 1, 2, \dots, K$) into a block diagonal matrix R_i .
5. Construct the rotation matrix R_i^a by rearranging the rows of R_i in order to match the order of attributes in F .

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1. Each regressor C_i is built by applying a given base learning algorithm to different training sets.
2. A block diagonal matrix is a block matrix which is a square matrix, and having main diagonal blocks square matrices, such that the off-diagonal blocks are zero matrices.

Pseudocode of RF ensemble method

Training Phase

Given

$\mathcal{L} = \{(\mathbf{x}_i, y_i)\}_{i=1}^N = [X Y]$ where X is an $N \times n$ matrix containing the input attribute values and Y is an N -dimensional column vector containing the outputs of each training instance.

- T : number of regressors that constitute the ensemble.
- K : number of attribute subsets (or M : number of input attributes contained in each subset).
- \mathcal{W} : a base learning algorithm.

For $i = 1, 2, \dots, T$

- Calculate the rotation matrix R_i^a for the i th regressor C_i
- Provide $[X R_i^a Y]$ as the input of \mathcal{W} to build a regressor C_i .

EndFor

Predicting Phase

- For a given data point \mathbf{x} , let $C_i(\mathbf{x}R_i^a)$ be the value predicted by the regressor C_i , then the prediction of \mathbf{x} can be calculated as

$$C^*(\mathbf{x}) = \frac{1}{T} \sum_{i=1}^T C_i(\mathbf{x}R_i^a).$$

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Rotation Forest to improve regressors

Rotation Forest regressor ensemble method

Description

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•  $T$ : number of regressors that constitute the ensemble.
•  $K$ : number of attribute subsets (or  $M$ : number of input attributes contained in each subset).
•  $\mathcal{W}$ : a base learning algorithm.
For  $i = 1, 2, \dots, T$ 
• Calculate the rotation matrix  $R_i^a$  for the  $i$ th regressor  $C_i$ .
• Provide  $[X R_i^a Y]$  as the input of  $\mathcal{W}$  to build a regressor  $C_i$ .
EndFor
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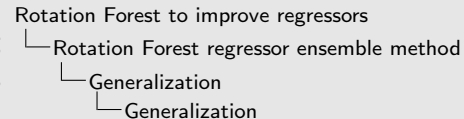
    
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Generalization

To achieve better generalization ability for an ensemble machine than a single predictor, it is critical that the ensemble machine consists of highly accurate members while at the same time disagree as much as possible.

- **Accuracy:** all the computed principal components are kept and the whole training set transformed through multiplying the rotation matrix is used to train each regressor
- **Diversity:** PCA is only applied on a subset of the training data set $X_{i,j}^l$ to obtain different principal component coefficients.

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Description

- Base learning algorithm: **single regression tree**
- Pruned and non-pruned regression trees
- Comparison with Bagging, Adaboost.R2, and Random Forest (also with base learning alg.)
- Implementation: *Stats* package in Matlab software (v7.1)

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Experimental studies

Description and Datasets

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Datasets

Table 1
 Summary of the used data sets

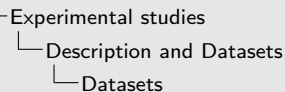
Data set	# Train	# Prune	# Test	# Attribute	
				Continuous	Discrete
Friedman #1	200	40	5000	10	0
Friedman #2	200	40	5000	4	0
Friedman #3	200	40	5000	4	0
Boston Housing	401	80	25	12	1
Servo	133	16	18	0	4

The Boston Housing and Servo data are available from the UCI repository
 The first three out of these data sets are synthetic Friedman_datasets

To convert discrete attributes into binary ones, each categorical attribute was replaced by s binary ones encoded numerically as 0 and 1, where s is the number of possible categories of the original attribute. Thus, Servo data set finally has 19 input attributes.

