S3. Classical and Modern Algebraic Geometry

Organizers:

- Laura Costa (Universitat de Barcelona, Spain)
- Angelo Felice Lopez (Università degli Studi Roma Tre, Italy)
- Roberto Muoz (Universidad Rey Juan Carlos, Spain)
- Paollo Stellari (Università degli Studi di Milano, Italy)

Speakers:

1. Miguel Angel Barja (Universitat Politècnica de Catalunya, Spain)
   Generalized Clifford-Severi inequality and the geography of irregular varieties
2. Cinzia Casagrande (Università di Torino, Italy)
   Locally unsplit families of rational curves on Fano manifolds
3. Paollo Cascini (Imperial College London, United Kingdom)
   Birational geometry in positive characteristic
4. Daniele Faenzi (Université de Pau et des Pays de l'Adour, France)
   Surfaces of minimal degree tame and wild representation type
5. Oscar García-Prada (Instituto de Ciencias Matemáticas, Spain)
   Involutions on Higgs bundle moduli spaces
6. Martí Lahoz (Institut de Mathématiques de Jussieu, France)
   Stable aCM bundles on cubic fourfolds
7. Gianluca Pacienza (Université de Strasbourg et CNRS, France)
   Families of rational curves on holomorphic symplectic varieties
8. Rita Pardini (Università di Pisa, Italy)
   Stable Gorenstein surfaces with $K^2 = 1$
9. José Carlos Sierra (Instituto de Ciencias Matemáticas, Spain)
   Some results on multisecant lines of smooth codimension two subvarieties of projective spaces
10. Yukinobu Toda (Kavli IPMU, University of Tokyo, Japan)
    Flops and S-duality conjecture

11. Jaroslaw Wisniewski (University of Warsaw, Poland)
On 81 symplectic resolutions of a 4-dimensional quotient by a group of order 32

Generalized Clifford-Severi inequality and the geography of irregular varieties

Miguel A. Barja

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I will present recent developments on numerical inequalities of linear systems on irregular varieties, the so called Generalized Clifford-Severi inequality, which applies to any nef line bundle. It provides a lower bound for the volume of any line bundle, which is sharp in some important cases. Also, it is an useful tool for the study of the geography and classification of these varieties.

[1] Barja, M.A., The generalized Clifford-Severi inequality and the volume of irregular varieties, *Duke Math. J.*, to appear; http://arxiv.org/abs/1303.3045.

Locally unsplit families of rational curves on Fano manifolds

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Let X be a smooth complex projective variety of dimension $n \geq 3$. We assume that X is uniruled, and consider a covering family of rational curves in X, that is: an irreducible component V of $\mathrm{Hilb}(X)$ such that the general point of V corresponds to an integral rational curve in X. The family V is called *locally unsplit* if non-integral curves of the family cover a *proper* closed subset of X. A uniruled variety always contains such a family, and the anticanonical degree d_V of curves of the family satisfies $2 \leq d_V \leq n+1$.

Cho, Miyaoka and Shepherd-Barron have shown that if $d_V = n+1$, then $X \cong P^n$ and V is the family of lines. In the next case $d_V = n$, Miyaoka has shown that if $\rho_X = 1$ (where ρ_X is the Picard number) and X does contain covering families of rational curves of degree smaller than n, then X is isomorphic to a quadric.

We consider the case where $d_V = n$ and $\rho_X > 1$, and moreover we assume that X is *Fano*. Using techniques from birational geometry, we show that $\rho_X \leq 3$, and we completely classify the possible (X, V) when $\rho_X = 3$. In this classification, we obtain some locally unsplit families whose variety of minimal tangents at a general point is singular. These are the first known examples of such behaviour.

This is a joint work with Stéphane Druel (Institut Fourier, Grenoble).

[1] Casagrande, C., Druel, S., Locally unsplit families of rational curves of large anticanonical degree on Fano manifolds, http://arxiv.org/abs/1212.5083.

