

## S29. Special functions, orthogonal polynomials and applications

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### Program:

1. Carlos Álvarez Fernández (Universidad Pontificia Comillas, Spain)  
*An algebraic approach to the calculation of Christoffel-Darboux formulas and recursion relations under a generalized Hankel symmetry*
2. Mirella Cappelletti Montano (University of Bari, Italy)  
*Asymptotic formulae for Bernstein-Schnabl operators associated to a Markov operator*
3. Roberto Cavoretto (University of Turin, Italy)  
*Numerical algorithms for multivariate approximation in convex domains*
4. Francesco Dell'Accio (University of Calabria, Italy)  
*Hermite-Birkhoff interpolation of scattered data by combined Shepard operators*
5. Filomena Di Tommaso (University of Calabria, Italy)  
*Multinode inverse distance methods for function approximation*
6. Edmundo Huertas Cejudo (Universidade de Coimbra, Portugal)  
*Algorithmic approach to the strong and ratio asymptotic expansions for Laguerre polynomials*
7. Erik Koelink (Radboud Universiteit, The Netherlands)  
*Matrix-valued Gegenbauer polynomials*
8. Vita Leonessa (University of Basilicata, Italy)  
*Approximation and shape preserving properties for Bernstein-Schnabl operators associated with Markov operators*

9. Mariarosa Mazza (University of Insubria, Italy)  
*On the constrained mock-Chebyshev least-squares*
10. Ester Pérez Sinusia (Universidad de Zaragoza, Spain)  
*Convergent and asymptotic expansions of solutions of second-order differential equations with a large parameter*
11. María Francisca Pérez Valero (Universidad Carlos III de Madrid, Spain)  
*On asymptotics of partial derivatives of diagonal Laguerre kernels and some applications to Sobolev orthogonal polynomials*
12. Dario Ramos López (Universidad de Almeria, Spain)  
*Comparison of two schemes for computing the diffraction integral of an optical system*
13. Vanesa Sánchez Canales (Universidad de Sevilla, Spain)  
*Discrete Rodrigues' Formulas for Orthogonal Matrix Polynomials Satisfying Second-Order Difference Equations*
14. Gabriele Santin (University of Padua, Italy)  
*Bases for Radial Basis Function approximation*

# An algebraic approach to the calculation of Christoffel-Darboux formulas and recursion relations under a generalized Hankel symmetry

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It is a classical result that orthogonal polynomials on the real line satisfy a three term recurrence and a Christoffel-Darboux formula. Both results can be considered as a consequence of the Hankel symmetry that is present in the moment matrix of the orthogonality measure. Both results have been extended to the case of the Szegő polynomials, to the case of multiple orthogonal polynomials on the real line [6], and to the orthogonal Laurent polynomials on the unit circle [5]. As both recursion relations and Christoffel-Darboux formulas are a consequence of the Hankel symmetry, it is possible to use that symmetry to revisit the known cases under different hypotheses. In addition it can be applied to a generalized (multigraded) Hankel symmetry that is a generalization of the Adler-van Moerbeke polynomials.

This communication is based on the results in [2, 3, 4], a joint work with Manuel Mañas and Ulises Fidalgo.

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# Asymptotic formulae for Bernstein-Schnabl operators associated with a Markov operator

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Let  $T$  be a Markov operator on  $C(K)$ ,  $K$  being a convex compact subset of  $\mathbf{R}^d$ ,  $d \geq 1$ ; under suitable assumptions on  $T$ , it is well defined a positive approximation process  $(B_n)_{n \geq 1}$  on  $C(K)$ , the so-called sequence of *Bernstein-Schnabl operators associated with  $T$* .

In this talk, we focus our attention on an asymptotic formula for the sequence  $(B_n)_{n \geq 1}$ . In particular, we prove that, for every  $u \in C^2(K)$ ,

$$\lim_{n \rightarrow \infty} n(B_n(u) - u) = W_T(u),$$

where  $W_T$  is the second-order elliptic differential operator such that, for every  $u \in C^2(K)$ ,  $W_T(u) := \frac{1}{2} \sum_{i,j=1}^d (T(pr_i pr_j) - pr_i pr_j) \frac{\partial^2 u}{\partial x_i \partial x_j}$ .