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1. Boston, igual reparto que el papel de Drucker. Servo, 80%+10%+10%
2. Without pruning, training =training+pruning

Effect of parameter M

Empirical analysis for each data set:

- Number of input attributes in each subset, $M = \{1 : 1 : n\}$
- Let $f = 0.75$ the ratio of the sample size of $X'_{i,j}$ to that of $X_{i,j}$
- Pruned and non-pruned regression tree as base learning algorithm
- Number of base predictors, $T = 50$ (for each value of M)
- Performance of RF: RMSE on the test dataset averaged over 100 trials
 - randomly generating training, pruning and testing instances for three synthetic sets, and
 - randomly split the original data set into three sets for training, pruning and testing for the two real-world datasets.

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Experimental studies

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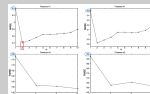
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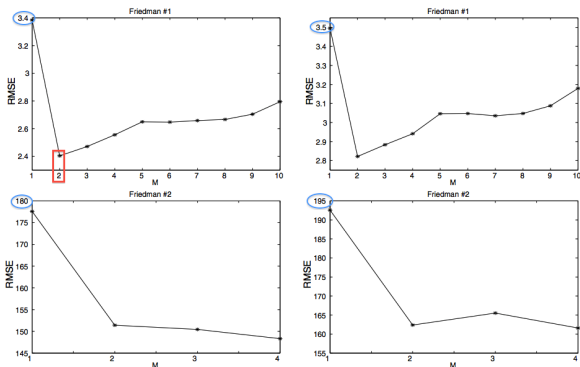
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1. In fact, we have run some extra experiments to see how the performance of Rotation Forest varies with the value of f and found that the variation is little.
2. RMSE: Root of mean squared error
3. For each value of M , the number of base predictors, namely T , was set to be 50 to construct Rotation Forest, and then the test RMSE was computed.
4. For each combination of the data set, the value of M and the base learning algorithm, the test RMSE was averaged over 100 trials



The averaged test RMSE versus the value of parameter M when using non-pruned trees (left plots) and pruned trees (right plots) to construct Rotation Forest.

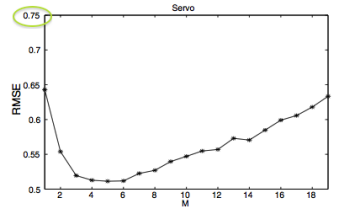
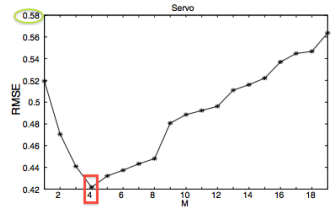
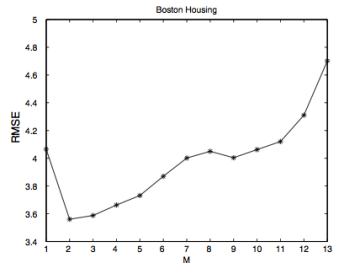
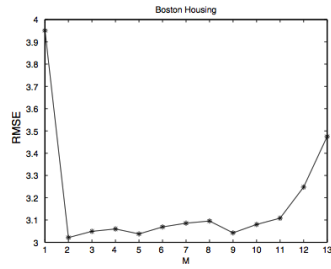
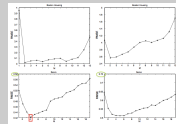


1. the averaged test RMSE decreases firstly, then it gradually exhibits a minimum and eventually rises as the value of M grows except for Friedman #2 and Friedman #3 data sets on which the test RMSE decreases monotonically with M.
2. comparing the left and right plots, respectively a non-pruned regression tree and a pruned regression tree as the base learning algorithm, we can find that there is not much difference between them: the **shape** of the plots is very similar, but ATTENTION the scales labeled on the vertical axes are different.
3. The performance of Rotation Forest can reach asymptotically optimal with $M = 2$ for the first four data sets and $M = 4$ for Servo data set. These values are fixed.

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Rotation Forest to improve regressors

- Experimental studies
 - Effect of parameter M



Comparison with other methods

Comparison with: Bagging, Random Forest and Adaboost.R2, and a single regression tree.

