Do Nonstrategic Considerations Matter for Behavior in Games? An Experimental Study Informed by Direct-sum Decompositions of Games*

Aleix Garcia-Galocha[†] Elena Iñarra[‡] Nagore Iriberri[§]

February 13, 2024

Abstract

We use the direct-sum decomposition proposed by Candogan et al. (2011) to decompose any normal-form finite game into the *strategic* and the *nonstrategic* components. Nash equilibrium is invariant to changes in the nonstrategic component. *Mutual-Max-Sum*, a new solution concept defined in this paper, depends *only* on the nonstrategic component, identifies the most relevant strategy profile in this component and it is invariant to changes in the strategic component. We design 3x3 games to empirically test, whether and when, manipulations in the nonstrategic component affect individual behavior and whether *Mutual-Max-Sum* is behaviorally relevant. We find that manipulations of the nonstrategic component affects individual behavior and that *Mutual-Max-Sum* is able to attract individual behavior only when it is Pareto efficient and in particular, payoff dominant. We conclude that Candogan et al. (2011)'s decomposition is informative to learn about individual behavior in games.

*Aleix Garcia-Galocha and Nagore Iriberri acknowledge funding by grant PID2019-106146GB-I00 funded by MCIN/AEI/10.13039/501100011033 and by "ERDF A way of making Europe". Elena Iñarra acknowledges funding by grant PID2019-107539GB-I00 funded by MCIN/AEI/10.13039/501100011033 and by "ERDF A way of making Europe". Aleix Garcia-Galocha, Elena Iñarra and Nagore Iriberri acknowledge funding by the Basque Government (IT1697-22).

[†]University of the Basque Country, UPV/EHU (`aleix.garciagalocha@gmail.com`)

[‡]University of the Basque Country, UPV/EHU (`elena.inarra@ehu.eus`)

[§]University of the Basque Country, UPV/EHU, and IKERBASQUE, Basque Foundation for Research (`nagore.iriberri@gmail.com`)

1 Introduction

Since economics in general, and game theory in particular, adopted the use of laboratory experiments, hundreds of experimental studies have shown that Nash equilibrium theory has clear limitations in regard to its ability to describe how people behave in strategic environments, see for example Thaler (1988), Nagel (1995), McKelvey and Palfrey (1992), Goeree and Holt (2001), Arad and Rubinstein (2012) among many others. If it is not only equilibrium thinking, then what determines individual behavior in games? Extensions of individual preferences to the so called social or interdependent preferences, e.g. Sobel (2005), and models of bounded rationality, e.g. Crawford et al. (2013), have been put forward to explain individual behavior in games. Yet, the determinants of individual behavior in games are not fully understood.

In this paper, we take a novel approach to have a better understanding of how individuals play games. We analyze the different pieces of information (considerations) contained within a game and study their impact on individual behavior. In particular, we use the direct-sum decomposition of games, proposed by Candogan et al. (2011), to connect individual behavior and different behavioral rules (solution concepts). Candogan et al. (2011) defined a direct-sum decomposition for any finite games in strategic form: games are decomposed into the *strategic* and *nonstrategic* components. The appealing attribute of this particular decomposition is that the strategic component, also referred to as the normalized game, captures all strategic, while the nonstrategic component, what is left, captures all nonstrategic considerations. In other words, this decomposition is the only one that filters out the strategic and nonstrategic information in two different components (see footnote 5 to understand the connection of this particular decomposition with other existing decompositions). Noncooperative games are solved using mainly strategic solution concepts. Among those, the canonical solution concept is the Nash equilibrium, which only takes the information contained in the strategic component. Therefore, from a game theory point of view, only the strategic component would be key in terms of predicting individual behavior and therefore individual behavior should remain constant in strategically equivalent games, i.e. games with the same strategic component, as defined in Candogan et al. (2011). What about considerations included in the nonstrategic component? They could indeed play a role in players' decision-making. In this paper we address whether the nonstrategic component of a game is relevant to behavior and if so, when.

To illustrate this idea, take the Prisoner's Dilemma (PD) game, and three additional modifications of this game, all shown in Figure 1. The four games have the same unique Nash

Prisoner's Dilemma I

	C	NC
C	6, 6	3, 7
NC	7, 3	4, 4

=

Strategic Component

	C	NC
C	-0.5, -0.5	-0.5, 0.5
NC	0.5, -0.5	0.5, 0.5

+

Nonstrategic Component

	C	NC
C	6.5, 6.5	3.5, 6.5
NC	6.5, 3.5	3.5, 3.5

Prisoner's Dilemma II

	C	NC
C	7.5, 7.5	1.5, 8.5
NC	8.5, 1.5	2.5, 2.5

=

Strategic Component

	C	NC
C	-0.5, -0.5	-0.5, 0.5
NC	0.5, -0.5	0.5, 0.5

+

Nonstrategic Component

	C	NC
C	8, 8	2, 8
NC	8, 2	2, 2

Prisoner's (non) Dilemma III

	C	NC
C	1.5, 1.5	7.5, 2.5
NC	2.5, 7.5	8.5, 8.5

=

Strategic Component

	C	NC
C	-0.5, -0.5	-0.5, 0.5
NC	0.5, -0.5	0.5, 0.5

+

Nonstrategic Component

	C	NC
C	2, 2	8, 2
NC	2, 8	8, 8

Prisoner's (non) Dilemma IV

	C	NC
C	4.25, 4.75	4.75, 5.75
NC	5.25, 4.25	5.75, 5.25

=

Strategic Component

	C	NC
C	-0.5, -0.5	-0.5, 0.5
NC	0.5, -0.5	0.5, 0.5

+

Nonstrategic Component

	C	NC
C	4.75, 5.25	5.25, 5.25
NC	4.75, 4.75	5.25, 4.75

Figure 1: Four Examples based on the Prisoner's Dilemma Game

equilibrium strategy profile, hereinafter referred to as *NE*, given by (NC, NC), which can be achieved by eliminating the strictly dominated strategy of C. Moreover, following Candogan et al. (2011), each of the four games can be decomposed into their strategic and nonstrategic components, as shown in Figure 1. For an easy illustration of how to decompose a game, we will start with the calculation of the payoffs of the nonstrategic component. In particular, in game I and for the row player, fixing column player's strategy C or NC and summing own payoffs (6+7) or (3+4) and dividing by 2, we obtain row player's payoffs of 6.5 or 3.5 for the nonstrategic component when the column player plays C or NC, respectively. Similarly, we can perform the same calculations to obtain the nonstrategic component's payoffs for the column player fixing strategies for the row player. As it is a direct-sum decomposition, the strategic component is then obtained by subtracting to each of the payoffs of the original game the payoff in the nonstrategic component.

On the one hand, note that all four games have exactly the same strategic component, which can be interpreted as a game on itself, and therefore, the four original games, as well as the four games represented by their respective strategic components will have the same *NE*. The same equivalence is true for other strategic solution concepts, such as *Quantal Response Equilibrium* (McKelvey and Palfrey, 1995) and level-*k* thinking rules (Stahl and Wilson, 1994, 1995; Nagel, 1995; Costa-Gomes et al., 2001; Camerer et al., 2004). For any games that have the same strategic component, Candogan et al. (2011) define them as strategically equivalent.

On the other, the nonstrategic component can be also interpreted as a game, although it is clear that there is no meaningful strategic consideration in this component because both strategies yield the same payoff for any player. Therefore, in the game represented by the nonstrategic component, all strategy profiles are Nash equilibria.

Does the addition or manipulation of a nonstrategic component affect individual behavior in the original game? In other words, is individual behavior constant in strategically equivalent games? This is the initial question we address in this paper. Although we have not taken these particular four games into the laboratory, we expect, as many readers will, that the answer will be positive. Moreover, we delve deeper into the analysis of the nonstrategic component, by defining a new solution concept, and by using carefully designed 3×3 games in two important ways. First, we show *when* the manipulation of the nonstrategic component will affect individual behavior *most* and second, we also show *how* individual behavior will be affected, i.e., which behavioral rule individuals will follow. The answer to these two questions in short is: individual behavior will be affected most when manipulations of the nonstrategic component change the Pareto optimality ordering of different outcomes in the original game, and individuals will mostly deviate to a behavioral rule that has Pareto optimality concerns.

We start analyzing the nonstrategic component. Going back to the example: What are the relevant considerations in the nonstrategic component? It is obvious that the outcomes given by the four strategy combinations in the nonstrategic components in Figure 1 can be partially ordered by Pareto optimality, see for example Mock (2011).[1] Most importantly, there is a unique strong Pareto optimal outcome, which coincides with the prediction by the social-welfare maximization or altruistic rule, hereafter A rule, which maximizes the sum of players' payoffs, as described by Charness and Rabin (2002). From now on, we will focus on A behavioral rule, whose outcome we see it as a refinement of the set of strong Pareto optimal outcomes.[2] Needless to say, and scanning all the four matrices shown by the nonstrategic components, the unique A is the sensible strategy profile to play. In particular, in the games represented by the nonstrategic components in I and II, (C,C) is the strategy profile selected by the A rule. In game III, the A rule selects (NC, NC), and finally, in game IV, the A prediction is given by (C, NC). What are the three different modifications of the nonstrategic components doing to the original PDs in II, III and IV in Figure 1? In game II, it is exacerbating the social dilemma that exists in the original PD, described in game I, making the Pareto dominance between (C,C) over (NC,NC) more extreme. By contrast, the nonstrategic component in games III and IV *destroys* the social dilemma that existed in the original PD, so that we cannot even label these last two games as PD games, since the unique NE is not Pareto dominated by (C,C). These four games clearly illustrate that strategy profiles selected by the A rule in the nonstrategic component will not necessarily coincide with those by the A in the original game. In particular, in game IV, the the strategy profile selected by A

[1]Payoff dominance, defined by Harsanyi et al. (1988), is a related concept, when an outcome is Pareto dominating another outcome. According to these authors the payoff dominance principle relies on the idea that "rational individuals will cooperate in pursuing their common interests if the conditions permit them to do so". There are multiple experimental investigations of when payoff dominance is important for individual behavior, in games where Nash equilibrium and payoff dominance are in conflict, as in the classical Prisoner's Dilemma, among the oldest experiments on games going back to Deutsch (1958), but also in games with multiple Nash equilibria, where payoff dominance is one selection criterion (Cooper et al. (1990), Cooper et al. (1992), Van Huyck et al. (1990), Van Huyck et al. (1991), Straub (1995), Haruvy and Stahl (2007) or Crawford et al. (2008)).

[2]In the nonstrategic component, a strong Pareto optimal and A or the social-welfare maximization rule will select the same strategy profile(s), which we will refer to as the A rule. However, in the original game, the set of strong or weak Pareto optimal outcomes and A rules' predictions will not necessarily fully coincide. Overall, Pareto optimality criterion can select multiple strategy profiles. In particular, any A profile will always be strong Pareto optimal, and consequently weak Pareto optimal, but there can be strong Pareto optimal profiles that are not selected by the A rule. Of all the strong Pareto optimal profiles, we will focus on A profile, so we see the A as a refinement of the set of Pareto optimal outcomes.

in the game represented by the nonstrategic component is (C,NC) but in the original game IV, the *A* rule selects (NC,NC), so the same behavioral rule can select different strategy profiles in the nonstrategic component and the original game. Consequently, to identify the importance of the nonstrategic component, separating the *A* rule predictions in the original and in the nonstrategic component game is crucial and an important contribution of this paper. We will now go on to explain this contribution using the examples in Figure 1.

The *NE* in the original game coincides with the *NE* in the game represented by the strategic component. However, when considering the *A* as the natural solution of the game represented by the nonstrategic component, the following question arises: How can we identify that strategy profile in the original game? It is clear that it may not necessarily be the *A* of the original game. To answer this question we define a new solution concept for two-player games, which we shall call the *Mutual-Max Sum*, *MMS* for short. The *MMS* coincides, in the original game, the strategy profile(s) identified by the *A* rule in the nonstrategic component. In particular, the *MMS* selects strategy profile(s) where players choose their strategies by maximizing the sum of the *other* player's payoffs. The *MMS* solution concept may be understood as a solution in which an empathetic player who chooses her strategy maximizing the sum of the *opponent's* payoffs, as if the other player would not be able to do so by herself. Indeed, under this interpretation, it captures an extreme form of altruism. Going back to the four games in Figure 1, the *MMS* for players 1 and 2 in PD I would choose C, because this strategy would yield a payoff of 6+7=13 for the other player (if she chose NC, then this strategy would yield a payoff of 3+4=7 for the other player). Similarly, in game II, the *MMS* profile would select (C, C), while this maximizes the sum of payoffs for the other player. However, in game III, the *MMS* prediction is given by (NC, NC) and by (C, NC) in game IV.

What are the appealing features of the *MMS* solution? We show that, in the original game, the *MMS* will always identify the *A* profile(s) in the nonstrategic component (Proposition 1). An important advantage of the *MMS* solution concept is that no decomposition is required to identify the *A* profile(s) of the nonstrategic component. Interestingly, as the *NE* is indifferent between any of the strategy profiles in the nonstrategic component, the *MMS* is also indifferent between any of the strategy profiles in the strategic component. Consequently, the *NE* will select at least one strategy profile of the strategic component while being indifferent between any of the strategy profiles in the nonstrategic component, and the *MMS* will select at least one of the strategy profiles of the nonstrategic component while being indifferent between any of the strategy profiles in the strategic component.

In addition to the *NE* and *MMS*, how do other solution concepts or behavioral rules depend on strategic and nonstrategic components? Focusing on *A* rule, we show that, in the original game, A profiles depend on both components and that, in principle, in the original game we can separate the strategy profiles selected by these main three solution concepts: *NE*, *MMS* and A. This is a very important result of our study, showing that *MMS* strategy profiles do not necessarily coincide with those by A. For example, in games I and II, *NE* is separated from A and *MMS* but the last two coincide. In game III, *NE*, *MMS* and A are all confounded. Finally, in game IV, *MMS* strategy profiles are different from those selected by *NE* and A but the last two coincide. To perfectly separate the three different rules, we then proceed to design 3×3 games to test whether *MMS* is behaviorally relevant. The question of interest in this regard is: when adding and manipulating a nonstrategic component, is *MMS* a good indicator of how the nonstrategic component affect individual behavior? This is a relevant question because the *MMS* identifies the altruistic profile, and the most sensible strategy profile, in the nonstrategic component.

To this end, we design two laboratory experiments to address the two questions mentioned above. First, is individual behavior constant in strategically equivalent games, when the only difference resides in the nonstrategic component? Second, is *MMS* behaviorally relevant, when fully separated from A rule and when joining forces with the A rule's predictions?

For the design of the experiments, we start with the direct-sum decomposition of games of normal-form by Candogan et al. (2011), which decomposes the game into the strategic and nonstrategic, and at the same time the strategic into the potential and harmonic components. We add to this decomposition the one proposed by Jessie and Saari (2015), which decomposes the nonstrategic into the behavioral and kernel components. This combination yields a four-component direct-sum decomposition of games: *potential*, *harmonic*, *behavioral* and *kernel* components (see Figure 2).

In the first experiment, and following Candogan et al. (2011), we use three different classes of games: *harmonic* games (those without a potential component), *potential* games (those without a harmonic component) and *constant-sum* games (games that have both potential and harmonic components). These further decompositions are useful to see when different behavioral rules' predictions will be separated or confounded. If two behavioral rules yield distinct strategy profiles for each player, then we say the two behavioral rules are fully separated. In contrast, if two behavioral rules yield the same strategy profiles for each player, then we say the two behavioral rules are fully confounded. Thus, different classes of games will be important to understand the experimental design of the games. Harmonic games are of some use for separating predictions by *MMS* from predictions by *NE*, although they are limited by the fact that predictions by *MMS* and A are fully confounded. Constant-sum games are the most useful for separating *NE* and *MMS* predictions, as their predictions will always be separated (Proposition 2), but by construction all strategy profiles will be compatible with the A rule. Finally, potential games are the most useful for separating MMS, A and NE predictions. With regard to the decomposition by Jessie and Saari (2015), we keep constant the *kernel* component in all variations, in contrast to Jessie and Kendall (2022), and change only the *behavioral* component. This is important because there is work showing that underlying stakes can also impact individual behavior, see for example Esteban-Casanelles and Gonçalves (2020).

In the second experiment, which is a follow up, we focus only on the class of potential games, which offer the highest separability between the three behavioral rules of interest: NE, MMS and A. We further explore cases in which the MMS and A show payoff dominance over the NE and cases in which there is no payoff dominance, to improve our understanding on when these behavioral rules will help explain deviations of individual behavior from the NE.

In the empirical test, we find that individual behavior may indeed show very important differences in strategically equivalent games, when changes occur only in the nonstrategic component. Although NE is a strong predictor of individual behavior in our games, changes in the nonstrategic component can clearly change individual behavior in harmonic and potential games when comparing individual behavior in games that are strategically equivalent but that differ in their nonstrategic component (more particularly, in their behavioral component keeping the kernel component constant). Indeed, we find that individual behavior is statistically different for both player roles in every comparison (both in harmonic games and in potential games).