Birational geometry in positive characteristic

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Many of the results in the Minimal Model Program depend on Kodaira vanishing theorem and its generalizations. On the other hand, because of the failure of these tools in positive characteristic, many of these results are still open in this case. I will survey some recent progress in the study of birational geometry of projective varieties defined over an algebraically closed field of positive characteristic.

Surfaces of minimal degree tame and wild representation type

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The representation type of a smooth projective variety X in P^n is a measure of the complexity of X, based on the category of maximal Cohen-Macaulay (MCM) modules over the homogeneous coordinate algebra R of X, which we assume to be a Cohen-Macaulay ring. Varieties supporting only finitely many indecomposable MCM are classified (Eisenbud-Herzog): all of them have minimal degree $deg(X) = codim(X) + 1$. On the other hand, many varieties are known to support arbitrarily large families of indecomposable MCM (wild type). We will look at the case of surfaces of minimal degree and show that two of them are of "tame type", i.e. indecomposable MCM's of a given rank form finitely many irreducible families of dimension 1 at most. This, together with the elliptic curve, conjecturally achieves the classification of all non-wild smooth varieties of posittive dimension. We will also classify rigid MCM's on some surface of minimal degree and wild type.

Joint work with F. Malaspina.

Involutions on Higgs bundle moduli spaces

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We study involutions on Higgs bundle moduli spaces over a compact Riemann surface, and describe their fixed points. We then explain their relation with representations of the fundamental group of the surface, and comment on their relevance in the study of branes in the context of mirror symmetry and Langlands duality for Higgs bundles.

Stable aCM bundles on cubic fourfolds

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We study the geometry of some moduli spaces of stable aCM bundles on cubic fourfolds. More precisely, I will present a (component of a) moduli space of stable aCM bundles which is birational to the hyperkähler manifold Z constructed by Lehn–Lehn–Sorger–van Straten in [2]. This is a joint work with Emanuele Macrì and Paolo Stellari. I will use some techniques developed in [1].

[1] Martí Lahoz, Emanuele Macrì, Paolo Stellari, ACM bundles on cubic threefolds and fourfolds containing a plane, `http://arxiv.org/abs/1303.6998`.
[2] Christian Lehn, Manfred Lehn, Christoph Sorger, Duco van Straten, Twisted cubics on cubic fourfolds, `http://arxiv.org/abs/1305.0178`.

Families of rational curves on holomorphic symplectic varieties

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I will report on a joint work with F. Charles, in which we study families of rational curves on certain irreducible holomorphic symplectic varieties. In particular, we prove that projective holomorphic symplectic fourfolds of $K3^{[2]}$-type contain uniruled divisors and rationally connected lagrangian surfaces. I will also mention some applications to the study of Chow groups of such varieties.

Stable Gorenstein surfaces with $K^2 = 1$

Rita Pardini (with M. Franciosi and S. Rollenske)

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The notion of stable surface generalizes the definition of (canonical model) of surface general type in the same way as the notion of stable curve generalizes the notion of smooth curve of genus $g > 1$: if one fixes the numerical invariants (the genus g in the case of curves, K^2 and $\chi(\mathcal{O})$ for surfaces), then the moduli space of the stable objects exists as a projective scheme and contains the usual moduli space as an open subset.

A possible approach to the study of stable surfaces, proposed by Kollár, consists in viewing them as obtained from a log-canonical pair $(\bar{X}, \bar{D})$ by glueing the normal surface $\bar{X}$ to itself via an involution of the (normalization of) the double locus $\bar{D}$. Alternatively, in the Gorenstein case one can use the classical method of studying the canonical ring and the pluricanonical maps.

In my talk I will focus on Gorenstein stable surfaces with $K^2 = 1$, explaining how a combination of the above two methods yields classification results.

[1] M. Franciosi, R. Pardini, S. Rollenske, Log-canonical pairs and Gorenstein stable with $K_X^2 = 1$; `http://arxiv.org/abs/1403.2159`.