- 1 Dependence of the performance on the number of base regressors
- 2 How well the ensemble methods perform on the given datasets

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└─ Experimental studies

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Dependence on the number of base regressors

Comparison with: Bagging, Random Forest and Adaboost.R2, and a single regression tree.

- 50 non-pruned and pruned regression trees to construct each ensemble
- the test RMSE was registered at every time that a tree was added into each ensemble
- 100 runs through randomly generating or splitting the experimental data

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Rotation Forest to improve regressors

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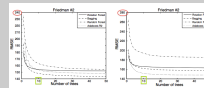
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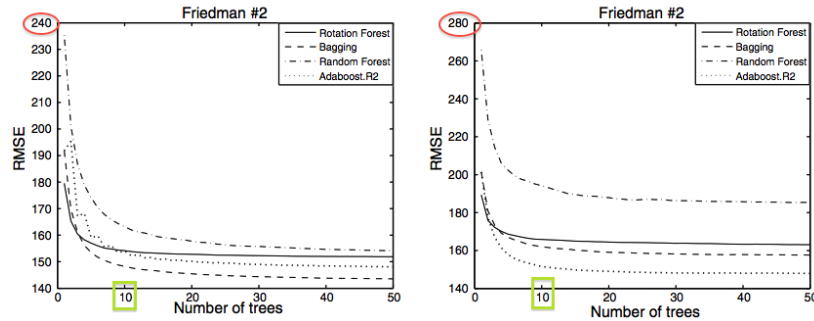
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- └ Experimental studies
 - └ Comparison with other methods

The dependence of the test RMSE averaged over 100 runs on the number of non-pruned trees (left plots) and pruned trees (right plots) that were used to construct the ensembles.



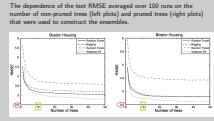
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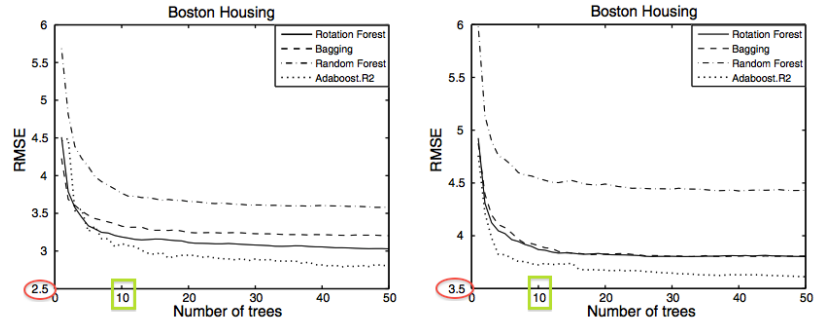
1. All ensembles are similar in their behavior, i.e., the test RMSEs of them decrease monotonically as the number of base regressors in the ensemble grows.
2. Rotation Forest generally performs worse than Adaboost.R2
3. Rotation Forest is better than Random Forest
4. Not a clear winner between Bagging and Rotation Forest
5. Pruning the tree seems to have some undue effect on the performance of the ensemble methods because the scales labeled on the vertical axes of the right plots are larger than those of left plots.
6. The performance of all ensemble methods begins to level off when the value of T lies in the vicinity of $T = 10$

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Rotation Forest to improve regressors
└ Experimental studies
└ Comparison with other methods



The dependence of the test RMSE averaged over 100 runs on the number of non-pruned trees (left plots) and pruned trees (right plots) that were used to construct the ensembles.



Performance of ensemble methods

- For each combination of the data set, the base learning algorithm and the ensemble construction method, we took 10 regression trees to construct an ensemble
- Evaluation with RMSE computed on the test set
- Three synthetic data sets: 100 times through randomly generating data in three sets used for training, pruning and testing.
- Boston Housing and Servo data sets: 100 times through randomly splitting the data into three sets used for training, pruning and testing.
- Calculate average and standard deviation of these 100 test RMSEs.