Moreover and most importantly, how does individual behavior change? Which rule do individuals follow? In constant-sum games the NE was by far the highest observed choice, followed by MMS. When MMS and A rules' predictions do not coincide and are different from the NE (necessarily, we need to focus on potential games), then NE prediction is the leading model to explain individual behavior, followed by the A rule and then to a much less extent MMS. However, when both types of altruism coincide (harmonic and potential games), meaning that the MMS and A rules' predictions coincide, and are different from NE's predictions, then the combination of both rules gains relevance and they are able to explain important deviations from the NE, in particular, when, in addition, there is payoff dominance over NE. We show these results comparing individual behavior across games but also through the estimation of mixture-of-types models.

Our results can also be interpreted in the following way: When does the altruistic solution attract individual behavior in games? Clearly, when both types of altruism, A and MMS, go hand in hand, then this combination will be able to explain important part of deviating behavior from the *NE*, in particular when the combination of *A* and *MMS* shows payoff dominance over the *NE* strategy profile. We do not provide a quantitative answer but rather offer a qualitative one for the class of potential games. It is noteworthy that the relevance of the *NE* diminishes when the *A* solution aligns with the *MMS* and this strategy profile payoff dominates that of the *NE*.

The most related papers are Jessie and Kendall (2022) and Kendall (2022), as they showed that individual behavior is significantly affected when manipulating the nonstrategic component of a game using several 2×2 games and stag-hunt games, respectively. However, in the designs by Jessie and Kendall (2022) and Kendall (2022), *MMS* strategy profile *always* coincided with that of the *A* rule, so they concluded that the *A* strategy profile in the behavioral component, in our definition the *MMS*, is very important for individual behavior. By contrast, in our design, where *MMS* can be separated from *A* (in particular, in potential games) comparing observed individual behavior directly but also through the use of a mixture-of-types econometric model estimation, we found that *A* and *MMS*'s empirical relevance heavily depends on whether they are joining forces or not, and when joining forces, whether there is payoff dominance over *NE*.

We conclude that Candogan et al. (2011) is useful to inform about individual behavior in games. As carefully observed in footnote 7 in by Candogan et al. (2011), the nonstrategic component can affect Pareto optimality ordering in games. We have elaborated on this idea by empirically showing that changes in the nonstrategic component will affect the Pareto ordering of different outcomes in the original game depending on the class of games. Furthermore, we empirically show that when the changes in the nonstrategic component make *MMS* and *A* rule to join forces and predict a different strategy combination than the *NE*, this is when individual behavior will depart the most from *NE*, in particular when *MMS* and *A* payoff dominate *NE*. On the contrary, when the three behavioral rules, *NE*, *A* and *MMS* are fully separated, then *NE* play will dominate, and *A* and *MMS* will be less behaviorally relevant. To summarize, going back to the four different versions of PD in Figure 1, our results would imply that, while individual behavior would follow the *MMS* and *A* prediction in games I, II and III, in game IV the *MMS* prediction would not explain much of the individual behavior.

The paper is organized as follows. Section 2 shows the four direct-sum decomposition of games, adding the decomposition by Jessie and Saari (2015) to the one by Candogan et al. (2011). This section also relates the decomposition to different behavioral rules, and to different classes of games. Section 3 describes the experimental design and procedures to empirically test whether and when the manipulations of the nonstrategic component will affect individual behavior and whether *MMS* is a behaviorally relevant rule. Section 4 shows the results and finally, Section 5 concludes.

2 Theoretical Framework

2.1 Preliminaries

We first introduce the general framework for two-person normal form games and their corresponding bimatrix representation.

Let $\mathscr{G} = \langle I, S, T, (u_i)_{\{i=1,2\}} \rangle$ be a two-person finite normal form game, where $I = \{1,2\}$ is the set of players, $S = \{s_1, \ldots, s_h\}$ and $T = \{t_1, \ldots, t_h\}$ are the sets of strategies for players 1 and 2, respectively, and $u_i : S \times T \to \mathbb{R}$ is player i $(i = 1,2)$ payoff function. A pair (s_i, t_j) $(i, j = 1, \ldots, h)$ denotes a strategy profile. A mixed strategy for player i $(i = 1,2)$ is a probability measure over her possible pure strategies, $\sigma \in \Delta(S)$ and $\tau \in \Delta(T)$. We will focus on games where players have the same number of strategies, although all results are easily generalizable to games in which players have a different number of strategies.

Game $\mathscr{G}$ can be written as a bimatrix square game (A,B). Matrix A corresponds to player 1's payoffs with elements a_{ij} $(i, j = 1, \ldots, h)$ where $a_{ij} = u_1(s_i, t_j)$. Matrix B corresponds to player 2's payoffs with elements b_{ij} $(i, j = 1, \ldots, h)$, where $b_{ij} = u_2(s_i, t_j)$. Since our study focuses on two-person games, we will use matrix notation when appropriate.

2.2 Direct-Sum Decomposition of Games

We start showing the direct-sum decomposition of games, proposed by Candogan et al. (2011) and then we add the decomposition of the nonstrategic component, proposed by Jessie and Saari (2015).[3] This combination leads to a four-component direct-sum decomposition, which is important to understand the underlying reasoning behind the experimental design, in particular, the selection of the games. Although in this section we will differentiate between the game and its corresponding components, note that each component can be understood as

[3] Candogan et al. (2011)'s decomposition was based on the *Helmholtz decomposition theorem*, which enables the decomposition of a flow on a graph into three components: globally consistent, locally consistent (but globally inconsistent), and locally inconsistent components, which are the potential, harmonic and nonstrategic components, respectively. For a more detailed theoretical description see Section 3 and for its application see Section 4 in Candogan et al. (2011). Jessie and Saari (2015)'s decomposition was based on the mathematics of symmetry groups and representation theory.

a payoff matrix of an independent game.[4]

Candogan et al. (2011) started normalizing the game by eliminating the nonstrategic information. In particular, the nonstrategic component is computed by taking the average of each player's own payoffs for each of their opponents' strategies. Then, in order to get the strategic component, this average is subtracted from the payoffs in the game, such that in the strategic component the sum of one player's payoffs, given the other players' strategies, is always zero. They further proposed a canonical direct-sum decomposition of the strategic component into two components: *potential* and *harmonic*.

Jessie and Saari (2015) build on Candogan et al. (2011) focusing on the nonstrategic component, which in turn was decomposed into what they called *behavioral* and *kernel* components. Although they defined the decomposition for 2×2 games, it is easily generalizable to $h \times h$ games.

The combination of the two proposed decompositions of bimatrix games yields the four-component decomposition represented in Figure 2.

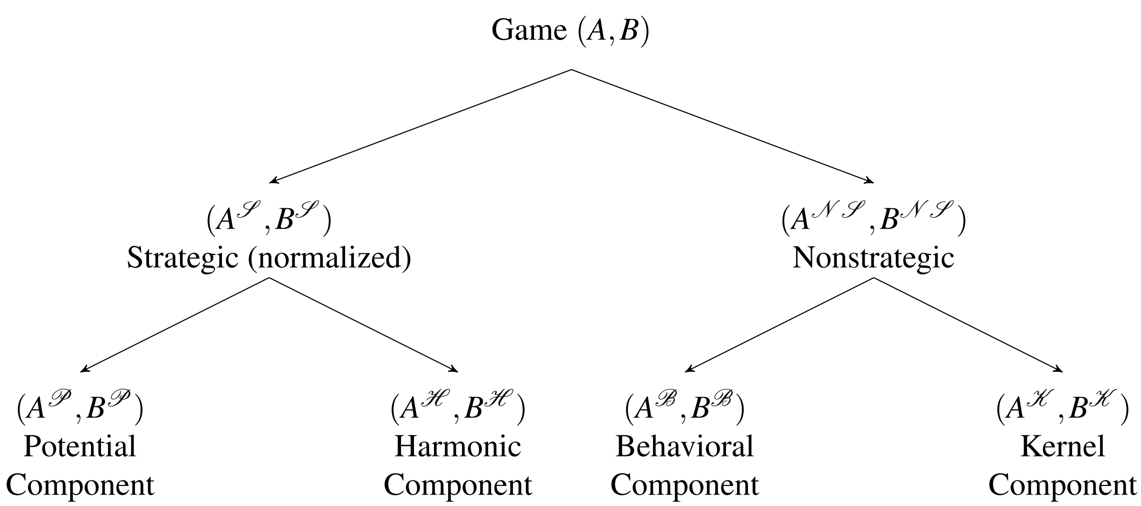

Figure 2: Diagram of Four-Component Direct-Sum Decomposition of Games

We will now describe the four-component decomposition for a bimatrix square game.

First, we consider the **nonstrategic component**. Denote the column vector of ones by $\mathbf{1}$ and its transpose by $\mathbf{1}^{\mathbf{T}}$. The nonstrategic component is then computed as follows:

$$(A^{\mathscr{N}\mathscr{S}}, B^{\mathscr{N}\mathscr{S}}) = ((\frac{1}{h})\mathbf{1}\mathbf{1}^{\mathbf{T}}A, (\frac{1}{h})B\mathbf{1}\mathbf{1}^{\mathbf{T}})$$

[4] The strategic and nonstrategic components correspond to the classes of nonstrategic and μ-normalized games introduced by Abdou et al. (2022) see Definition 2.2 and Section 2.3 for details.

Then, we further decompose the nonstrategic component into the kernel component, and the behavioral component, denoted by $(A^{\mathscr{K}}, B^{\mathscr{K}})$ and $(A^{\mathscr{B}}, B^{\mathscr{B}})$, respectively.

The *kernel component* is a matrix of payoffs computed by taking the average of all payoffs for each player of game (A, B). Formally, the kernel component can be computed as follows:

$$(A^{\mathscr{K}}, B^{\mathscr{K}}) = ((\frac{1}{h^2})\mathbf{11^T}A\mathbf{11^T}, (\frac{1}{h^2})\mathbf{11^T}B\mathbf{11^T})$$

This component can be interpreted as an "inflationary term" or the underlying stakes that can vary by player.

The *behavioral component* is obtained as the difference between the nonstrategic and the kernel components.

$$(A^{\mathscr{B}}, B^{\mathscr{B}}) = (A^{\mathscr{N}\mathscr{S}}, B^{\mathscr{N}\mathscr{S}}) - (A^{\mathscr{K}}, B^{\mathscr{K}})$$

All rows of matrix $A^{\mathscr{B}}$ and all columns of matrix $B^{\mathscr{B}}$ have equal payoffs, meaning that both players are strategically indifferent between their strategies in the behavioral component. Also, since this component is normalized, there must always be at least one positive payoff in each row (and column). Therefore, strategy profiles in the behavioral component can be ordered according to Pareto optimality criterion, Mock (2011). We are interested in the strategy profile selected by the A rule on this component. This is also the case for the nonstrategic component.

There are two explanatory comments that we would like to make. First, throughout the paper we will use nonstrategic and behavioral components interchangeably, as the kernel component is just a constant term for each player (and we make sure we keep this term constant in all our manipulations in the experimental design). Second, given what our paper reveals, our preference would be to change the term "behavioral component" to "efficiency component", as it can affect the efficiency or Pareto optimality of the outcomes in the original game. However, given this name was originally proposed by Jessie and Saari (2015), we decided to follow their labeling of these components.

Then, we consider the decomposition of the **strategic component**. We start by identifying the strategic component, which can be obtained as the difference between (A, B) and its nonstrategic component $(A^{\mathscr{N}\mathscr{S}}, B^{\mathscr{N}\mathscr{S}})$.

$$(A^{\mathscr{S}}, B^{\mathscr{S}}) = (A - (\frac{1}{h})\mathbf{11^T}A, B - (\frac{1}{h})B\mathbf{11^T}).$$

Then, the potential and the harmonic components are obtained by first calculating the following matrices: $M = \frac{1}{2}(A^{\mathscr{S}} + B^{\mathscr{S}})$, $D = \frac{1}{2}(A^{\mathscr{S}} - B^{\mathscr{S}})$ and $\Gamma = \frac{1}{2h}(A^{\mathscr{S}}\mathbf{11^T} - \mathbf{11^T}B^{\mathscr{S}})$.

The *potential component* is, then:

$$(A^{\mathscr{P}}, B^{\mathscr{P}}) = (M + \Gamma, M - \Gamma)$$

while the *harmonic component* is:

$$(A^{\mathscr{H}}, B^{\mathscr{H}}) = (D - \Gamma, -D + \Gamma)$$

These two components are also normalized, hence $\mathbf{1}^T A^{\mathscr{P}} = 0, B^{\mathscr{P}}\mathbf{1}$=0 and $\mathbf{1}^T A^{\mathscr{H}} = 0, B^{\mathscr{H}}\mathbf{1}$=0.

This decomposition separates the cyclical and the acyclical parts of the strategic component giving rise to the harmonic and the potential components, respectively. By construction, the harmonic part is a zero-sum payoff matrix. Consequently, starting from any of its payoff profiles, there exists a sequence of deviations for a single player that strictly increases her payoff until the same payoff profile is reached again. By contrast, this iteration always ends in some strategy profile in the potential component.[5]

This completes the introduction of the four-component direct-sum decomposition of games. From now on, when a particular component is a matrix of zeros, we say so or alternatively, that it does not have that particular component. To illustrate the calculation of the four-component direct-sum decomposition, please find a detailed step by step calculation for a particular game, as well as the 11 and 15 experimental games from the laboratory experiments we use in the empirical test decomposed into the four direct-sum components in the Appendix A.

2.3 Decomposition and Solution Concepts: *Nash Equilibrium*, *Altruistic*, *Mutual-Max-Sum* and Other Rules

A solution concept or behavioral rule can be understood as a prediction of how agents will play a game. We start by describing the two solution concepts: *NE* and *A*. Then, we formally introduce a new solution concept, the *MMS*. Finally, we also describe other behavioral rules

[5]Kalai and Kalai (2013) proposed a decomposition of a two-person normal-form game into an identical common-interest component, which is potential, and a zero-sum component which is not necessarily a harmonic component. Clearly, this decomposition is in line with the decomposition of the strategic component proposed by Candogan et al. (2011) whenever each cell of payoffs of the auxiliary matrix Γ is zero. Demuynck et al. (2022) develops an index of cooperativeness and competitiveness based on the common-interest and competitiveness components, respectively, by the Kalai and Kalai (2013) decomposition and shows that individual behavior is more/less cooperative and less/more competitive consistent with this index. Hwang and Rey-Bellet (2020) show that any two-person normal form game can be uniquely decomposed into a zero-sum normalized game, a zero-sum equivalent potential game, and an identical interest normalized game.

that have shown useful to explain individual behavior in games.

The canonical solution concept is the *NE*. A strategy profile is said to be a *NE* if no player can gain by altering its strategy, given the existing strategies of other players. Thus, a *NE* represents a best response by any player to the given strategies of other players.

Other solution concepts can be better understood as if they were selected by an external observer whose aim is to identify the best outcomes for the two players. Pareto optimality or efficiency stands out as the most popular criterion. With a weak Pareto optimal outcome, any change will make at least one player no better off, but may not make any party worse off. With a strong Pareto optimal, any change will make at least one player worse off. Often there will be multiple strategy combinations that lead to Pareto outcomes. The most salient Pareto optimal outcome, and therefore our focus, is the *altruistic*, or social welfare maximizing behavioral rule (Charness and Rabin, 2002), A, one which can be viewed as an implicit agreement between players who select the strategy profile that maximizes the sum of their payoffs. So, when choosing her strategy, the A behavioral rule simply sums her own and opponent's payoffs in each cell of the payoff matrix, and applies the max operator. In such a solution, rather than trying to predict her decision, both players implicitly assume that the other player is also altruistic (Costa-Gomes et al., 2001).

We now introduce a novel solution concept that we call, *Mutual-Max-Sum* (*MMS*). This solution can be understood as the reciprocal behavior that may take place in bilateral encounters between empathetic players. Thus, each player when choosing her strategy considers, not her own payoffs, but instead the payoffs of her opponent. Therefore, it can be understood as an extreme form of altruism.[6]

DEFINITION 1. *Let $\mathcal{G}$ be a two-person normal-form game. A strategy profile $(\widetilde{s},\widetilde{t}) \in S \times T$ is Mutual-Max-Sum if:*

$$\widetilde{s} \in \arg\max_{s_i \in S} \sum_{t_j \in T} u_2(s_i,t_j) \text{ and } \widetilde{t} \in \arg\max_{t_j \in T} \sum_{s_i \in S} u_1(s_i,t_j).$$

Note that this is not an equilibrium concept, as players are not mutually best responding to each other. Indeed, individuals choose their strategies independently of the behavior of their opponent but we assume both players are doing this.

[6] This definition shares similarities with the *mutual-max* solution defined by Rabin (1993). The difference relies on the fact that for the selection of a strategy in the mutual-max solution each player considers the maximum payoff of her opponent while in the *MMS* solution each player selects the maximum *sum* of the payoffs of her opponent. As it is the case for the *mutual-max* solution, the *MMS* does not satisfy invariance to the deletion of dominated strategies. However, in contrast to the *mutual-max* solution, the *MMS* satisfies the invariance to affine transformations.