Moreover, we show that  $W_T$  (pre-)generates a Markov semigroup on  $C(K)$ , which may be approximated by iterates of the  $B_n$ 's. (see [1, 2]).

Finally (see [2]), we present a modification of the sequence  $(B_n)_{n \geq 1}$ , in order to obtain an asymptotic formula (and generation results) for the second-order differential operator

$$V_T(u) = W_T(u) + \sum_{i=1}^d \beta_i \frac{\partial u}{\partial x_i} + \gamma u \quad (u \in C^2(K), \beta_1, \dots, \beta_d, \gamma \in C(K)).$$

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# Numerical algorithms for multivariate approximation in convex domains

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In this paper we present new numerical algorithms for interpolating multivariate scattered data in convex domains  $\mathcal{D} \subseteq \mathbb{R}^N$ , for any  $N \geq 2$ . In order to organize the (generally) large number of interpolation points contained in  $\mathcal{D}$ , we build space-partitioning data structures either using cell-based or  $kd$ -tree procedures [1, 2, 3]. These techniques allow us to efficiently apply a global partition-of-unity interpolant, which is here locally combined with radial basis function approximants and compactly supported weight functions [4, 5]. Complexity analysis and numerical results show efficiency of the implemented procedures and applicability of such algorithms in several real-life situations.

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# Hermite-Birkhoff interpolation of scattered data by combined Shepard operators

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Methods approaching the problem of the Hermite-Birkhoff interpolation of scattered data by combining Shepard operators with local interpolating polynomials are not new in literature [1, 2, 3, 4].

In [3] combinations of Shepard operators with bivariate Hermite-Birkhoff local interpolating polynomials are introduced to increase the algebraic degree of precision (polynomial reproduction degree) of Shepard operators. These polynomials are of total degree and must interpolate the Hermite-Birkhoff data in proper subsets of the data set, but the definition of such sets requires to fix some special ordering of the node set which assures the existence of the local interpolating polynomials.

In [1] the most general problem of Hermite-Birkhoff interpolation of scattered data is solved, but the local polynomial interpolant could have low algebraic degree of precision or even have not any algebraic degree of precision, for example in the case when in some node the value of the function is unknown. This lack badly affects the accuracy of the approximation of the combination in that cases.

The method here proposed, obtained by combining Shepard operators with three point Hermite-Birkhoff interpolation polynomials, is an attempt to overcome the weaknesses of the aforesaid methods. Methods [2, 4] are particular cases of this new general procedure.

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- [3] Coman, G., Shepard operators of Birkhoff-type, *Calcolo* **35** (1998), 197–203.
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# Multinode inverse distance methods for function approximation

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The Hermite interpolation problem consists of determining a continuous function, belonging to a given interpolation space, such that its values and successive derivatives at each interpolation node match certain given values, called data values. The Birkhoff interpolation problem [1] is a generalization of the Hermite one, which differs only for the fact that selected derivatives data may not be successive (such data are then called lacunary or irregular). Moreover, while an Hermite interpolation problem can be always solved, a Birkhoff interpolation problem is not always solvable even in the appropriate polynomial space. We propose a method that split up the initial problem in subproblems having a unique polynomial solution and use multinode rational basis functions in order to obtain a global interpolant.

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# Algorithmic approach to the strong and ratio asymptotic expansions for Laguerre polynomials

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In this work, we consider the strong asymptotic behavior of Laguerre polynomials in the complex plane. The leading behavior is well known from Perron and Mehler–Heine formulas, but higher order coefficients, which are important in the context of Krall–Laguerre, and Laguerre–Sobolev type orthogonal polynomials, are notoriously difficult to compute. Here, we propose the use of an alternative expansion, due to Buchholz, in terms of Bessel functions of the first kind. The coefficients in this expansion can be obtained in a straightforward way using symbolic computation (Wolfram Mathematica, MAPLE, etc). As an application, we derive extra terms in the asymptotic expansion of ratios of Laguerre polynomials in  $\mathbb{C} \setminus [0, \infty)$ .