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1. As for Adaboost.R2, the results computed with linear, square and exponential loss functions were all listed here for a complete comparison.

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Rotation Forest to improve regressors

Experimental studies

Comparison with other methods

Table 2
 Comparison of performance of different methods computed with non-pruned regression trees

Data set	Rotation Forest	Single tree	Bagging	Random Forest	Adaboost.R2		
					Linear	Square	Exponential
Friedman #1	2.547 ± 0.107	3.396 ± 0.122	• 2.574 ± 0.081	• 3.063 ± 0.144	• 2.647 ± 0.081	• 2.675 ± 0.083	• 2.651 ± 0.079
Friedman #2 (×10 ²)	1.523 ± 0.046	1.786 ± 0.077	• 1.486 ± 0.036	• 1.628 ± 0.067	• 1.551 ± 0.046	• 1.567 ± 0.051	• 1.544 ± 0.047
Friedman #3	0.167 ± 0.009	0.201 ± 0.010	• 0.163 ± 0.008	○ 0.168 ± 0.008	○ 0.164 ± 0.009	○ 0.164 ± 0.009	○ 0.164 ± 0.008
Boston Housing	3.115 ± 0.851	4.205 ± 1.473	• 3.225 ± 0.931	• 3.443 ± 0.973	• 3.121 ± 0.960	• 3.095 ± 0.908	• 3.093 ± 0.948
Servo	0.464 ± 0.244	0.577 ± 0.372	• 0.537 ± 0.398	• 0.730 ± 0.276	• 0.366 ± 0.197	○ 0.398 ± 0.206	○ 0.365 ± 0.194

Legend:
 "•": Rotation Forest is significantly better.
 "○": Rotation Forest is significantly worse.
 Significance level α = 0.05.

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"○": Rotation Forest is significantly worse.
 "•": Rotation Forest is significantly better.
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one-tailed paired *t* test

1. Rotation Forest performs significantly better than a single tree and Random Forest except for Friedman #3 data set on which the difference between Rotation Forest and Random Forest is not significant when the base learning algorithm is a non-pruned tree.
2. When adopting a pruned tree as the base learning algorithm, Adaboost.R2 is seen to significantly outperform Rotation Forest in almost all cases

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Rotation Forest to improve regressors
 Experimental studies
 Comparison with other methods

Table 3
 Comparison of performance of different methods computed with pruned regression trees

Data set	Rotation Forest			Adaboost.R2		
	Single tree	Bagging	Random Forest	Linear	Square	Exponential
Friedman #1	2.928 ± 0.131	3.481 ± 0.143	2.931 ± 0.122	3.563 ± 0.149	2.876 ± 0.091	2.876 ± 0.114
Friedman #2	1.644 ± 0.062	1.935 ± 0.121	1.625 ± 0.082	1.948 ± 0.110	1.570 ± 0.053	1.586 ± 0.061
Friedman #3	0.180 ± 0.009	0.213 ± 0.012	0.177 ± 0.010	0.186 ± 0.011	0.174 ± 0.009	0.174 ± 0.010
Boston Housing	3.894 ± 0.978	4.311 ± 1.468	3.833 ± 1.058	4.982 ± 1.435	3.423 ± 0.732	3.494 ± 0.821
Servo	0.531 ± 0.258	0.690 ± 0.384	0.616 ± 0.334	0.817 ± 0.267	0.473 ± 0.226	0.477 ± 0.257

Legend:
 "o": Rotation Forest is significantly worse.
 "•": Rotation Forest is significantly better.
 Significance level $\alpha = 0.05$.