In an interesting and useful result, demonstrated in Appendix B, we relate *MMS* in the original game and the *A* payoff profile(s) in the behavioral component. In particular, the *MMS* in the (A,B) always identifies the payoff profile(s) that is *A* in the behavioral component.

PROPOSITION 1. *Let (A,B) be a bimatrix game and let $(A^{\mathscr{B}},B^{\mathscr{B}})$ be its behavioral component. Then the MMS solution(s) of (A,B) will coincide with the A payoff profile(s) of $(A^{\mathscr{B}},B^{\mathscr{B}})$.*

It is worth noting two points. First, this result allows us to apply the *MMS* solution to game (A,B) and identify the most relevant payoff profile(s) in the behavioral component without having to decompose the game. Second, although the *MMS* solution may not be unique, in our experimental exercise we will only consider the case where this is unique or the trivial case, where the behavioral component is zero, such that any strategy combination is trivially *MMS*.

These are the main three behavioral rules we will focus on. However, we will also briefly describe other behavioral rules that have shown useful to explain individual behavior. Among the non-equilibrium solution concepts, the level-k thinking model excels. In the so-called level-k model, each player of type level-$k = 0,1,...,$ corresponds to the number of steps of reasoning the player is able to perform. Thus, a level-*0* agent chooses her strategies randomly while a level-*1* agent assumes her opponent will act as a level-*0* agent and best responds. Alternatively, level-*1* players 1 and 2 sum their own payoffs across columns and rows, respectively, and take the strategy that yields the maximum sum of payoffs.[7] Finally, also following Costa-Gomes et al. (2001), we consider both the *Pessimistic* and the *Optimistic* behavioral rules. The *Pessimistic* (P) can be understood as a conservative player who, when choosing her strategy, maximizes her minimum payoff. The *Optimistic* behavioral rule (O) on the other hand, when choosing her strategy, maximizes her maximum payoff.[8]

All these behavioral rules that we have presented are defined in the original game (A,B). Now, if we were to make changes to any of the two main strategic or nonstrategic components, would their predictions change? We can identify strategic, nonstrategic, and mixed behavioral rules based on their dependence on the strategic and nonstrategic components. On the one

[7] $k = 2$ or higher are similarly defined such that level-k best response to level-*k-1* behavior.

[8] In addition, and in the spirit of the *A* behavioral rule, we could also define a version of inequity aversion, who simply takes the absolute difference between her own and opponent's payoffs in each cell of the payoff matrix, and applies the minmin operator. However, given other studies have found little evidence of such extreme inequity aversion, for example in the work by Garcia-Pola and Iriberri (2019), we decided not to include it in our study. This consideration of inequity aversion was suggested during the refereeing process and we did check for its relevance. We included it in our estimation exercise in Section 4.3. However, we did not find any evidence for it so we decided not to include it. Results are available upon request.

hand, *NE* and any level-k behavioral rules are strategic rules, such that any change in the nonstrategic component will never affect their predicted behavior. On the other hand, *MMS* is a nonstrategic rule, such that it is invariant to any changes in the strategic component. Finally, predictions by A, P and O rules can be affected by any changes in any of the two main components, meaning that we will refer them as mixed behavioral rules.

Looking at this in more detail, we can see that the *NE* prediction in the original game (A,B) will always coincide with its prediction in the strategic component. This is also the case for any behavioral rule that is strategic. In other words, as the strategic component contains all the strategic information of the original game, the strategic solutions remain invariant between the original game and the strategic component. Furthermore, the predictions by every strategic behavioral rule would be trivially indifferent for any of the strategy profiles in the nonstrategic components. Interestingly, the *MMS* solution is the mirror image such that its prediction in the original game (A,B) will always coincide with its prediction in the nonstrategic component and, further, its prediction in the strategic component will be trivially indifferent for any of the strategy profiles. In short, the strategic component isolates all the strategic considerations, while the nonstrategic component isolates all the nonstrategic considerations. These results are summarized in the following remark.

REMARK 1. *(i) The Nash equilibria of* $(A^{\mathscr{S}},B^{\mathscr{S}})$ *coincides with the Nash equilibria of* (A,B)*, while every strategy profile of* $(A^{\mathscr{S}},B^{\mathscr{S}})$ *is a Mutual-Max-Sum. (ii) The Mutual-Max-Sum solution* $(A^{\mathscr{N}\mathscr{S}},B^{\mathscr{N}\mathscr{S}})$ *coincides with the Mutual-Max-Sum of* (A,B)*, while every strategy profile in the* $(A^{\mathscr{N}\mathscr{S}},B^{\mathscr{N}\mathscr{S}})$ *is a Nash equilibrium.*

Finally, note that we cannot make any similar statements for mixed rules, such as the A, P and O rules. They can select different strategy profiles for different components and, therefore, their predictions in the original game are not invariant to changes in any of the components.

2.4 Decomposition and Classes of Games: Harmonic, Potential and Constant-Sum Games

Candogan et al. (2011), see their Theorem 5.1, allows to reformulate the classes of potential and harmonic games in terms of their components as follows: the absence of a harmonic component defines a potential game, while the absence of a potential component defines a harmonic game. A *potential game* admits at least one Nash equilibrium in pure strategies, while a square *harmonic game* admits only a uniformly mixed Nash equilibrium.[9] Any other class of games always has both components. Looking at these other games with both components, we will focus on *constant-sum* games. These are games of conflict where the sum of all players' payoffs remains constant for each strategy profile, meaning that the gain for one player is always at the expense of her opponent. All constant-sum games have both potential and harmonic components, except the class of matching pennies (rock-paper-scissors for three strategies).[10]

Our main objective is to understand whether and when the manipulations in the nonstrategic component will affect individual behavior and also whether the *MMS* solution is relevant to behavior. Therefore, we use this classification of games to assess when the *MMS* solution will make a different prediction from other relevant behavioral rules. To proceed, we start by saying that any two solution concepts or behavioral rules are *separable* if they can provide a different predicted probability of choosing each of the strategies and for each of the players, while they are *separated* if they never coincide. Below is a table that summarizes the separability between *MMS*, *NE*, and *A* by the classes of games.

Table 1: Separability between *MMS*, *NE* and *A*, by Class of Game

	MMS vs NE	MMS vs A
Harmonic Games	Separated	Not separable
Constant-sum Games	Separated	Not separable*
Potential Games	Separable	Separable

* In CSG all strategy profiles are *A*.

Harmonic games. The unique Nash equilibrium prediction in these games is the uniformly mixed strategy profile. Therefore, the *MMS* prediction is always separated in these games. However, such a prediction cannot be separated by the *A* behavioral rule. Consequently, harmonic games will not be useful when it comes to separating the *MMS* solution from the *A* behavioral rule.

Constant-sum games. These games are particularly interesting because *NE* and *MMS* are

[9]As pointed out by Candogan et al. (2011), harmonic games have appeared in earlier publications but have not been defined as a class.

[10]Furthermore, a constant-sum game can be transformed into a *zero-sum* game by subtracting half of the value of the constant from each payoff in the initial game so that in the former the kernel is positive instead of 0. Zero-sum games generalize the *generalized rock-paper-scissors games* whose decomposition was analyzed by Candogan et al. (2011).

always perfectly separated as shown by the proposition below. However, they will not to be useful when it comes to separating *MMS* predictions from *A* rule predictions.

PROPOSITION 2. *Let (A,B) be a constant-sum game with a unique Nash equilibrium in pure strategies and a unique MMS solution. Then, the NE and the MMS solution will never coincide.*

Potential games. These games offer the highest degree of separability. Interestingly, manipulating the behavioral component we can lead to three situations:

(1) All three rules point to the same strategy profile, reinforcing each other. No separation at all.

(2) All three rules point toward a different strategy profile, such that there is perfect separation. There can be two additional subcases, when *MMS* and *A* payoff dominate the *NE* or when there is no payoff dominance among these three behavioral rules.

(3) The *MMS* and *A* rules choose the same strategy profile but their prediction is different from the prediction by the *NE*. There can be two additional subcases, when *MMS* and *A* payoff dominate the *NE* or when there is no payoff dominance among these three behavioral rules.

As summarized in Table 1, although it is relatively easy to separate *MMS* predictions from *NE* predictions, it is not trivial to separate *MMS* predictions from *A* rule's predictions. Potential games allow for this separation. In our design, we do separate them, and we show that this separation leads to a very different interpretation of the results on the importance of both the nonstrategic component and the *MMS*.

3 Experimental Study

Is individual behavior constant in strategically equivalent games? Do individuals follow *MMS* predictions? If not, what does this solution concept contribute regarding the strategic behavior of the players? As these are empirical questions, we carried out a laboratory experiment. Potentially, we could use existing empirical studies and games to answer this question. However, the games in existing studies were not designed with our research questions in mind, and, as such, they would not provide the most informative answer. Therefore, we designed our own games guided by the four direct-sum decompositions of games.

3.1 Procedures

Using the ORSEE system (Greiner, 2015), we recruited 400 subjects for two different experiments. The laboratory sessions, which lasted around 1 hour and a half, were conducted using the computer software z-tree (Fischbacher, 2007). The 10 sessions, with around 40 participants each, took place in April 2022 and November 2023 in the Laboratory of Experimental Analysis (Bilbao Labean) at the University of the Basque Country UPV/EHU.[11]

We started with general instructions that informed subjects that payments would depend on their own and other participants' decisions in the same session, as well as on luck. After that, the participants were given detailed instructions explaining the task in hand, including examples of games, how their own and the other players' decisions could affect the payments and how they were going to be matched. Before subjects started the task, we posed a set of three questions to ensure the correct understanding of the payoff-matrix representation of games and payments. Appendix D includes a translated version of the instructions.

In the first experiment, 200 subjects played the same eleven 3×3 normal-form two-player games in the same order, twice, once as a row player and once as a column player, leading to a total of 22 decisions per subject. In the second experiment, 200 subjects played the same fifteen 3×3 normal-form two-player games in the same order, twice, once as a row player and once as a column player, leading to a total of 30 decisions per subject. Hence, the two experiments differ in the games and in the number of decisions.

When the subjects had finished all their decisions, the computer randomly matched subjects in pairs and selected one game per pair, in each of the two parts (the first 11 or 15 decisions and the second 11 or 15 decisions). This ensured that each subject was paid for one game played in each of the two player roles. After we informed subjects about their payments, the subjects completed a non-incentivized questionnaire regarding demographic data, risk preferences following Eckel and Grossman (2002), and a cognitive reflection test. Table 2 shows the descriptive statistics for all these variables. The majority of the subjects were mostly Spanish, aged between 18 and 22, with a higher presence of women (64%). The latter is consistent with there being a higher proportion of women studying social sciences, particularly Business Administration and Management and Economics, which represent more than 60%. Participants are also risk averse, as the most frequent choice is the safe option. We also requested free-format responses regarding their explanations of how they made their choices and their expectations of how other subjects made their choices. To finish the session, each

[11]The CEISH-UPV/EHU Ethics Committee issued a favorable report for carrying out the experiment. Ref.: M10_2022_102

subject was paid privately according to the two games selected plus a 3 euros attendance fee. The average payment was 17.06 euros, with a standard deviation of 3.71, in the first experiment, and 17.28 euros, with a standard deviation of 5.77, in the second experiment.

Table 2: Descriptive Statistics

Variables	First Experiment Mean Values (SD)	Second Experiment Mean Values (SD)
Women	0.635	0.640
Age	20.75	21.29
	(2.817)	(3.126)
Spanish	0.96	0.96
University Entry Grade (out of 10)	7.831	7.693
	(2.269)	(1.099)
Business and Economics Degree	0.625	0.640
Distribution over risk choices:		
1.5€ with 0.50 or 1.5€ with 0.50	0.350	0.375
1.3€ with 0.50 or 1.8€ with 0.50	0.170	0.115
1.1€ with 0.50 or 2.1€ with 0.50	0.195	0.150
0.9€ with 0.50 or 2.4€ with 0.50	0.080	0.075
0.7€ with 0.50 or 2.7€ with 0.50	0.070	0.105
0.6€ with 0.50 or 2.8€ with 0.50	0.020	0.025
0.4€ with 0.50 or 2.9€ with 0.50	0.020	0.015
0€ with 0.50 or 3€ with 0.50	0.095	0.140
Cognitive reflection test:		
Q1. Percent correct answer	0.295	0.285
Q1. Percent intuitive answer	0.210	0.250
Q2. Percent correct answer	0.375	0.220
Q2. Percent intuitive answer	0.370	0.495
Q3. Percent correct answer	0.600	0.495
Q3. Percent intuitive answer	0.280	0.305

Notes: The second column displays the mean values of the first experiment and the third column displays the mean values for the second experiment. Women is a dummy variable which takes a value of 1 if the subject is female. Age is measured in years. Spanish is a dummy variable which takes a value of 1 if the subject is Spanish. University Entry Grade is normalized to a grade out of 10. Risk choices are ordered from safest to riskiest and were elicited via Eckel and Grossman (2002). Finally, the cognitive reflection test includes questions from Toplak et al. (2014). The questions are as follows: 1. If John can drink one barrel of water in 6 days, and Mary can drink one barrel of water in 12 days, how long would it take them to drink one barrel of water together? (correct answer 4 days; intuitive answer 9); 2. Jerry received both the 15th highest and the 15th lowest mark in the class. How many students are in the class? (correct answer 29 students; intuitive answer 30); 3. A man buys a pig for 60, sells it for 70, buys it back for 80, and sells it finally for 90. How much has he made? (correct answer 20; intuitive answer 10).

3.2 Experimental Design: Player Roles, Games, Behavioral Rule Predictions and Separability

The specific structure of the experiments was as follows. The computer randomly divided the participants into two types, *Type* 1 and *Type* 2. *Type* 1 subjects started the first eleven and fifteen decisions playing as row players and, then, in the second part of the task, they played as column players. *Type* 2 subjects played the opposite way round, first as column players and then as row players. The subjects were never informed about their types or even about the existence of types, but at the beginning of the experimental task they were told they would be presented with 11 and 15 payoff-matrices, one at a time. Only when these 11 and 15 decisions had been taken were they told that they would be presented with an additional set of 11 and 15 payoff matrices. The subjects did know there would be participants playing as row and column players, but they were not explicitly told that the total of 22 and 30 matrix payoffs came from the same 11 and 15 games. In order to facilitate the reading of the games, we showed all the games to all subjects from the perspective of row players, transposing the games when the subject was a column player. There were no time restrictions for making decisions.

When designing the games, the main goal was to separate *MMS* predictions from the predictions of other behavioral rules, particularly the predictions by the *NE* and *A* behavioral rules. Therefore, we chose 3×3 normal-form games instead of 2×2 normal-form games, as 2×2 games make it impossible to perfectly separate out the predictions of *three* different behavioral rules.

Figures 3 and 4 display the eleven and fifteen 3×3 normal-form two-player games designed for the first and second experiments, respectively. We presented the games to the subjects in a randomized order, but in the same order to all subjects.[12] By design no game has dominated strategies in pure strategies.

In the first experiment, the eleven games can be separated into 3 different sets of games, shown in Figure 3. G1 to G3 are strategically equivalent harmonic games, where G2 and G3 have a behavioral component, and the *MMS* points towards a different strategy profile each, while G1 has no behavioral component. G4 and G5 are the two experimental constant-sum games we designed. These are interesting because by definition the predictions of *NE* and *MMS* are always fully separated. Finally, G6 to G8 and G9 to G11 are the two sets of

[12] The actual order of the games was G5, G9, G7, G11, G2, G6, G8, G4, G10, G3, G1 in the first experiment and G7, G13, G8, G3, G11, G15, G4, G10, G12, G9, G2, G6, G14, G5, G1 in the second experiment. The goal of randomizing was to prevent the subjects from observing the similarity in some particular games.