This is a joint work with Alfredo Deaño and Francisco Marcellán ([1]).

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# Matrix-valued Gegenbauer polynomials

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Matrix-valued orthogonal polynomial analogues of the classical scalar valued Gegenbauer polynomials of parameter  $\nu$  are discussed for any size of the matrix. For these matrix-valued polynomials we have a number of very explicit properties, such as three-term recursion, expression in terms of matrix-valued hypergeometric functions, differential operators, etc. The matrix-valued Gegenbauer polynomials can be obtained from the matrix-valued Chebyshev polynomials, i.e. from the case  $\nu = 1$ , which originally were obtained from group representations. We focus on two aspects of the matrix-valued Gegenbauer polynomials, namely its expression in terms of a Rodrigues formula and its expression in terms of scalar-valued Gegenbauer and Racah polynomials.

The results are joint work with Ana M. de los Ríos and Pablo Román.

# Approximation and shape preserving properties for Bernstein-Schnabl operators associated with Markov operators

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Let  $K$  be a convex compact subset of some (not necessarily finite dimensional) locally convex Hausdorff space and fix an arbitrary Markov operator  $T$  on the space of all continuous functions on  $K$ .

The aim of this talk is to introduce a sequence of positive linear operators, called Bernstein-Schnabl operators, associated with  $T$  and study their approximation as well as shape preserving properties. Examples of such operators are the classical Bernstein operators on the unit interval, on the multidimensional hypercubes and on simplices.

All results are new ([2]) and generalize similar ones obtained when the generating operator is in particular a Markov projection (see [1, Chapter 6] and the references contained in the relevant notes).

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# On the constrained mock-Chebyshev least-squares

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The algebraic polynomial interpolation on uniformly distributed nodes is affected by the Runge phenomenon, also when the function to be interpolated is analytic. Among all techniques that have been proposed to defeat this phenomenon, there is the mock-Chebyshev interpolation which is an interpolation made on a subset of the given nodes whose elements mimic *as well as possible* the Chebyshev-Lobatto points [2]. In this work we use the constrained least-squares theory [1] to combine the previous technique with a polynomial regression, in order to increase the accuracy of the approximation of a given analytic function. By using a result contained in [3], we give indications on how to select the degree of the simultaneous regression in order to obtain polynomial approximant good in the uniform norm and provide a sufficient condition to improve, in that norm, the accuracy of the mock-Chebyshev interpolation with a simultaneous regression. Comparisons with some Radial Basis Functions, Hermite Function interpolation and Floater-Hormann barycentric interpolation are provided.

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# Convergent and asymptotic expansions of solutions of second-order differential equations with a large parameter

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We consider the second-order linear differential equation

$$y'' = \left[ \frac{\Lambda^2}{x^\alpha} + g(x) \right] y,$$

where  $x \in [0, X]$ ,  $X > 0$ ,  $\alpha \in (-\infty, 2]$ ,  $\Lambda$  is a large complex parameter and  $g$  is a continuous function. The asymptotic method designed by F. W. J. Olver [1] gives the Poincaré-type asymptotic expansion of two independent solutions of the equation in inverse powers of  $\Lambda$ . We add initial conditions to the differential equation and consider the corresponding initial value problem. By using the Green's function of an auxiliary problem, we transform the initial value problem into a Volterra integral equation of the second kind. Then, using a fixed point theorem, we construct a sequence of functions that converges to the unique solution of the problem. This sequence has also the property of being an asymptotic expansion for large  $\Lambda$  (not of Poincaré-type) of the solution of the problem. Moreover, we show that the idea may be applied also to nonlinear differential equations with a large parameter. A more detailed discussion and examples for the cases  $\alpha = -1, 0, 1, 2$  are presented.

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# On asymptotics of partial derivatives of diagonal Laguerre kernels and some applications to Sobolev orthogonal polynomials

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We will establish some new results corresponding to the asymptotic behavior of the partial derivatives of diagonal Laguerre kernels. As a consequence, we will obtain the outer and inner relative asymptotics for certain Laguerre-Sobolev type polynomials, i.e. we will compare the asymptotic behavior of the Laguerre-Sobolev polynomials and the classical ones both inside and outside  $[0, \infty)$ .