Table 3
 Comparison of performance of different methods computed with pruned regression trees

Data set	Rotation Forest			Adaboost.R2		
	Single tree	Bagging	Random Forest	Linear	Square	Exponential
Friedman #1	2.928 ± 0.131	3.481 ± 0.143	2.931 ± 0.122	3.563 ± 0.149	2.876 ± 0.091	2.876 ± 0.114
Friedman #2	1.644 ± 0.062	1.935 ± 0.121	1.625 ± 0.082	1.948 ± 0.110	1.570 ± 0.053	1.586 ± 0.061
Friedman #3	0.180 ± 0.009	0.213 ± 0.012	0.177 ± 0.010	0.186 ± 0.011	0.174 ± 0.009	0.174 ± 0.010
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- the scoring matrix gives the average relative performance (expressed in %) of one procedure over another procedure for the considered data sets.
- $SM_{i,j}$ the average performance of the i th method (labeled in row) over the j th method (labeled in column):

$$SM_{i,j} = \frac{1}{N} \sum_{k=1}^N \frac{RMSE_{k,i} - RMSE_{k,j}}{\max(RMSE_{k,i}, RMSE_{k,j})}$$

where N is the number of data sets

Another comparison: Scoring Matrix

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Table 4
 Scoring matrix for different methods with non-pruned tree (values are expressed in %)

Method	Single tree	Bagging	Random Forest	Adaboost.R2			Rotation Forest	Total
				Linear	Square	Exponential		
Single tree	0	-18.03	-6.45	-23.19	-21.86	-23.42	-20.43	-113.38
Bagging	18.03	0	12.09	-5.50	-4.07	-5.77	-2.65	12.13
Random Forest	6.45	-12.09	0	-15.98	-14.88	-16.23	-13.97	-66.70
Adaboost.R2 (Linear)	23.19	5.50	15.98	0	1.85	-0.29	3.43	49.67
Adaboost.R2 (Square)	21.86	4.07	14.88	-1.85	0	-2.14	1.81	38.63
Adaboost.R2 (Exponential)	23.42	5.77	16.23	0.29	2.14	0	3.71	51.57
Rotation Forest	20.43	2.65	13.97	-3.43	-1.81	-3.71	0	28.09

Table 5
 Scoring matrix for different methods with pruned tree (values are expressed in %)

Method	Single tree	Bagging	Random Forest	Adaboost.R2			Rotation Forest	Total
				Linear	Square	Exponential		
Single tree	0	-14.11	3.86	-21.32	-20.71	-21.62	-15.83	-89.72
Bagging	14.11	0	17.36	-8.17	-7.48	-8.50	-1.90	5.42
Random Forest	-3.86	-17.36	0	-23.71	-23.16	-23.94	-18.70	-110.73
Adaboost.R2 (Linear)	21.32	8.17	23.71	0	0.78	-0.45	6.53	60.05
Adaboost.R2 (Square)	20.71	7.48	23.16	-0.78	0	-1.22	5.82	55.17
Adaboost.R2 (Exponential)	21.62	8.50	23.94	0.45	1.22	0	6.91	62.64
Rotation Forest	15.83	1.90	18.70	-6.53	-5.82	-6.91	0	17.18

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Rotation Forest to improve regressors

- Experimental studies
- Comparison with other methods

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Adaboost.R2 (Linear)	23.19	5.50	15.98	0	1.85	-0.29	3.43
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- the methods ranked by scoring from highest to lowest are Adaboost.R2 (exponential), Adaboost.R2 (linear), Adaboost.R2 (square), Rotation Forest, Bagging, Random Forest and Single tree when the base learning algorithm is a non-pruned regression tree.
- Random Forest performs even worse than a single tree when a pruned tree is adopted as the base learning algorithm.

Conclusions

Results

- Pruning has some bad effect on the performance of all the considered methods.
- Rotation Forest:
 - Number of attributes in each subset : some influence
 - Ensemble size (not too small): trivial
- Adaboost.R2 generally outperforms Rotation Forest and both of them are better than Random Forest and a single tree. There is not a clear winner between Bagging and Rotation Forest.
- Further work: Rotation Forest with neural network as base learning algorithm

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Rotation Forest to improve regressors

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Rotation Forest to improve regressors

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