Harmonic Games

G1

6.62* 6.56*	7.62* 5.56*	5.62* 7.56*
5.62* 7.56*	6.62* 6.56*	7.62* 5.56*
7.62* 5.56*	5.62* 7.56*	6.62* 6.56*

G2

6.62* 11.56*	7.62* 0.56*	5.62* 7.56*
0.62* 12.56*	1.62* 1.56*	2.62* 5.56*
12.62* **10.56***	10.62* 2.56*	11.62* 6.56*

G3

6.62* 1.56*	7.62* 5.56*	5.62* 12.56*
0.62* 2.56*	1.62* 6.56*	2.62* 10.56*
12.62* 0.56*	10.62* 7.56*	**11.62*** **11.56***

Constant-sum Games

G4

4.55* 5.45*	3.77 6.23	1.67 8.33
5.82 4.18	3.15 6.85	**7.76** **2.24**
4.89 5.11	9.78 0.22	0.98 9.02

G5

3.11 6.89	4.80 5.20	**6.67** **3.33**
3.95 6.05	3.97* 6.03*	1.90 8.10
7.72 2.28	5.74 4.26	0.15 9.85

Potential Games. First set

G6

8.53 7.18	7.63 4.05	8.59 6.66
8.29 6.82	8.43 4.75	8.03 5.97
5.11 5.49	12.55* 10.71*	7.08 6.87

G7: *MMS* payoff dominated

9.68 8.44	8.78 1.45	9.74 8.00
9.73 8.08	9.87 2.15	**9.47** **7.31**
2.52 6.75	9.96* 8.11*	4.49 8.21

G8: *NE* payoff dominated

8.33 6.98	7.43 0.05	8.39 10.86
12.49 6.62	12.63 0.75	**12.23** **10.17**
1.11 5.29	8.55* 6.71*	3.08 11.07

Potential Games. Second Set

G9

5.56 7.53	5.22 6.88	5.72 7.41
6.32* 9.15*	5.32 7.84	4.85 7.40
4.03 5.82	6.30 7.78	6.18 7.69

G10: *MMS* payoff dominated

5.86 6.63	5.52 7.28	**6.02** **7.91**
6.12* 8.25*	5.12 8.24	4.65 7.90
3.93 4.92	6.20 8.18	6.08 8.19

G11: *NE* payoff dominated

9.86 3.53	9.52 6.58	**10.02** **11.71**
2.32* 5.15*	1.32 7.54	0.85 11.70
3.73 1.82	6.00 7.48	5.88 11.99

Notes: For each game, outcomes compatible with the *NE* play are denoted by *, those compatible with the *MMS* are in **bold**, and those compatible with the *A* play are underlined. For simplicity purposes, *MMS* is only shown when the behavioral component of the game is non-zero, so it is not shown in G1, G6, and G9. In the experiment, we show only two decimals as in the figure. The actual payoffs of the games are displayed in Figure 5 in Appendix A.

Figure 3: 11 Experimental Games in the First Experiment

First Set

G1 (V0)

$\underline{34}$* $\underline{34}$*	12 21	29 31
31 25	24 27	20 16
21 16	23 27	31 28

G2 (V1: no payoff dominance)

31* 29*	9 25	26 32
33 20	**26** **31**	22 17
22 11	24 31	$\underline{32}$ $\underline{29}$

G3 (V2: *NE* payoff dominated)

28* 27*	6 26	23 33
36 18	**29** **32**	25 18
22 9	24 32	$\underline{32}$ $\underline{30}$

G4 (V3: no payoff dominance)

29* 26*	7 28	24 32
35 17	$\underline{\mathbf{28}}$ $\underline{\mathbf{34}}$	24 17
22 8	24 34	32 29

G5 (V4: *NE* payoff dominated)

29* 29*	7 31	24 26
41 20	$\underline{\mathbf{34}}$ $\underline{\mathbf{37}}$	30 11
16 11	18 37	26 23

Second Set

G6 (V0)

15 23	29 31	$\underline{31}$* $\underline{38}$*
26 27	22 17	27 27
27 25	35 27	13 10

G7 (V1: no payoff dominance)

11 26	25 31	27* 35*
29 **30**	25 17	30 24
28 28	$\underline{36}$ $\underline{27}$	14 7

G8 (V2: *NE* payoff dominated)

11 29	25 34	27* 29*
29 **33**	25 20	30 18
28 31	$\underline{36}$ $\underline{30}$	14 1

G9 (V3: no payoff dominance)

11 28	25 30	27* 34*
$\underline{\mathbf{31}}$ $\underline{\mathbf{32}}$	27 16	32 23
26 30	34 26	12 6

G10 (V4: *NE* payoff dominated)

10 33	24 26	26* 33*
$\underline{\mathbf{36}}$ $\underline{\mathbf{37}}$	32 12	37 22
22 35	30 22	8 5

Third Set

G11 (V0)

30 30	14 21	31 29
31 27	24 27	20 14
19 18	21 27	$\underline{35}$* $\underline{32}$*

G12 (V1: no payoff dominance)

$\underline{31}$ $\underline{29}$	15 27	32 24
33 26	**26** **33**	22 9
16 17	18 33	32* 27*

G13 (V2: *NE* payoff dominated)

$\underline{34}$ $\underline{32}$	18 27	35 21
36 29	**29** **33**	25 6
10 20	12 33	26* 24*

G14 (V3: no payoff dominance)

28 29	12 27	29 24
$\underline{37}$ $\underline{26}$	$\underline{\mathbf{30}}$ $\underline{\mathbf{33}}$	26 9
15 17	17 33	31* 27*

G15 (V4: *NE* payoff dominated)

25 25	9 31	26 24
41 22	$\underline{\mathbf{34}}$ $\underline{\mathbf{37}}$	30 9
14 13	16 37	30* 27*

Notes: For each game, outcomes compatible with the *NE* play are denoted by *, those compatible with the *MMS* are in **bold**, and those compatible with the *A* play are $\underline{\text{underlined}}$. For simplicity purposes, *MMS* is only shown when the behavioral component of the game is non-zero, so it is not shown in G1, G6 and G11. The decomposition for each of the experimental games is displayed in Figures 6, 7 and 8 in Appendix A.

Figure 4: 15 Experimental Games in the Second Experiment

strategically equivalent potential games. Both sets have the same structure. The first game has no behavioral component, meaning that the behavioral component is composed of all 0s, and the *NE* and *A* behavioral predictions coincide in the same strategy profile. Then, in the second game of each set, we added a behavioral component where the *MMS*, *A* and *NE* predictions are all separated. Finally, in the last game of each potential set, we increased the magnitude of the behavioral component to obtain a game where the *MMS* prediction will also coincide with the *A* rule's prediction. However, these two are separated from the *NE* predictions, which is Pareto dominated. With regard to the actual chosen payoff numbers, we opted for having three digit numbers in order to increase separability between different behavioral rules and also avoid round numbers.

In the second experiment, we focused only on potential games, as they offer the highest separability between *MMS* and *A* behavioral types and also used payoffs with no decimals, which simplified subjects' decision making. The payoffs in the second experiment represented points that were then translated into euros, in particular, 1 point represented 0.25 euros. We designed three sets of potential games, shown in each of the columns in Figure 4. In each set, all five potential games are strategically equivalent, such that the *NE* predictions remain exactly the same and only differ in the existence and addition of behavioral components (as the kernel component is also kept constant), leading to five different games within each set. The first game, V0, has no behavioral component and the unique Nash equilibrium profile is also the *A* prediction. In the next two versions, V1 and V2, the three behavioral rules, *MMS*, *NE* and *A* are perfectly separated, and in addition, in V1, while there is no payoff dominance among the three behavioral rules, in V2, both the *MMS* and *A* payoff dominate the *NE*. Finally, in the last two versions, *MMS* and *A* coincide, and are separated from *NE*. In addition, in V3, there is no payoff dominance between the *NE* and *MMS* and *A*, while in the last version, V4, *MMS* and *A*, payoff dominate the *NE*.

Table 3: Predicted Strategies by Different Behavioral Rules for the 11 and 15 Games

First Experiment

Behavioral Rules	Roles	*G1*	*G2*	*G3*	*G4*	*G5*	*G6*	*G7*	*G8*	*G9*	*G10*	*G11*
NE: Nash Equilibrium	R	1,2,3	1,2,3	1,2,3	1	2	3	3	3	2	2	2
	C	1,2,3	1,2,3	1,2,3	1	2	2	2	2	1	1	1
MMS: Mutual-Max Sum	R	1,2,3	3	3	2	1	1,2,3	2	2	1,2,3	1	1
	C	1,2,3	1	3	3	3	1,2,3	3	3	1,2,3	3	3
A: Altruistic	R	1,2,3	3	3	1,2,3	1,2,3	3	1	2	2	3	1
	C	1,2,3	1	3	1,2,3	1,2,3	2	1	3	1	2	3
L1: Level-1	R	1,2,3	1,2,3	1,2,3	1	2	3	3	3	2	2	2
	C	1,2,3	1,2,3	1,2,3	2	1	2	2	2	2	2	2
P: Pessimistic	R	1,2,3	3	2	1	2	3	3	3	2	2	2
	C	1,2,3	3	3	1	2	2	2	2	2	2	1
O: Optimistic	R	1,2,3	2	1	3	3	3	1	3	2	2	3
	C	1,2,3	1	1	2	1	2	2	2	1	2	3

Second Experiment

Behavioral Rules	Roles	*G1*	*G2*	*G3*	*G4*	*G5*	*G6*	*G7*	*G8*	*G9*	*G10*	*G11*	*G12*	*G13*	*G14*	*G15*
NE: Nash Equilibrium	R	1	1	1	1	1	1	1	1	1	1	3	3	3	3	3
	C	1	1	1	1	1	3	3	3	3	3	3	3	3	3	3
MMS: Mutual-Max Sum	R	1,2,3	2	2	2	2	1,2,3	2	2	2	2	1,2,3	2	2	2	2
	C	1,2,3	2	2	2	2	1,2,3	1	1	1	1	1,2,3	2	2	2	2
A: Altruistic	R	1	3	3	2	2	1	3	3	2	2	3	1	1	2	2
	C	1	3	3	2	2	3	2	2	1	1	3	1	1	1,2	2
L1: Level-1	R	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
	C	1	1	1	1	1	2	2	2	2	2	3	3	3	3	3
P: Pessimistic	R	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
	C	1	1,3	3	3	3	2	2	2	2	2	3	3	3	3	3
O: Optimistic	R	1	1	1	2,3	2,3	1	1	1	1	2	3	2,3	2,3	2,3	2,3
	C	1	1	1	1	1	2	2	2	2	3	3	1	1	1	1

Notes: The table reports the strategies predicted by all the behavioral rules we consider; 1, 2 and 3 refer to the first, second and third strategies, respectively. In a few instances, a behavioral rule is indifferent between multiple strategies, so we assume the behavioral rule will predict any of those strategies with equal probability.

Table 3 shows the predicted strategies by different behavioral rules. We can comment on the predicted choices by the *MMS*. In the games with no behavioral component, such that these games only have the strategic component and the kernel component, we can observe that *MMS* is indifferent between any of the strategies (see games G1, G6 and G9, in experiment 1, or G1, G6 and G11, in experiment 2). In any other case, i.e., when the behavioral component is positive, then the games are designed such that the *MMS* will have a unique prediction.

Table 4: Separation between Different Behavioral Rules

First Experiment

	MMS	*NE*	*A*	*L1*	*P*	*O*
MMS	0	**0.6061**	**0.3636**	**0.6061**	**0.5455**	**0.5455**
NE	0.6061	0	0.4848	0.1515	0.1818	0.3636
A	0.3636	0.4848	0	0.4848	0.4242	0.3636
L1	0.6061	0.1515	0.4848	0	0.2121	0.3030
P	0.5455	0.1818	0.4242	0.2121	0	0.3636
O	0.5455	0.3636	0.3636	0.3030	0.3636	0

Second Experiment

	MMS	*NE*	*A*	*L1*	*P*	*O*
MMS	0	**0.6667**	**0.4111**	**0.6667**	**0.6778**	**0.5778**
NE	0.6667	0	0.5444	0.2222	0.3000	0.3111
A	0.4111	0.5444	0	0.5000	0.4667	0.4111
L1	0.6667	0.2222	0.5000	0	0.0778	0.3556
P	0.6778	0.3000	0.4667	0.0778	0	0.4333
O	0.5778	0.3111	0.4111	0.3556	0.4333	0

Notes: The table reports the proportions of decisions across all 22 decisions in which the different behavioral rules predict different strategies. The minimum possible separation value is 0, which occurs when two behavioral rules prescribe the same strategy in all 22 decisions, and the maximum possible separation value is 1, which occurs when the two models predict a different strategy in each of the 22 decisions. When one behavioral rule's prediction is 1,2,3, meaning playing each of the strategies with equal probability, and another behavioral rule's prediction is 1,2, meaning playing the first two strategies with equal probability, the separation value is equal to 1/3, as these two behavioral rules can be separated only 1/3 of the times, particularly, when a subject plays the third strategy.

Finally, we measure how successful we were in separating *MMS* predictions from the predictions of any other behavioral rules. Table 4 shows the separation between different behavioral rules. The values in the table represent the proportion of games $\times$ player roles, i.e. decisions in which the predictions of two behavioral rules are separated. The numbers can take any value between 0 (no separation at all, such that two behavioral rules predict exactly the same strategy in each of the 22 and 30 decisions) and 1 (full separation, such that two behavioral rules predict a different strategy in each of the 22 and 30 decisions). The most interesting row in the table is the one referring to the *MMS*, shown in bold, as the main goal when designing the games was to have the highest separation between *MMS* and the rest of the behavioral rules. All separation values for *MMS* are above 50%, as desired, with the exception of the separation between *MMS* and *A* behavioral rules, which is the hardest to separate. This is closely linked to the results shown in Table 1, as harmonic games and constant-sum games are not qualified to separate predictions by *A* and *MMS*. In the second experiment, these values are improved, as this was one of the objectives of carrying out the second experiment.

As far as the separability between other behavioral rules is concerned, we can conclude that these games are far from ideal in terms of separating predictions by *NE* and *L*1 and predictions by *NE* and *P* with perfect confounds between some of these behavioral rules and particular classes of games. However, the goal was to separate *MMS* from *NE* and from *A* rules. We will come back to this when interpreting the empirical results.

4 Results

We will start off by performing some preliminary analysis testing for whether different sessions can be pooled and for the effects of player role order, as half of the subjects played first as row players and then as column players, while the other half played in the reverse order. We will then analyze how the subjects played game by game to understand whether the manipulation and addition of a behavioral component affect individual behavior. Finally, we will carry out mixture-of-types model estimations, across all games and by sets of games, to get conclusions about the empirical relevance of *MMS*.

4.1 Preliminaries: Testing for the Effects of Player Role Order

We held 5 different sessions in each of the two experiments and in each of them we had subjects playing the games in each of the two roles.

We start off by testing whether we can pool all 5 sessions, both overall, and by player role order. Table A1 shows the *p*-values for Chi-Square test performed for the overall participants in each session, and for the subsets of participants corresponding to each player type in the experiment. We cannot reject the null hypothesis of no significant differences at the 1% significance level between each of the sessions and the rest. Therefore, we are able to pool all 5 sessions in each of the experiments.

Due to the two-part design of the experiment, and two types of subjects (Type 1 and Type 2, as described in Section 3.2), we next check whether there was any kind of effect from player role order when participants chose their strategies, i.e., whether subjects behaved differently when they started playing as row players instead of as column players. Table A2 displays p-value for the Chi-Square test. We cannot reject the null hypothesis of equal behavior across different player role orders. This allows us to use the data for each subject in both roles, and not only in the first role they performed the task. Consequently, we are able to use 200 observations per game in each of the two experiments.

4.2 Individual Behavior Game by Game: Is Individual Behavior Constant in Strategically Equivalent Games?

We start by analyzing individual behavior game by game. Table 5 shows the frequencies of play of each of the three strategies in each of the player roles game by game, by all 200 participants in each of the two experiments. The strategies that are in the *NE* profile, *MMS* and *A* are denoted by *, in bold and underlined, respectively, for each game. For simplicity, we have only marked the *MMS* prediction when the behavioral component is non-zero.