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# Comparison of two schemes for computing the diffraction integral of an optical system

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According to Fourier optics, given an optical system, the diffraction integral depending on a defocus parameter  $f$  (see equation below) transforms the complex pupil function (CPF)  $P(\rho, \theta)$  into the impulse-response function  $U(r, \phi; f)$ , which can be analyzed to determine the optical properties of the system along with metrics of image quality:

$$U(r, \phi; f) = \frac{1}{\pi} \int_0^1 \int_0^{2\pi} [\exp(iff\rho^2)P(\rho, \theta) \exp((2\pi i\rho r \cos(\theta - \phi)))] \rho d\theta d\rho$$

The evaluation of this integral may require computer-intensive methods, especially when the range of values for  $f$  is wide. A semi-analytical method consisting in the fit of the CPF by means of the Zernike orthogonal polynomials was proposed in [1] and improved in [2], yielding explicit formulas for function  $U$  once the Zernike representation of the CPF is known.

However, this technique presents some practical problems and limitations, due to its restriction to symmetrical systems. Thus, an alternative scheme has been developed in [3], by making use of the Gaussian radial basis functions instead of the Zernike polynomials.

In this talk, the description and comparison of these two procedures are carried out, showing the benefits of the new scheme for real applications.

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# Discrete Rodrigues' Formulas for Orthogonal Matrix Polynomials Satisfying Second-Order Difference Equations

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The first examples of orthogonal matrix polynomials which are eigenfunctions of a second order difference operator of the form

$$S_{-1}F_{-1}(t) + S_0F_0(x) + S_1F_1(x)$$

(with left eigenvalues and where  $S_l$  denotes the shift operator  $S_l(f) = f(x+l)$ ) have appeared in the last few years (see [2], [1], [3]).

These examples have been essentially found by solving the following set of commuting and difference equations

$$F_0W = WF_0^*, \quad F_1(x-1)W(x-1) = W(x)F_{-1}^*(x), \quad (1)$$

where  $W$  is the weight matrix.

In this talk, we discuss the existence of discrete Rodrigues' formulas for these families of orthogonal matrix polynomials, that is, assuming that  $W$  satisfies the commuting and difference equations (1) we show an efficient method to find functions  $R_n$  such that

$$P_n = \Delta^n(R_n)W^{-1}$$

is a sequence of orthogonal matrix polynomials with respect to  $W$ . The only condition we impose to the functions  $R_n$  is that they have to be simple enough to allow the explicit calculation of  $P_n$ .

Using this approach, we produce Rodrigues' formulas for a couple of illustrative examples.

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2. Durán, A. J., The algebra of difference operators associated to a family of orthogonal polynomials, *J. Approx. Theory* **164** (2012), 586–610.
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# Bases for Radial Basis Function Approximation.

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In the setting of Radial Basis Functions (RBF) approximation it is well known that several problems arise when dealing with large amount of (scattered) data or when some parameters are not properly chosen.

The recent work [3] gives a quite general way to build stable, orthonormal bases for the *native Hilbert space* of a RBF kernel  $K$  on a compact set  $\Omega \in \mathbb{R}^d$ .

Starting from that setting we described in [1] a particular orthonormal, discretely orthogonal basis that arises from a weighted singular value decomposition of the so called *kernel matrix*. This basis is related to a discretization of the compact operator  $T_K : L_2(\Omega) \rightarrow L_2(\Omega)$ ,

$$T_K[f](x) = \int_{\Omega} K(x, y)f(y)dy \quad \forall x \in \Omega,$$

and provides a connection with the continuous basis that arises from an eigen-decomposition of  $T_K$ .

Furthermore, this basis allows the extraction of a suitable truncated basis which preserves the approximation capability of the full one, while giving better results in terms of stability of the approximant. In this view, we will also present some new results (see [3]) which allow to construct in a fast way a suitable approximation of this reduced basis.

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