Some results on multisecant lines of smooth codimension two subvarieties of projective spaces

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We will show how to extend in several ways the classical results of Ascione [1], Severi [4] and Marletta [3], as well as the modern result of Aure [2], concerning the trisecant lines of smooth complex projective surfaces in $\mathbf{P}^4$. If time permits, we will also discuss higher-dimensional generalizations.

[1] Ascione, E., Sulle superficie immerse in un S_4, le cui trisecanti costituiscono complessi di 1° ordine, *Atti Accad. Naz. Lincei Rend. Cl. Sci. Fis. Mat. Natur. (5)* **6** (1897), 162–169.
[2] Aure, A. B., The smooth surfaces in $\mathbf{P}^4$ without apparent triple points, *Duke Math. J.* **57** (1988), 423–430.
[3] Marletta, G., Le superficie generali dell' S_4 dotate di due punti tripli apparenti, *Rend. Circ. Mat. Palermo* **34** (1912), 179–186.
[4] Severi, F., Intorno ai punti doppi impropri di una superficie generale dello spazio a quattro dimensioni, e a' suoi punti tripli apparenti, *Rend. Circ. Mat. Palermo* **15** (1901), 33–51.

Flops and S-duality conjecture

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In this talk, I give the transformation formula of Donaldson-Thomas invariants counting two dimensional torsion sheaves on Calabi-Yau 3-folds under flops. The error term is described by the Dedekind eta function and the Jacobi theta function, and the result gives evidence of a 3-fold version of Vafa-Witten's S-duality conjecture. As an application, I show that the generating series of Euler characteristics of Hilbert schemes of points on any algebraic surface with at worst A-type singularities is a Fourier development of a meromorphic modular form.

On 81 symplectic resolutions of a 4-dimensional quotient by a group of order 32

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Symplectic resolutions of complex quotient singularities are interesting for a number of reasons. For example, they can be used in the generalized Kummer contruction, see [1], which possibly may lead to constructing new hyper-Kähler manifolds. The existence of a symplectic resolution of a quotient singularity can be established by finding its smoothing by Poisson deformation, see [7] and [6]. This method was used by Bellamy and Schedler to find a group of order 32 acting linearly on a 4-dimensional space whose quotient admits a symplectic resolution, [3].

In a joint project with Maria Donten-Bury we find a description of all symplectic resolutions of this quotient singularity. Our method is based on the theory of total coordinate rings, see [2], and it has been used already for solving surface quotient singularities, [4], [5]. Once the total coordinate ring of a symplectic resolution is determined, all the symplectic resolutions are constructed as GIT quotients of the ring in question. In the case under consideration, we link the geometry of the resolutions to the geometry of the projective plane blown-up in four points, which is a component of the special fiber of a symplectic resolution.

[1] Marco Andreatta and Jarosław A. Wiśniewski, On the Kummer construction, *Rev. Mat. Complut.* **23** (1) (2010), 191–215.

[2] Ivan Arzhantsev, Ulrich Derenthal, Jürgen Hausen, and Antonio Laface, Cox Rings (2010), http://arxiv.org/abs/1003.4229.

[3] Gwyn Bellamy and Travis Schedler, A new linear quotient of $\mathbf{C}^4$ admitting a symplectic resolution, *Math. Z.* **273** (3-4) (2013), 753–769.

[4] S. Cacciola, M. Donten-Bury, O. Dumitrescu, A. Lo Giudice, and J. Park, Cones of divisors of blow-ups of projective spaces, *Matematiche (Catania)* **66** (2) (2011), 153–187.

[5] Maria Donten-Bury, Cox rings of minimal resolutions of surface quotient singularities (2013); http://arxiv.org/abs/1301.2633.

[6] Victor Ginzburg and Dmitry Kaledin, Poisson deformations of symplectic quotient singularities, *Adv. Math.* **186** (1) (2004), 1–57.

[7] Yoshinori Namikawa, Poisson deformations of affine symplectic varieties, *Duke Math. J.* **156** (1) (2011), 51–85.