Table 5: Frequencies of Strategy Choices, by Player Role and by Game

	Row Players			Column Players		
Experiment 1	1	2	3	1	2	3
G1	$\underline{0.225}^\star$	$\underline{0.545}^\star$	$\underline{0.230}^\star$	$\underline{0.205}^\star$	$\underline{0.570}^\star$	$\underline{0.225}^\star$
G2	0.105*	0.200*	$\underline{\mathbf{0.695}}^\star$	$\underline{\mathbf{0.310}}^\star$	0.165*	0.525*
G3	0.300*	0.420*	$\underline{\mathbf{0.280}}^\star$	0.105*	0.255*	$\underline{\mathbf{0.640}}^\star$
G4	$\underline{0.805}^\star$	**0.160**	$\underline{0.035}$	$\underline{0.775}^\star$	$\underline{0.205}$	$\underline{\mathbf{0.020}}$
G5	$\underline{\mathbf{0.265}}$	$\underline{0.700}^\star$	$\underline{0.035}$	$\underline{0.360}$	$\underline{0.605}^\star$	$\underline{\mathbf{0.035}}$
G6	0.030	0.015	$\underline{0.955}^\star$	0.040	$\underline{0.920}^\star$	0.040
G7 (*MMS* payoff dominated)	$\underline{0.100}$	**0.060**	0.840*	$\underline{0.105}$	0.770*	**0.125**
G8 (*NE* payoff dominated)	0.055	$\underline{\mathbf{0.135}}$	0.810*	0.045	0.735*	$\underline{\mathbf{0.220}}$
G9	0.045	$\underline{0.900}^\star$	0.055	$\underline{0.565}^\star$	0.345	0.090
G10 (*MMS* payoff dominated)	**0.075**	0.795*	$\underline{0.130}$	0.455*	$\underline{0.365}$	**0.180**
G11 (*NE* payoff dominated)	$\underline{\mathbf{0.380}}$	0.555*	0.065	0.380*	0.235	$\underline{\mathbf{0.385}}$
Experiment 2	1	2	3	1	2	3
G1	$\underline{0.975}^\star$	0.015	0.010	$\underline{0.930}^\star$	0.005	0.065
G2 (V1: no payoff dominance)	0.875*	**0.060**	$\underline{0.065}$	0.825*	**0.010**	$\underline{0.165}$
G3 (V2: *NE* payoff dominated)	0.820*	**0.125**	$\underline{0.055}$	0.775*	**0.020**	$\underline{0.205}$
G4 (V3: no payoff dominance)	0.800*	$\underline{\mathbf{0.125}}$	0.075	0.715*	$\underline{\mathbf{0.050}}$	0.235
G5 (V4: *NE* payoff dominated)	0.795*	$\underline{\mathbf{0.185}}$	0.020	0.640*	$\underline{\mathbf{0.095}}$	0.265
G6	$\underline{0.935}^\star$	0.040	0.025	0.015	0.560	$\underline{0.425}^\star$
G7 (V1: no payoff dominance)	0.745*	**0.165**	$\underline{0.090}$	**0.110**	$\underline{0.520}$	0.370*
G8 (V2: *NE* payoff dominated)	0.745*	**0.125**	$\underline{0.130}$	**0.055**	$\underline{0.650}$	0.295*
G9 (V3: no payoff dominance)	0.850*	$\underline{\mathbf{0.130}}$	0.020	$\underline{\mathbf{0.120}}$	0.560	0.320*
G10 (V4: *NE* payoff dominated)	0.760*	$\underline{\mathbf{0.230}}$	0.010	$\underline{\mathbf{0.225}}$	0.480	0.295*
G11	0.585	0.035	$\underline{0.380}^\star$	0.210	0.005	$\underline{0.785}^\star$
G12 (V1: no payoff dominance)	$\underline{0.675}$	**0.100**	0.225*	$\underline{0.260}$	**0.055**	0.685*
G13 (V2: *NE* payoff dominated)	$\underline{0.745}$	**0.050**	0.205*	$\underline{0.340}$	**0.050**	0.610*
G14 (V3: no payoff dominance)	0.735	$\underline{\mathbf{0.120}}$	0.145*	$\underline{0.195}$	$\underline{\mathbf{0.120}}$	0.685*
G15 (V4: *NE* payoff dominated)	0.590	$\underline{\mathbf{0.215}}$	0.195*	0.240	$\underline{\mathbf{0.100}}$	0.660*

Notes: $1, 2, 3$ denote the first, second, and third strategies of the game respectively for each role. For each game, strategies in the *NE* strategy profile are denoted by *, those in the *MMS* strategy profile are in **bold**, and those in the *A* strategy profile are underlined. For simplicity purposes, *MMS* is only shown when the behavioral component of the game is non-zero.

A straightforward way to analyze whether manipulation of the behavioral component affects individual behavior is to compare individual behavior across subsets of strategically equivalent games (G1-G2-G3, G6-G7-G8, G9-G10-G11 in the first experiment and G1-G2-G3-G4-G5, G6-G7-G8-G9-G10, and G11-G12-G13-G14-G15 in the second experiment). We check whether the observed differences are significant or not by performing Chi-Square test between any strategically equivalent games. Table A3 in the appendix contains the corresponding p-value for each of those tests, which show that the behavior is significantly different across strategically equivalent games. Hence, the first result is that behavior is not constant across strategically equivalent games, such that, modifying behavioral component affects individual behavior. We now comment these differences by each class of games.

We start with the three strategically equivalent harmonic games, G1 to G3, where each of them has a unique uniformly mixed NE. First, for neither of the two roles the observed frequencies are equal to the theoretical predictions (of $1/3$ for each strategy), as the subjects playing in both player roles show a bias towards the central strategy. This bias is consistent with experimental work on related zero-sum games, see Rubinstein et al. (1997), Rosenthal et al. (2003) and Crawford and Iriberri (2007). Second, in G2 and G3, once a non-zero behavioral component is added such that there is a unique MMS prediction, the strategy choice frequencies change for both of the player roles. In more detail, for the row player, the frequency of playing the third strategy increases from 0.230 to 0.695 (increment of 200%) and to 0.280 (increment of 22%) in G2 and G3, respectively. For the column player, the significance of the effect is similar. The observed frequency of playing the MMS strategy increases from 0.205 to 0.310 in G2 (increment of 55%) and from 0.225 to 0.640 in G3 (increment of 184%). As shown by the p-values in Table A3, the changes in the strategy choices from G1 to G2 and from G1 to G3 are significant for both player roles. Therefore, the addition of a behavioral component with a unique MMS (which is at the same time an A strategy profile) strategy profile does indeed modify individual behavior in harmonic games. However, it is worth remembering that in harmonic games, the MMS will always coincide with A rule predictions, so we cannot conclude that MMS by itself is relevant for behavior.

We observe a similar pattern for the five strategically equivalent sets of potential games.

For both sets of potential games in experiment 1, we start with a game with no behavioral component, G6 and G9, where the NE and the A solutions coincide, such that the observed frequencies are clearly the highest: 0.955 and 0.920 in G6, and 0.900 and 0.565 in G9, for row and column roles, respectively.[13] In the first modification, G7 and G10, where all three behavioral rules are perfectly separated, we observe that the frequencies of the NE strategies

[13] In G9 and the column player, the second strategy also gets some prevalence with a frequency of 0.34, which is compatible with $L1$ and P outcomes.

decrease, while the strategy choices by *MMS* and *A* increase. In G7 and G10, when *MMS* and *A* are directly competing with each other, both gain adherence, although *A* gets if anything more frequency than *MMS*, but are not able to deviate the behavior from *NE* by a lot. In more detail, *NE* predicted strategy changes from 0.955 to 0.84 and from 0.92 to 0.77 for G7 and from 0.90 to 0.795 and from 0.565 to 0.455 for G10, for each of the player roles, respectively. Finally, in the second modification, G8 and G11, when we keep modifying the behavioral components, *MMS* and *A* fully coincide and compete with the *NE* prediction, which they also payoff dominate, then the frequency of play for the strategies prescribed by the *NE* decrease even further, bringing the frequency of play by *MMS* and *A* rules' predictions close to the frequency of *NE*. In more detail, we observe that the play for the *NE* strategies decreases down to 0.810 and 0.735 in G8, and down to 0.555 and 0.380 in G11, for row and column players, respectively. One weakness of the design of games G8 and G11 is that on top of coinciding both *A* and *MMS* it is also the case where these two behavioral rules payoff dominate the *NE*. Thus, for potential games, we conclude that *MMS* is most relevant for behavior when it coincides with the predictions of the *A* type and when both dominate the *NE*.

The games in experiment 2 dig deeper onto the empirical relevance of *NE*, *MMS* and *A* rules. As in the two potential games in experiment 1, we start with a potential game without any behavioral component in which *NE* and *A* behavioral rules coincide (by construction) and we find, as expected, and consistent with the results in experiment 1, that this strategy profile gets very high support. In G1, 97% of row and 93% of column players coordinate on this strategy profile. In G6, 93% of the row players and 42% of column players play the *NE*, and in G11, 38% of row players and 78% of the column players play the *NE* profile.[14] What occurs when behavioral component is added? Table A3 shows that in every of the four versions, when the behavioral component is added, participants' play is significantly changed for both player roles, such that clearly individual behavior is not immune to the modification of the behavioral component. When the three behavioral rules are fully separated, see games G2-G3, G7-G8, and G12-G13, then both the *A* and the *MMS* are able to attract individual behavior such that *NE* loses frequency of play and at the same time, in the competition between *A* and *MMS*, the *A* rule wins over *MMS* with a few exceptions (row column in G3 and in G7, where *MMS* gets slightly higher frequency of play). When *A* and *MMS* coincide but there is no payoff dominance over *NE*, in G4, G9 and G14, then the combination steals even higher frequency from the *NE*, even though the majority of players still coordinate

[14] In G6, column players' most played strategy is the second strategy, and in G11 the row players' most played strategy is the first strategy, both compatible with $L1$'s prediction.

on the *NE*. Finally, as in experiment 1, in games G5, G10 and G15, when *NE* is payoff dominated by the strategy profile in which *MMS* and *A* coincide, this is when *NE* loses its adherence the most and in which the combination of *MMS* and *A* gets the highest support.

Finally, it is worth remembering that the constant-sum games we considered for the experiment were independent games of each other. By contrast with the harmonic and potential games, we did not modify and add any behavioral component.[15] Despite this, we can remark an important aspect of the observed behavior. For both games, G4 and G5, the strategy in the *NE* strategy profile was by far the highest observed choice with frequencies between 0.605 to 0.805, which is in line with the results in Rey-Biel (2009) (please see the next section to note the lack of separability between *P* rule and *NE* rules in constant-sum games). Interestingly, for row players the strategy predicted by the *MMS* profile is the second highest observed frequency, while it is the lowest for the column role.

To sum up, adding a behavioral component where we have a unique *MMS* seems to affect individual behavior because the observed behavior between strategically equivalent games changes significantly. However, and more importantly, as shown by our potential games, these changes are most relevant when the *MMS* and *A* behavioral rules predictions coincide and they payoff dominate the *NE*, leading us to conclude that it is not the *MMS* itself which has the most impact on behavior but the combination of both types of altruism such that they can even payoff dominate the *NE*. This is an important contribution over the findings of Jessie and Kendall (2022) and Kendall (2022). This result will be more clearly confirmed in the following section.

4.3 Mixture-of-types Model Estimation: Do Individuals Follow the *MMS* Behavioral Rule?

Mixture-of-types models, which are probabilistic models for representing the presence of sub-populations within an overall population, are useful to understand the prevalence of each behavioral rule on the subject sample. In this section we carry out mixture-of-types models estimation, using all 11 and 15 games where we allow for the three behavioral models we focus on: *NE*, *MMS* and *A*. In the Appendix C, Table A4, we include additional estimation results where we include more behavioral rules and also estimation results by subsets of games, that we briefly comment at the end of this section.

We assume that a subject i employing rule k follows type-k's predicted decision with

[15] Note that if we start with a constant-sum game and modify the behavioral component, the resulting game will no longer be a constant-sum game.

probability $(1-\varepsilon_k)$ but with a probability of making a mistake of $\varepsilon_k \in [0,1]$. In such a case, the individual would play each of the three available strategies uniformly at random. As in most mixture-of-types model applications, we assume that the errors are identically and independently distributed across games and that they are type-specific. The first assumption facilitates the statistical treatment of the data, while the second considers that some behavioral rules may be more difficult to follow and thus make more errors than others.

The likelihood of a particular individual of a particular type can be constructed as follows. First, let $P_k^{g,j}$ be type-k's predicted choice probability for strategy j in game g. Some rules may predict more than one strategy in a particular game. This characteristic is reflected in the vector $P_k^g = (P_k^{g,1}, P_k^{g,2}, P_k^{g,3})$ with $\sum_j P_k^{g,j} = 1$. When multiple strategies belong to the predicted set, the predicted choice probabilities are defined as choosing uniformly randomly over the predicted set. For each individual in each game, we observe the chosen strategy and whether it is consistent with k. Let $x_i^{g,j} = 1$ if strategy j is chosen by subject i in game g in the experiment and $x_i^{g,j} = 0$ otherwise. The likelihood of observing a sample $x_i = (x_i^{g,j})_{g,j}$ given type k and subject i is then:

$$L_i^k(\varepsilon_k|x_i) = \prod_g \prod_j \left[(1-\varepsilon_k)P_k^{g,j} + \frac{\varepsilon_k}{3}\right]^{x_i^{g,j}} \tag{1}$$

Second, the likelihood function is given by the sum of all the behavioral types that are considered. We include $K = 7$ behavioral models: *MMS*, *NE*, *A*, *PO*, *L1*, *P* and *O*; where p_k assigns probabilities $p = (p_1, p_2, ..., p_K)$ to each behavioral rule. Finally, and as we are interested in the behavioral rule's frequency at the sample of subjects in the experiment, we sum the log likelihood over all 200 subjects.

$$lnL(p,\varepsilon|x_i) = \sum_i ln \sum_k p_k L_i^k(\varepsilon_k|x_i) \tag{2}$$

The output from these models are the estimated frequencies for each of the behavioral models we consider, $p = (p_1, p_2, ..., p_K)$, as well as their noise levels, $\varepsilon = (\varepsilon_1, \varepsilon_2, ..., \varepsilon_K)$.

Table 6: Estimation Results

Exp 1: All 11 Games			Exp2: All 15 Games		
	p_k	ε_k		p_k	ε_k
Rules	(1)	(2)	Rules	(3)	(4)
NE	0.85	0.34	*NE*	0.79	0.46
MMS	0.05	0.80	*MMS*	0.00	—
A	0.10	0.37	*A*	0.21	0.70
LL	3695.05		*LL*	5280.73	

Notes: The table reports the estimation results for the uniform error specification for all 11 in columns 1 and 2 and for all 15 games in columns 3 and 4. Columns 1 and 3 present the estimated frequencies of each behavioral model, while columns 2 and 4 show the estimated error for each of the behavioral models.

The main results are shown in Table 6. Consistent with the results in the previous section, the most frequent behavior is that of *NE*, followed by the large majority of participants, followed by the *A* behavior. We find little evidence that the *MMS* is a relevant behavioral rule. Notice that this does not rule out the fact that the addition and modification of the behavioral component is able to modify individual behavior, which is indeed the case. However, *MMS* is able to attract individual behavior only when it fully coincides with the *A* behavior and in particular, when both *A* and *MMS* payoff dominate the *NE*. We therefore conclude that the *MMS* behavioral rule is *most* relevant for explaining individual behavior and deviations from *NE*, when it coincides with with behavioral rules with efficiency concerns, such as *A*.

Additional estimation results are shown in Table A4 in the Appendix C. Two comments are warranted. First, with regard to other behavioral rules on top of the three main we have focused on, and consistent with existing work, other two behavioral rules are important: $L1$ and P rules. $L1$ rules are important for constant-sum and potential games, while P is behaviorally very relevant for harmonic and constant-sum games. However, regarding the relevance of *NE*, *MMS* and *A*, results are qualitatively the same. Second, although we have mentioned that *A* is a refinement of strong Pareto optimal and our main focus, we could keep both strong Pareto and *A* in the econometric specification as there is some separability between them. If we include both *A* and strong Pareto, then, as expected, some of the behavior explained before by *A* is now explained by strong Pareto optimal outcomes, but the main findings remain. These results are available upon request.

5 Conclusions

In this paper, we empirically test two main questions. First, whether and when changes in the nonstrategic component of games of normal form are relevant for individual behavior. Second, after defining the *MMS*, whether *MMS* predictions are behaviorally relevant when they are clearly separated from *NE* and *A* behavioral rules and when *MMS* and *A* go hand in hand together. As they are empirical questions, we carry out two laboratory experiments.

Regarding the first question, and consistent with the work by Jessie and Kendall (2022) and Kendall (2022), we find that additions and manipulations of nonstrategic component indeed can change individual behavior, particularly when the Pareto optimality ordering of different outcomes is changed in the original game. In particular, this is the case when we put side by side, an initial game without any behavioral component and any of the versions when adding the behavioral component. In other words, individual behavior can vary substantially in strategically equivalent games. Regarding the second question, in relation to *MMS*, which captures the most important considerations of the nonstrategic component of the game, we find that its empirical relevance crucially depends on whether it reinforces *A* behavioral rule's prediction or not. How useful is then the *MMS*? From a theoretical point of view, it is an extreme form of altruism, and importantly, it is the *only* behavioral rule that depends only on the nonstrategic component. Moreover, it is very likely that *MMS* predictions will be coinciding with the *A* behavioral rule, which shows Pareto efficiency concerns and indeed the manipulations of the nonstrategic component could change the efficiency of different outcomes, as mentioned by Candogan et al. (2011). In those cases, when both forms of altruism go hand in hand, and in addition when they payoff dominate the unique *NE*, they would be most relevant to explain individual behavior and deviations from Nash equilibrium play. When *MMS* can be separated from the *A* rule predictions and they point toward a different strategy profile than the prediction by the *NE*, then we find some empirical relevance for altruism but very little empirical evidence for *MMS*. We conclude that the decomposition proposed by Candogan et al. (2011) is a useful tool to explain deviations from the *NE*, in particular when the efficiency ordering of strategy profiles is changed when the non-strategic component is added or modified.

We see two avenues for further research. First, the empirical analysis could also be applied to games with more than 3 strategies, as this would expand the possibility of separating out more than the three behavioral rules we have focused on: *NE*, *MMS* and *A*. Second, and more challenging, the analysis could be extended to games with more than two players, where a potential re-definition of *MMS* is needed.

References

Abdou, J., N. Pnevmatikos, M. Scarsini, and X. Venel (2022). Decomposition of games: some strategic considerations. *Mathematics of Operations Research 47*(1), 176–208.

Arad, A. and A. Rubinstein (2012). The 11-20 money request game: A level-k reasoning study. *American Economic Review 102*(7), 3561–73.

Camerer, C. F., T.-H. Ho, and J.-K. Chong (2004). A cognitive hierarchy model of games. *The Quarterly Journal of Economics 119*(3), 861–898.

Candogan, O., I. Menache, A. Ozdaglar, and P. A. Parrilo (2011). Flows and decompositions of games: Harmonic and potential games. *Mathematics of Operations Research 36*(3), 474–503.

Charness, G. and M. Rabin (2002). Understanding social preferences with simple tests. *The quarterly journal of economics 117*(3), 817–869.

Cooper, R., D. V. DeJong, R. Forsythe, and T. W. Ross (1992). Communication in coordination games. *The Quarterly Journal of Economics 107*(2), 739–771.

Cooper, R. W., D. V. DeJong, R. Forsythe, and T. W. Ross (1990). Selection criteria in coordination games: Some experimental results. *The American Economic Review 80*(1), 218–233.

Costa-Gomes, M., V. P. Crawford, and B. Broseta (2001). Cognition and behavior in normal-form games: An experimental study. *Econometrica 69*(5), 1193–1235.

Crawford, V. P., M. A. Costa-Gomes, and N. Iriberri (2013). Structural models of nonequilibrium strategic thinking: Theory, evidence, and applications. *Journal of Economic Literature 51*(1), 5–62.

Crawford, V. P., U. Gneezy, and Y. Rottenstreich (2008). The power of focal points is limited: Even minute payoff asymmetry may yield large coordination failures. *American Economic Review 98*(4), 1443–58.

Crawford, V. P. and N. Iriberri (2007). Fatal attraction: Salience, naivete, and sophistication in experimental" hide-and-seek" games. *American Economic Review 97*(5), 1731–1750.

Demuynck, T., C. Seel, and G. Tran (2022). An index of competitiveness and cooperativeness for normal-form games. *American Economic Journal: Microeconomics 14*(2), 215–239.

Deutsch, M. (1958). Trust and suspicion. *Journal of conflict resolution 2*(4), 265–279.

Eckel, C. C. and P. J. Grossman (2002). Sex differences and statistical stereotyping in attitudes toward financial risk. *Evolution and human behavior 23*(4), 281–295.

Esteban-Casanelles, T. and D. Gonçalves (2020). The effect of incentives on choices and beliefs in games: An experiment. Technical report, Mimeo.

Fischbacher, U. (2007). z-tree: Zurich toolbox for ready-made economic experiments. *Experimental economics 10*(2), 171–178.

Garcia-Pola, B. and N. Iriberri (2019). Naivete and sophistication in initial and repeated play in games.

Goeree, J. K. and C. A. Holt (2001). Ten little treasures of game theory and ten intuitive contradictions. *American Economic Review 91*(5), 1402–1422.

Greiner, B. (2015). Subject pool recruitment procedures: organizing experiments with orsee. *Journal of the Economic Science Association 1*(1), 114–125.

Harsanyi, J. C., R. Selten, et al. (1988). A general theory of equilibrium selection in games. *MIT Press Books 1*.

Haruvy, E. and D. O. Stahl (2007). Equilibrium selection and bounded rationality in symmetric normal-form games. *Journal of Economic Behavior & Organization 62*(1), 98–119.

Hwang, S.-H. and L. Rey-Bellet (2020). Strategic decompositions of normal form games: Zero-sum games and potential games. *Games and Economic Behavior 122*, 370–390.

Jessie, D. and R. Kendall (2022). Decomposing models of bounded rationality. *Unpublished manuscript. Available at: https://pdfs. semanticscholar. org/f2e7/042e31e85b7b6affa82258536a25ee00b874. pdf*.

Jessie, D. T. and D. G. Saari (2015). Strategic and behavioral decomposition of games.

Kalai, A. and E. Kalai (2013). Cooperation in strategic games revisited. *The Quarterly Journal of Economics 128*(2), 917–966.

Kendall, R. (2022). Decomposing coordination failure in stag hunt games. *Experimental Economics*, 1–37.

McKelvey, R. D. and T. R. Palfrey (1992). An experimental study of the centipede game. *Econometrica: Journal of the Econometric Society*, 803–836.

McKelvey, R. D. and T. R. Palfrey (1995). Quantal response equilibria for normal form games. *Games and economic behavior 10*(1), 6–38.

Mock, W. B. T. (2011). *Pareto Optimality*, pp. 808–809. Dordrecht: Springer Netherlands.

Nagel, R. (1995). Unraveling in guessing games: An experimental study. *The American economic review 85*(5), 1313–1326.

Rabin, M. (1993). Incorporating fairness into game theory and economics. *The American economic review*, 1281–1302.

Rey-Biel, P. (2009). Equilibrium play and best response to (stated) beliefs in normal form games. *Games and Economic Behavior 65*(2), 572–585.

Rosenthal, R. W., J. Shachat, and M. Walker (2003). Hide and seek in arizona. *International Journal of Game Theory 32*(2), 273–293.

Rubinstein, A., A. Tversky, and D. Heller (1997). Naive strategies in competitive games. In *Understanding strategic interaction*, pp. 394–402. Springer.

Sobel, J. (2005). Interdependent preferences and reciprocity. *Journal of economic literature 43*(2), 392–436.

Stahl, D. O. and P. W. Wilson (1994). Experimental evidence on players' models of other players. *Journal of economic behavior & organization 25*(3), 309–327.

Stahl, D. O. and P. W. Wilson (1995). On players models of other players: Theory and experimental evidence. *Games and Economic Behavior 10*(1), 218–254.

Straub, P. G. (1995). Risk dominance and coordination failures in static games. *The Quarterly Review of Economics and Finance 35*(4), 339–363.

Thaler, R. H. (1988). Anomalies: The ultimatum game. *Journal of economic perspectives 2*(4), 195–206.

Toplak, M. E., R. F. West, and K. E. Stanovich (2014). Assessing miserly information processing: An expansion of the cognitive reflection test. *Thinking & Reasoning 20*(2), 147–168.

Van Huyck, J. B., R. C. Battalio, and R. O. Beil (1990). Tacit coordination games, strategic uncertainty, and coordination failure. *The American Economic Review 80*(1), 234–248.

Van Huyck, J. B., R. C. Battalio, and R. O. Beil (1991). Strategic uncertainty, equilibrium selection, and coordination failure in average opinion games. *The Quarterly Journal of Economics 106*(3), 885–910.

A Appendix: Decomposition of Games, An Example and the Case for the 11 and 15 Experimental Games

We start by showing an example of how to compute the four-components of the decomposition of a game. We specifically decompose G4, and Figure 5 displays the decomposition of the 11 experimental games.[16]

G4 is represented in matrix notation as follows:

$$A = \begin{pmatrix} 5.45 & 6.23 & 8.33 \\ 4.18 & 6.85 & 2.24 \\ 5.11 & 0.22 & 9.02 \end{pmatrix}; \; B = \begin{pmatrix} 4.55 & 3.77 & 1.67 \\ 5.82 & 3.15 & 7.76 \\ 4.89 & 9.78 & 0.98 \end{pmatrix}$$

Recall that we can decompose a game starting either from the *strategic* component or from the *nonstrategic* components. We will start by obtaining the nonstrategic components. The kernel component is obtained as $A^{\mathscr{K}} = (\frac{1}{h^2})\mathbf{1}\mathbf{1}^{\mathbf{T}}A\mathbf{1}\mathbf{1}^{\mathbf{T}}$, and $B^{\mathscr{K}} = (\frac{1}{h^2})\mathbf{1}\mathbf{1}^{\mathbf{T}}B\mathbf{1}\mathbf{1}^{\mathbf{T}}$, for row and column player, respectively, which can be computed, alternatively, as follows:

$$k_1 = \frac{5.45+6.23+8.33+4.18+6.85+2.24+5.11+0.22+9.02}{9} = 5.29$$

$$k_2 = \frac{4.55+3.77+1.67+5.82+3.15+7.76+4.89+9.78+0.98}{9} = 4.71$$

With the kernel payoffs, which are the average of each player's payoffs, the resulting matrix of the kernel component, for each player, is:

$$A^{\mathscr{K}} = \begin{pmatrix} k_1 & k_1 & k_1 \\ k_1 & k_1 & k_1 \\ k_1 & k_1 & k_1 \end{pmatrix} = \begin{pmatrix} 5.29 & 5.29 & 5.29 \\ 5.29 & 5.29 & 5.29 \\ 5.29 & 5.29 & 5.29 \end{pmatrix}; \; B^{\mathscr{K}} = \begin{pmatrix} k_2 & k_2 & k_2 \\ k_2 & k_2 & k_2 \\ k_2 & k_2 & k_2 \end{pmatrix} = \begin{pmatrix} 4.71 & 4.71 & 4.71 \\ 4.71 & 4.71 & 4.71 \\ 4.71 & 4.71 & 4.71 \end{pmatrix}$$

As the game is a 3×3 game, we have 3 behavioral payoffs for each player. To obtain each value, for each of the opponent's strategies, we just compute the average payoff, keeping constant the opponent's strategy, and subtract the own kernel value. That is,

$$b_1^1 = \frac{5.45+4.18+5.11}{3} - 5.29 = -0.38$$

$$b_1^2 = \frac{6.23+6.85+0.22}{3} - 5.29 = -0.86$$

[16]In both cases, for simplicity, we rounded up all payoffs to two decimals, as in the experiment.

$$b_1^3 = \frac{8.33+2.24+9.02}{3} - 5.29 = 1.24$$

Analogously we obtain the behavioral payoffs for column player: $b_2^1 = -1.38$, $b_2^2 = 0.87$, and $b_2^3 = 0.51$.

The matrices of the behavioral component, given the behavioral payoffs obtained above, are:

$$A^{\mathscr{B}} = \begin{pmatrix} b_1^1 & b_1^2 & b_1^3 \\ b_1^1 & b_1^2 & b_1^3 \\ b_1^1 & b_1^2 & b_1^3 \end{pmatrix} = \begin{pmatrix} -0.38 & -0.86 & 1.24 \\ -0.38 & -0.86 & 1.24 \\ -0.38 & -0.86 & 1.24 \end{pmatrix}; \; B^{\mathscr{B}} = \begin{pmatrix} b_2^1 & b_2^1 & b_2^1 \\ b_2^2 & b_2^2 & b_2^2 \\ b_2^3 & b_2^3 & b_2^3 \end{pmatrix} = \begin{pmatrix} -1.38 & -1.38 & -1.38 \\ 0.87 & 0.87 & 0.87 \\ 0.51 & 0.51 & 0.51 \end{pmatrix}$$

To obtain the strategic component, we can either normalize the original game or take the differences between the original game and the sum of the nonstrategic component. The strategic component of the game is:

$$A^{\mathscr{S}} = \begin{pmatrix} 0.54 & 1.80 & 1.80 \\ -0.73 & 2.42 & -4.29 \\ 0.20 & -4.21 & 2.49 \end{pmatrix}; \; B^{\mathscr{S}} = \begin{pmatrix} 1.22 & 0.44 & -1.66 \\ 0.24 & -2.43 & 2.18 \\ -0.33 & 4.56 & -4.24 \end{pmatrix}$$

Once we obtain the strategic component, denoted by $A^{\mathscr{S}}$ and $B^{\mathscr{S}}$ for row and column player, respectively, we can compute the potential and harmonic components. To do so, we need to calculate first three auxiliary matrices: $M = \frac{1}{2}(A^{\mathscr{S}} + B^{\mathscr{S}})$, $D = \frac{1}{2}(A^{\mathscr{S}} - B^{\mathscr{S}})$, and $\Gamma = \frac{1}{2h}(A\mathbf{1}\mathbf{1}^{\mathbf{T}} - \mathbf{1}\mathbf{1}^{\mathbf{T}}B)$. In our case,

$$M = \frac{1}{2}\begin{pmatrix} 0.54+1.22 & 1.80+0.44 & 1.80-1.66 \\ -0.73+0.24 & 2.42-2.43 & -4.29+2.18 \\ 0.20-0.33 & -4.21+4.56 & 2.49-4.24 \end{pmatrix} = \begin{pmatrix} 0.88 & 1.12 & 0.07 \\ -0.25 & -0.01 & -1.05 \\ -0.06 & 0.17 & -0.87 \end{pmatrix}$$

$$D = \frac{1}{2}\begin{pmatrix} 0.54-1.22 & 1.80-0.44 & 1.80+1.66 \\ -0.73-0.24 & 2.42+2.43 & -4.29-2.18 \\ 0.20+0.33 & -4.21-4.56 & 2.49+4.24 \end{pmatrix} = \begin{pmatrix} -0.34 & 0.68 & 1.73 \\ -0.49 & 2.42 & -3.24 \\ 0.26 & -4.39 & 3.36 \end{pmatrix}$$

To obtain the matrix Γ we need first two more auxiliaries matrices, denoted by Γ^A and Γ^B, when $\Gamma^A = A\mathbf{1}\mathbf{1}^{\mathbf{T}}$, and $\Gamma^B = \mathbf{1}\mathbf{1}^{\mathbf{T}}B$

$$\Gamma^A = \begin{pmatrix} 0.54+1.80+1.80 & 0.54+1.80+1.80 & 0.54+1.80+1.80 \\ -0.73+2.42-4.29 & -0.73+2.42-4.29 & -0.73+2.42-4.29 \\ 0.20-4.21+2.49 & 0.20-4.21+2.49 & 0.20-4.21+2.49 \end{pmatrix} = \begin{pmatrix} 4.13 & 4.13 & 4.13 \\ -2.61 & -2.61 & -2.61 \\ -1.53 & -1.53 & -1.53 \end{pmatrix}$$

$$\Gamma^B = \begin{pmatrix} 1.22+0.24-0.33 & 0.44-2.43+4.56 & -1.66+2.18-4.24 \\ 1.22+0.24-0.33 & 0.44-2.43+4.56 & -1.66+2.18-4.24 \\ 1.22+0.24-0.33 & 0.44-2.43+4.56 & -1.66+2.18-4.24 \end{pmatrix} = \begin{pmatrix} 1.14 & 2.58 & -3.71 \\ 1.14 & 2.58 & -3.71 \\ 1.14 & 2.58 & -3.71 \end{pmatrix}$$

Then,

$$\Gamma = \frac{1}{2h}(\Gamma^A - \Gamma^B) = \frac{1}{6} \begin{pmatrix} 4.13-1.14 & 4.13-2.58 & 4.13+3.71 \\ -2.61-1.14 & -2.61-2.58 & -2.61+3.71 \\ -1.53-1.14 & -1.53-2.58 & -1.53+3.71 \end{pmatrix} = \begin{pmatrix} 0.50 & 0.26 & 1.31 \\ -0.62 & -0.86 & 0.18 \\ -0.44 & -0.68 & 0.36 \end{pmatrix}$$

Finally, the potential component is obtained as $(M+\Gamma, M-\Gamma)$ and the harmonic component as $(D-\Gamma, -D+\Gamma)$.

$$A^{\mathscr{P}} = \begin{pmatrix} 0.88+0.50 & 1.12+0.26 & 0.07+1.31 \\ -0.25-0.62 & -0.01-0.86 & -1.05+0.18 \\ -0.06-0.44 & 0.17-0.68 & -0.87+0.36 \end{pmatrix} = \begin{pmatrix} 1.38 & 1.38 & 1.38 \\ -0.87 & -0.87 & -0.87 \\ -0.51 & -0.51 & -0.51 \end{pmatrix}$$

$$B^{\mathscr{P}} = \begin{pmatrix} 0.88-0.50 & 1.12-0.26 & 0.07-1.31 \\ -0.25+0.62 & -0.01+0.86 & -1.05-0.18 \\ -0.06+0.44 & 0.17+0.68 & -0.87-0.36 \end{pmatrix} = \begin{pmatrix} 0.38 & 0.86 & -1,24 \\ 0.38 & 0.86 & -1.24 \\ 0.38 & 0.86 & -1.24 \end{pmatrix}$$

$$A^{\mathscr{H}} = \begin{pmatrix} -0.34-0.50 & 0.68-0.26 & 1.73-1.31 \\ -0.49+0.62 & -0.49+0.86 & -3.24-0.18 \\ 0.26+0.44 & -4.39+0.68 & 3.36-0.36 \end{pmatrix} = \begin{pmatrix} -0.84 & 0.42 & 0.42 \\ 0.14 & 3.29 & -3.42 \\ 0.71 & -3.70 & 3.00 \end{pmatrix}$$

$$B^{\mathscr{H}} = \begin{pmatrix} 0.34+0.50 & -0.68+0.26 & -1.73+1.31 \\ 0.49-0.62 & 0.49-0.86 & 3.24+0.18 \\ -0.26-0.44 & 4.39-0.68 & -3.36+0.36 \end{pmatrix} = \begin{pmatrix} 0.84 & -0.42 & -0.42 \\ -0.14 & -3.29 & 3.42 \\ -0.71 & 3.70 & -3.00 \end{pmatrix}$$

The final decomposition for G4 is:

$$\begin{aligned} \begin{pmatrix} 5.45,4.55 & 6.23,3.77 & 8.33,1.67 \\ 4.18,5.82 & 6.85,3.15 & 2.24,7.76 \\ 5.11,4.89 & 0.22,9.78 & 9.02,0.98 \end{pmatrix} &= \begin{pmatrix} 1.38,0.38 & 1.38,0.86 & 1.38,-1.24 \\ -0.87,0.38 & -0.87,0.86 & -0.87,-1.24 \\ -0.51,0.38 & -0.51,0.86 & -0.51,-1.24 \end{pmatrix} \\ &+ \begin{pmatrix} 0.84,-0.84 & 0.42,-0.42 & 0.42,-0.42 \\ 0.14,-0.14 & 3.29,-3.29 & -3.42,3.42 \\ 0.71,-0.71 & -3.70,3.70 & 3.00,-3.00 \end{pmatrix} \\ &+ \begin{pmatrix} -0.38,-1.38 & -0.86,-1.38 & 1.24,-1.38 \\ -0.38,0.87 & -0.86,0.87 & 1.24,0.87 \\ -0.38,0.51 & -0.86,0.51 & 1.24,0.51 \end{pmatrix} \\ &+ \begin{pmatrix} 5.29,4.71 & 5.29,4.71 & 5.29,4.71 \\ 5.29,4.71 & 5.29,4.71 & 5.29,4.71 \\ 5.29,4.71 & 5.29,4.71 & 5.29,4.71 \end{pmatrix} \end{aligned}$$

Harmonic Games

G1			Potential			Harmonic			Behavioral			Kernel		
6.62 6.56	7.62 5.56	5.62 7.56	0 0	0 0	0 0	0 0	1 -1	-1 1	0 0	0 0	0 0	6.62 6.56	6.62 6.56	6.62 6.56
5.62 7.56	6.62 6.56	7.62 5.56	0 0	0 0	0 0	-1 1	0 0	1 -1	0 0	0 0	0 0	6.62 6.56	6.62 6.56	6.62 6.56
7.62 5.56	5.62 7.56	6.62 6.56	0 0	0 0	0 0	1 -1	-1 1	0 0	0 0	0 0	0 0	6.62 6.56	6.62 6.56	6.62 6.56

G2			Potential			Harmonic			Behavioral			Kernel		
6.62 11.56	7.62 0.56	5.62 7.56	0 0	0 0	0 0	0 0	-1 1	1 -1	5 0	-5 0	0 0	6.56 6.62	6.56 6.62	6.56 6.62
0.62 12.56	1.62 1.56	2.62 5.56	0 0	0 0	0 0	1 -1	0 0	-1 1	5 -5	-5 -5	0 -5	6.56 6.62	6.56 6.62	6.56 6.62
12.62 10.56	10.62 2.56	11.62 6.56	0 0	0 0	0 0	-1 1	1 -1	0 0	5 5	-5 5	0 5	6.56 6.62	6.56 6.62	6.56 6.62

G3			Potential			Harmonic			Behavioral			Kernel		
1.56 6.62	5.56 7.62	12.56 5.62	0 0	0 0	0 0	0 0	1 -1	-1 1	0 -5	0 0	0 5	6.62 6.56	6.62 6.56	6.62 6.56
2.56 0.62	6.56 1.62	10.56 2.62	0 0	0 0	0 0	-1 1	0 0	1 -1	-5 -5	-5 0	-5 5	6.62 6.56	6.62 6.56	6.62 6.56
0.56 12.62	7.56 10.62	11.56 11.62	0 0	0 0	0 0	1 -1	-1 1	0 0	5 -5	5 0	5 5	6.62 6.56	6.62 6.56	6.62 6.56

Constant-Sum Games

G4			Potential			Harmonic			Behavioral			Kernel		
4.55 5.45	3.77 6.23	1.67 8.33	0.38 1.38	0.86 1.38	-1.24 1.38	0.84 -0.84	-0.42 0.42	-0.42 0.42	-1.38 -0.38	-1.38 -0.86	-1.38 1.24	4.71 5.29	4.71 5.29	4.71 5.29
5.82 4.18	3.15 6.85	7.76 2.24	0.38 -0.87	0.86 -0.87	-1.24 -0.87	-0.14 0.14	-3.29 3.29	3.42 -3.42	0.87 -0.38	0.87 -0.86	0.87 1.24	4.71 5.29	4.71 5.29	4.71 5.29
4.89 5.11	9.78 0.22	0.98 9.02	0.38 -0.51	0.86 -0.51	-1.24 -0.51	-0.71 0.71	3.70 -3.70	-3.00 3.00	0.51 -0.38	0.51 -0.86	0.51 1.24	4.71 5.29	4.71 5.29	4.71 5.29

G5			Potential			Harmonic			Behavioral			Kernel		
3.11 6.89	4.80 5.20	6.67 3.33	0.70 -0.64	0.61 -0.64	-1.32 -0.64	-2.45 2.45	-0.67 0.67	3.13 -3.13	0.64 -0.70	0.64 -0.61	0.64 1.32	4.22 5.78	4.22 5.78	4.22 5.78
3.95 6.05	3.97 6.03	1.90 8.10	0.70 0.95	0.61 0.95	-1.32 0.95	-0.03 0.03	0.08 -0.08	-0.06 0.06	-0.95 -0.70	-0.95 -0.61	-0.95 1.32	4.22 5.78	4.22 5.78	4.22 5.78
7.72 2.28	5.74 4.26	0.15 9.85	0.70 -0.31	0.61 -0.31	-1.32 -0.31	2.48 -2.48	0.59 -0.59	-3.07 3.07	0.31 -0.70	0.31 -0.61	0.31 1.32	4.22 5.78	4.22 5.78	4.22 5.78

Potential Games: First Set

G6			Potential			Harmonic			Behavioral			Kernel		
$8.529\overline{4}$ $7.182\overline{7}$	$7.62\overline{4}$ $4.05\overline{1}$	$8.592\overline{7}$ $6.656\overline{1}$	$0.280\overline{5}$ $0.682\overline{7}$	$-0.62\overline{4}$ $-2.44\overline{8}$	$0.343\overline{8}$ $0.156\overline{1}$	0 0	0 0	0 0	0 0	0 0	0 0	8.25 6.50	8.25 6.50	8.25 6.50
$8.28\overline{7}$ $6.82\overline{4}$	$8.432\overline{7}$ $4.742\overline{7}$	$8.026\overline{1}$ $5.982\overline{7}$	$0.03\overline{8}$ $0.32\overline{4}$	$0.183\overline{8}$ $-1.757\overline{2}$	$-0.222\overline{7}$ $-0.527\overline{2}$	0 0	0 0	0 0	0 0	0 0	0 0	8.25 6.50	8.25 6.50	8.25 6.50
$5.112\overline{7}$ $5.492\overline{7}$	$12.552\overline{7}$ $10.706\overline{1}$	$7.08\overline{3}$ $6.87\overline{1}$	$-3.136\overline{1}$ $-1.007\overline{2}$	$4.303\overline{8}$ $4.206\overline{1}$	$-1.1\overline{6}$ $0.37\overline{1}$	0 0	0 0	0 0	0 0	0 0	0 0	8.25 6.50	8.25 6.50	8.25 6.50

G7			Potential			Harmonic			Behavioral			Kernel		
$9.680\overline{5}$ $8.442\overline{7}$	$8.77\overline{5}$ $1.45\overline{1}$	$9.743\overline{8}$ $7.996\overline{1}$	$0.280\overline{5}$ $0.682\overline{7}$	$-0.62\overline{4}$ $-2.44\overline{8}$	$0.343\overline{8}$ $0.156\overline{1}$	0 0	0 0	0 0	1.15 1.26	1.15 -2.60	1.15 1.34	8.25 6.50	8.25 6.50	8.25 6.50
$9.72\overline{8}$ $8.08\overline{4}$	$9.873\overline{8}$ $2.142\overline{7}$	$9.467\overline{2}$ $7.312\overline{7}$	$0.03\overline{8}$ $0.32\overline{4}$	$0.183\overline{8}$ $-1.757\overline{2}$	$-0.222\overline{7}$ $-0.527\overline{2}$	0 0	0 0	0 0	1.44 1.26	1.44 -2.60	1.44 1.34	8.25 6.50	8.25 6.50	8.25 6.50
$2.523\overline{8}$ $6.752\overline{7}$	$9.963\overline{8}$ $8.106\overline{1}$	$4.49\overline{2}$ $8.2\overline{1}$	$-3.136\overline{1}$ $-1.007\overline{2}$	$4.303\overline{8}$ $4.206\overline{1}$	$-1.1\overline{6}$ $0.37\overline{1}$	0 0	0 0	0 0	-2.59 1.26	-2.59 -2.60	-2.59 1.34	8.25 6.50	8.25 6.50	8.25 6.50

G8			Potential			Harmonic			Behavioral			Kernel		
$8.330\overline{5}$ $6.982\overline{7}$	$7.42\overline{5}$ $0.05\overline{1}$	$8.393\overline{8}$ $10.856\overline{1}$	$0.280\overline{5}$ $0.682\overline{7}$	$-0.62\overline{4}$ $-2.44\overline{8}$	$0.343\overline{8}$ $0.156\overline{1}$	0 0	0 0	0 0	-0.20 -0.20	-0.20 -4.00	-0.20 4.20	8.25 6.50	8.25 6.50	8.25 6.50
$12.4\overline{8}$ $6.62\overline{4}$	$12.633\overline{8}$ $0.742\overline{7}$	$12.22\overline{7}$ $10.172\overline{7}$	$0.03\overline{8}$ $0.32\overline{4}$	$0.183\overline{8}$ $-1.757\overline{2}$	$-0.222\overline{7}$ $-0.527\overline{2}$	0 0	0 0	0 0	4.20 -0.20	4.20 -4.00	4.20 4.20	8.25 6.50	8.25 6.50	8.25 6.50
$1.113\overline{8}$ $5.292\overline{7}$	$8.553\overline{8}$ $6.706\overline{1}$	$3.08\overline{2}$ $11.07\overline{1}$	$-3.136\overline{1}$ $-1.007\overline{2}$	$4.303\overline{8}$ $4.206\overline{1}$	$-1.1\overline{6}$ $0.37\overline{1}$	0 0	0 0	0 0	-4.00 -0.20	-4.00 -4.00	-4.00 4.20	8.25 6.50	8.25 6.50	8.25 6.50

Potential Games: Second Set

G9			Potential			Harmonic			Behavioral			Kernel		
5.56 7.53	5.22 6.88	5.72 7.41	0.06 0.03	-0.28 -0.62	0.22 -0.09	0 0	0 0	0 0	0 0	0 0	0 0	5.50 7.50	5.50 7.50	5.50 7.50
$6.32\overline{3}$ 9.15	$5.32\overline{3}$ 7.84	$4.85\overline{3}$ 7.40	$0.82\overline{3}$ 1.65	$-0.17\overline{6}$ 0.34	$-0.64\overline{6}$ -0.10	0 0	0 0	0 0	0 0	0 0	0 0	5.50 7.50	5.50 7.50	5.50 7.50
$4.02\overline{6}$ 5.82	$6.29\overline{6}$ 7.78	$6.17\overline{6}$ 7.69	$-1.47\overline{3}$ -1.68	$0.79\overline{6}$ 0.28	$0.67\overline{6}$ 0.19	0 0	0 0	0 0	0 0	0 0	0 0	5.50 7.50	5.50 7.50	5.50 7.50

G10			Potential			Harmonic			Behavioral			Kernel		
5.86 6.63	5.52 7.28	6.02 7.91	0.06 0.03	-0.28 -0.62	0.22 -0.09	0 0	0 0	0 0	0.30 -0.90	0.30 0.40	0.30 0.50	5.50 7.50	5.50 7.50	5.50 7.50
$6.12\overline{3}$ 8.25	$5.12\overline{3}$ 8.24	$4.65\overline{3}$ 7.90	$0.82\overline{3}$ 1.65	$-0.17\overline{6}$ 0.34	$-0.64\overline{6}$ -0.10	0 0	0 0	0 0	-0.20 -0.90	-0.20 0.40	-0.20 0.50	5.50 7.50	5.50 7.50	5.50 7.50
$3.92\overline{6}$ 4.92	$6.19\overline{6}$ 8.18	$6.07\overline{6}$ 8.19	$-1.47\overline{3}$ -1.68	$0.79\overline{6}$ 0.28	$0.67\overline{6}$ 0.19	0 0	0 0	0 0	-0.10 -0.90	-0.10 0.40	-0.10 0.50	5.50 7.50	5.50 7.50	5.50 7.50

G11			Potential			Harmonic			Behavioral			Kernel		
9.86 3.53	9.52 6.58	10.02 11.71	0.06 0.03	-0.28 -0.62	0.22 -0.09	0 0	0 0	0 0	4.30 -4.00	4.30 -0.30	4.30 4.30	5.50 7.50	5.50 7.50	5.50 7.50
$2.32\overline{3}$ 5.15	$1.32\overline{3}$ 7.54	$0.85\overline{3}$ 11.70	$0.82\overline{3}$ 1.65	$-0.17\overline{6}$ 0.34	$-0.64\overline{6}$ -0.10	0 0	0 0	0 0	-4.00 -4.00	-4.00 -0.30	-4.00 4.30	5.50 7.50	5.50 7.50	5.50 7.50
$3.72\overline{6}$ 1.82	$5.99\overline{6}$ 7.48	$5.87\overline{6}$ 11.99	$-1.47\overline{3}$ -1.68	$0.79\overline{6}$ 0.28	$0.67\overline{6}$ 0.19	0 0	0 0	0 0	-0.30 -4.00	-0.30 -0.30	-0.30 4.30	5.50 7.50	5.50 7.50	5.50 7.50

Notes: The figure displays the 11 experimental games used in the experiments and the corresponding four-components decomposition. In some games and components the actual value is periodic, $1.23\overline{4}$ denotes the periodicity of the third decimal.

Figure 5: Decomposition of the 11 Experimental Games

G1				Potential				Harmonic				Behavioral				Kernel		
34 / 34	12 / 21	29 / 31		9 / 9	-13 / -4	4 / 6		0 / 0	0 / 0	0 / 0		0 / 0	0 / 0	0 / 0		25 / 25	25 / 25	25 / 25
31 / 25	24 / 27	20 / 16	=	6 / 0	-1 / 2	-5 / -9	+	0 / 0	0 / 0	0 / 0	+	0 / 0	0 / 0	0 / 0	+	25 / 25	25 / 25	25 / 25
21 / 16	23 / 27	31 / 28		-4 / -9	-2 / 2	6 / 3		0 / 0	0 / 0	0 / 0		0 / 0	0 / 0	0 / 0		25 / 25	25 / 25	25 / 25

G2				Potential				Harmonic				Behavioral				Kernel		
31 / 29	9 / 25	26 / 32		9 / 9	-13 / -4	4 / 6		0 / 0	0 / 0	0 / 0		-3 / -5	-3 / 4	-3 / 1		25 / 25	25 / 25	25 / 25
33 / 20	26 / 31	22 / 17	=	6 / 0	-1 / 2	-5 / -9	+	0 / 0	0 / 0	0 / 0	+	2 / -5	2 / 4	2 / 1	+	25 / 25	25 / 25	25 / 25
22 / 11	24 / 31	32 / 29		-4 / -9	-2 / 2	6 / 3		0 / 0	0 / 0	0 / 0		1 / -5	1 / 4	1 / 1		25 / 25	25 / 25	25 / 25

G3				Potential				Harmonic				Behavioral				Kernel		
28 / 27	6 / 26	23 / 33		9 / 9	-13 / -4	4 / 6		0 / 0	0 / 0	0 / 0		-6 / -7	-6 / 5	-6 / 2		25 / 25	25 / 25	25 / 25
36 / 18	29 / 32	25 / 18	=	6 / 0	-1 / 2	-5 / -9	+	0 / 0	0 / 0	0 / 0	+	5 / -7	5 / 5	5 / 2	+	25 / 25	25 / 25	25 / 25
22 / 9	24 / 32	32 / 30		-4 / -9	-2 / 2	6 / 3		0 / 0	0 / 0	0 / 0		1 / -7	1 / 5	1 / 2		25 / 25	25 / 25	25 / 25

G4				Potential				Harmonic				Behavioral				Kernel		
29 / 26	7 / 28	24 / 32		9 / 9	-13 / -4	4 / 6		0 / 0	0 / 0	0 / 0		-5 / -8	-5 / 7	-5 / 1		25 / 25	25 / 25	25 / 25
35 / 17	28 / 34	24 / 17	=	6 / 0	-1 / 2	-5 / -9	+	0 / 0	0 / 0	0 / 0	+	4 / -8	4 / 7	4 / 1	+	25 / 25	25 / 25	25 / 25
22 / 8	24 / 34	32 / 29		-4 / -9	-2 / 2	6 / 3		0 / 0	0 / 0	0 / 0		1 / -8	1 / 7	1 / 1		25 / 25	25 / 25	25 / 25

G5				Potential				Harmonic				Behavioral				Kernel		
29 / 29	7 / 31	24 / 26		9 / 9	-13 / -4	4 / 6		0 / 0	0 / 0	0 / 0		-5 / -5	-5 / 10	-5 / -5		25 / 25	25 / 25	25 / 25
41 / 20	34 / 37	30 / 11	=	6 / 0	-1 / 2	-5 / -9	+	0 / 0	0 / 0	0 / 0	+	10 / -5	10 / 10	10 / -5	+	25 / 25	25 / 25	25 / 25
16 / 11	18 / 37	26 / 23		-4 / -9	-2 / 2	6 / 3		0 / 0	0 / 0	0 / 0		-5 / -5	-5 / 10	-5 / -5		25 / 25	25 / 25	25 / 25

Figure 6: Decomposition of the 15 Experimental Games: First Set of 5

Second Set

G6				Potential				Harmonic				Behavioral				Kernel		
15 / 23	29 / 31	31 / 38		-10 / -2	4 / 6	6 / 13		0 / 0	0 / 0	0 / 0		0 / 0	0 / 0	0 / 0		25 / 25	25 / 25	25 / 25
26 / 27	22 / 17	27 / 27	=	1 / 2	-3 / -8	2 / 2	+	0 / 0	0 / 0	0 / 0	+	0 / 0	0 / 0	0 / 0	+	25 / 25	25 / 25	25 / 25
27 / 25	35 / 27	13 / 10		2 / 0	10 / 2	-12 / -15		0 / 0	0 / 0	0 / 0		0 / 0	0 / 0	0 / 0		25 / 25	25 / 25	25 / 25

G7				Potential				Harmonic				Behavioral				Kernel		
11 / 26	25 / 31	27 / 35		-10 / -2	4 / 6	6 / 13		0 / 0	0 / 0	0 / 0		-4 / 3	-4 / 0	-4 / -3		25 / 25	25 / 25	25 / 25
29 / 30	25 / 17	30 / 24	=	1 / 2	-3 / -8	2 / 2	+	0 / 0	0 / 0	0 / 0	+	3 / 3	3 / 0	3 / -3	+	25 / 25	25 / 25	25 / 25
28 / 28	36 / 27	14 / 7		2 / 0	10 / 2	-12 / -15		0 / 0	0 / 0	0 / 0		1 / 3	1 / 0	1 / -3		25 / 25	25 / 25	25 / 25

G8				Potential				Harmonic				Behavioral				Kernel		
11 / 29	25 / 34	27 / 29		-10 / -2	4 / 6	6 / 13		0 / 0	0 / 0	0 / 0		-4 / 6	-4 / 3	-4 / -9		25 / 25	25 / 25	25 / 25
29 / 33	25 / 20	30 / 18	=	1 / 2	-3 / -8	2 / 2	+	0 / 0	0 / 0	0 / 0	+	3 / 6	3 / 3	3 / -9	+	25 / 25	25 / 25	25 / 25
28 / 31	36 / 30	14 / 1		2 / 0	10 / 2	-12 / -15		0 / 0	0 / 0	0 / 0		1 / 6	1 / 3	1 / -9		25 / 25	25 / 25	25 / 25

G9				Potential				Harmonic				Behavioral				Kernel		
11 / 28	25 / 30	27 / 34		-10 / -2	4 / 6	6 / 13		0 / 0	0 / 0	0 / 0		-4 / 5	-4 / -1	-4 / -4		25 / 25	25 / 25	25 / 25
31 / 32	27 / 16	32 / 23	=	1 / 2	-3 / -8	2 / 2	+	0 / 0	0 / 0	0 / 0	+	5 / 5	5 / -1	5 / -4	+	25 / 25	25 / 25	25 / 25
26 / 30	34 / 26	12 / 6		2 / 0	10 / 2	-12 / -15		0 / 0	0 / 0	0 / 0		-1 / 5	-1 / -1	-1 / -4		25 / 25	25 / 25	25 / 25

G10				Potential				Harmonic				Behavioral				Kernel		
10 / 33	24 / 26	26 / 33		-10 / -2	4 / 6	6 / 13		0 / 0	0 / 0	0 / 0		-5 / 10	-5 / -5	-5 / -5		25 / 25	25 / 25	25 / 25
36 / 37	32 / 12	37 / 22	=	1 / 2	-3 / -8	2 / 2	+	0 / 0	0 / 0	0 / 0	+	10 / 10	10 / -5	10 / -5	+	25 / 25	25 / 25	25 / 25
22 / 35	30 / 22	8 / 5		2 / 0	10 / 2	-12 / -15		0 / 0	0 / 0	0 / 0		-5 / 10	-5 / -5	-5 / -5		25 / 25	25 / 25	25 / 25

Figure 7: Decomposition of the 15 Experimental Games: Second Set of 5

Third Set

G11 = Potential + Harmonic + Behavioral + Kernel

G11

30 / 30	14 / 21	31 / 29
31 / 27	24 / 27	20 / 14
19 / 18	21 / 27	35 / 32

Potential

5 / 5	-11 / -4	6 / 4
6 / 2	-1 / 2	-5 / -11
-6 / -7	-4 / 2	10 / 7

Harmonic

0 / 0	0 / 0	0 / 0
0 / 0	0 / 0	0 / 0
0 / 0	0 / 0	0 / 0

Behavioral

0 / 0	0 / 0	0 / 0
0 / 0	0 / 0	0 / 0
0 / 0	0 / 0	0 / 0

Kernel

25 / 25	25 / 25	25 / 25
25 / 25	25 / 25	25 / 25
25 / 25	25 / 25	25 / 25

G12 = Potential + Harmonic + Behavioral + Kernel

G12

31 / 29	15 / 27	32 / 24
33 / 26	26 / 33	22 / 9
16 / 17	18 / 33	32 / 27

Potential

5 / 5	-11 / -4	6 / 4
6 / 2	-1 / 2	-5 / -11
-6 / -7	-4 / 2	10 / 7

Harmonic

0 / 0	0 / 0	0 / 0
0 / 0	0 / 0	0 / 0
0 / 0	0 / 0	0 / 0

Behavioral

1 / -1	1 / 6	1 / -5
2 / -1	2 / 6	2 / -5
-3 / -1	-3 / 6	-3 / -5

Kernel

25 / 25	25 / 25	25 / 25
25 / 25	25 / 25	25 / 25
25 / 25	25 / 25	25 / 25

G13 = Potential + Harmonic + Behavioral + Kernel

G13

34 / 32	18 / 27	35 / 21
36 / 29	29 / 33	25 / 6
10 / 20	12 / 33	26 / 24

Potential

5 / 5	-11 / -4	6 / 4
6 / 2	-1 / 2	-5 / -11
-6 / -7	-4 / 2	10 / 7

Harmonic

0 / 0	0 / 0	0 / 0
0 / 0	0 / 0	0 / 0
0 / 0	0 / 0	0 / 0

Behavioral

4 / 2	4 / 6	4 / -8
5 / 2	5 / 6	5 / -8
-9 / 2	-9 / 6	-9 / -8

Kernel

25 / 25	25 / 25	25 / 25
25 / 25	25 / 25	25 / 25
25 / 25	25 / 25	25 / 25

G14 = Potential + Harmonic + Behavioral + Kernel

G14

28 / 29	12 / 27	29 / 24
37 / 26	30 / 33	26 / 9
15 / 17	17 / 33	31 / 27

Potential

5 / 5	-11 / -4	6 / 4
6 / 2	-1 / 2	-5 / -11
-6 / -7	-4 / 2	10 / 7

Harmonic

0 / 0	0 / 0	0 / 0
0 / 0	0 / 0	0 / 0
0 / 0	0 / 0	0 / 0

Behavioral

-2 / -1	-2 / 6	-2 / -5
6 / -1	6 / 6	6 / -5
-4 / -1	-4 / 6	-4 / -5

Kernel

25 / 25	25 / 25	25 / 25
25 / 25	25 / 25	25 / 25
25 / 25	25 / 25	25 / 25

G15 = Potential + Harmonic + Behavioral + Kernel

G15

25 / 25	9 / 31	26 / 24
41 / 22	34 / 37	30 / 9
14 / 13	16 / 37	30 / 27

Potential

5 / 5	-11 / -4	6 / 4
6 / 2	-1 / 2	-5 / -11
-6 / -7	-4 / 2	10 / 7

Harmonic

0 / 0	0 / 0	0 / 0
0 / 0	0 / 0	0 / 0
0 / 0	0 / 0	0 / 0

Behavioral

-5 / -5	-5 / 10	-5 / -5
10 / -5	10 / 10	10 / -5
-5 / -5	-5 / 10	-5 / -5

Kernel

25 / 25	25 / 25	25 / 25
25 / 25	25 / 25	25 / 25
25 / 25	25 / 25	25 / 25

Figure 8: Decomposition of the 15 Experimental Games: Third Set of 5

B Appendix: Proofs

A two-person game $\mathscr{G}$ can be written as a $h \times h$ bimatrix game (A,B) with $a_{ij} = u_1(s_i,t_j)$ $(i,j = 1,...,h)$ and $b_{ij} = u_2(s_i,t_j)$ $(i,j = 1,...,h)$. To be specific,

$$A = \begin{pmatrix} a_{11} & \cdots & a_{1h} \\ \vdots & \ddots & \vdots \\ a_{h1} & \cdots & a_{hh} \end{pmatrix} \text{ and } B = \begin{pmatrix} b_{11} & \cdots & b_{1h} \\ \vdots & \ddots & \vdots \\ b_{h1} & \cdots & b_{hh} \end{pmatrix}$$

Proof of Proposition 1. A strategy profile $(\widetilde{s_i},\widetilde{t_j}) \in S \times T$ in game $\mathscr{G}$ is *mutual-max-sum MMS* if:

$$\widetilde{s_i} \in \arg\max_{s_i \in S} \sum_{t_j \in T} u_2(s_i,t_j) \text{ and } \widetilde{t_j} \in \arg\max_{t_j \in T} \sum_{s_i \in S} u_1(s_i,t_j).$$

We now define the following matrices $\widetilde{A}$, $\widetilde{B}$ as follows: the players add their opponent's payoffs for each of their own strategies.

$$\widetilde{A} = \begin{pmatrix} \sum_j b_{1j} & \cdots & \sum_j b_{1j} \\ \vdots & \ddots & \vdots \\ \sum_j b_{hj} & \cdots & \sum_j b_{hj} \end{pmatrix} \text{ and } \widetilde{B} = \begin{pmatrix} \sum_i a_{i1} & \cdots & \sum_i a_{ih} \\ \vdots & \ddots & \vdots \\ \sum_i a_{i1} & \cdots & \sum_i a_{ih} \end{pmatrix}$$

The *MMS* can be alternatively defined by requiring from each player to choose the strategies with the highest payoff in matrices $\widetilde{A}$ and $\widetilde{B}$.

Next, the nonstrategic component $(A^{\mathscr{NS}},B^{\mathscr{NS}})$ can be displayed as follows:

$$A^{\mathscr{NS}} = \frac{1}{h}\begin{pmatrix} \sum_i a_{i1} & \cdots & \sum_i a_{ih} \\ \vdots & \ddots & \vdots \\ \sum_i a_{i1} & \cdots & \sum_i a_{ih} \end{pmatrix} \text{ and } B^{\mathscr{NS}} = \frac{1}{h}\begin{pmatrix} \sum_j b_{1j} & \cdots & \sum_j b_{1j} \\ \vdots & \ddots & \vdots \\ \sum_j b_{hj} & \cdots & \sum_j b_{hj} \end{pmatrix}$$

Observe that $\widetilde{B} = \frac{1}{h} \times A^{\mathscr{NS}}$ and $\widetilde{A} = \frac{1}{h} \times B^{\mathscr{NS}}$. Since the altruistic solution in $(A^{\mathscr{NS}},B^{\mathscr{NS}})$ is computed by selecting the strategy profile that maximizes the sum of both players payoffs, then we can state that the *MMS* solution in (A,B), computing by selecting the strategies with the highest payoffs in $\widetilde{A}$ and $\widetilde{B}$, coincides with the altruistic solution in $(A^{\mathscr{NS}},B^{\mathscr{NS}})$. Finally, the behavioral component is defined by,

$$(A^{\mathscr{B}},B^{\mathscr{B}}) = (A^{\mathscr{NS}},B^{\mathscr{NS}}) - (A^{\mathscr{K}},B^{\mathscr{K}}).$$

Given that subtracting the kernel component $(A^{\mathscr{K}},B^{\mathscr{K}})$ means subtracting a fixed amount for each player then the *altruistic* solution in $(A^{\mathscr{N}\mathscr{S}}, B^{\mathscr{N}\mathscr{S}})$ coincides with the *altruistic* solution in $(A^{\mathscr{B}}, B^{\mathscr{B}})$ and consequently with the *MMS* solution in (A, B). □

Proof of Proposition 2. Let (A, B) be a *constant-sum game* in which an amount $C > 0$ is to be divided between players 1 and 2. W.l.o.g. let (a_{11}, b_{11}) be the Nash equilibrium and the MMS outcome of the game.

First, we show that $a_{1j} > a_{i1}$, $i = j \neq 1$.

Since (a_{11}, b_{11}) is the NE outcome, necessarily $b_{1j} \leq b_{11}$ and $a_{i1} \leq a_{11}$ for i, $j = 2, ..., h$. By definition of the constant-sum-game, $a_{1j} + b_{1j} = a_{11} + b_{11} = C$. As $b_{1j} \leq b_{11}$ we have $a_{11} \leq a_{1j}$ and by NE outcome we have that $a_{i1} \leq a_{11} \leq a_{1j}$. Therefore, by transitivity we have:

$$a_{1j} \geq a_{i1},\ i = j \neq 1 \tag{3}$$

Second, given (a_{11}, b_{11}) is the *MMS* outcome, by definition we have:

$$\begin{aligned} \text{for each } i &= 2, ..., h,\ \sum_{j=1}^{h} b_{1j} > \sum_{j=1}^{h} b_{ij} \text{ , and} \\ \text{for each } j &= 2, ..., h,\ \sum_{i=1}^{h} a_{i1} > \sum_{i=1}^{h} a_{ij} \end{aligned}$$

Summing over these two expressions we obtain:

$$(h-1)\sum_{j=1}^{h} b_{1j} + (h-1)\sum_{i=1}^{h} a_{i1} > \sum_{j=1}^{h} b_{2j} + ... + \sum_{j=1}^{h} b_{hj} + \sum_{i=1}^{h} a_{i2} + ... + \sum_{i=1}^{h} a_{ih}$$

Considering that for each i, j, $b_{ij} = C - a_{ij}$ and substituting it in the previous expression, by simple algebraic manipulations, we obtain:

$$\sum_{i=2}^{h} a_{i1} > \sum_{j=2}^{h} a_{1j}$$

which contradicts condition (3). □

C Appendix: Additional Tables

Table A1: Poolability of Sessions: p-value of Chi-Square Test

	First Experiment			Second Experiment		
H_0	Overall	Type 1	Type 2	Overall	Type 1	Type 2
S1 = Rest	0.7580	0.7982	0.9310	0.9987	0.9980	0.9982
S2 = Rest	0.6353	0.7301	0.2378	0.9992	0.9964	0.9998
S3 = Rest	0.7592	0.4227	0.9338	0.9957	0.9973	0.9936
S4 = Rest	0.5858	0.5763	0.6310	0.9993	0.9995	0.9988
S5 = Rest	0.9864	0.1004	0.0693	0.9994	0.9978	0.9983

Notes: The null hypotheses are $H_0 : \mu_1 = \mu_2$ where μ_1 and μ_2 correspond to the means of the distributions of the strategy choices for participants for a given session and for the remaining sessions jointly, respectively. For p-values lower than the significance level, the null hypothesis is rejected in favor of the alternative, $H_1 : \mu_1 \neq \mu_2$. S1, S2,..., S5 refer to different sessions, while Rest refers to the remaining sessions pooled together. Type 1 and 2 refer to those subjects who played first as row players and then as column players and then the other way round, respectively. Results are robust to using paired two-sided t-test and Kolgomorov-Smirnof test.

Table A2: Player Role Order Effects: p-value of Chi-Square Test

	H_0	Overall
Experiment 1	Type 1 = Type 2	0.2616
Experiment 2	Type 1 = Type 2	0.2834

Notes: The null hypotheses are $H_0 : \mu_1 = \mu_2$ where μ_1 and μ_2 correspond to the means of the distributions of the strategy choices for participants labeled as type 1 (started as a row player and then as a column players) and type 2 (started as a column player and then as a row player), respectively, for a given session. For p-values lower than the significance level, the null hypothesis is rejected in favor of the alternative, $H_1 : \mu_1 \neq \mu_2$. Results are robust to using paired two-sided t-test and Kolgomorov-Smirnof test.

Table A3: Significance of the Behavioral Effects, p-value of Chi-Square Test

	Row Players	Column Players
Experiment 1		Harmonic
G1-G2	$2.2e-16$	$2.2e-16$
G1-G3	0.0415	$5.35e-13$
		Potential 1
G6-G7	0.0007	0.0002
G6-G8	0.0099	0.0005
		Potential 2
G9-G10	0.0118	0.0144
G9-G11	$8.77e-13$	$3.67e-08$
Experiment 2		Potential 1
G1-G2	0.0005	0.0026
G1-G3	0.0001	0.0001
G1-G4	0.0001	0.0001
G1-G5	0.0001	0.0001
		Potential 2
G6-G7	0.0001	0.0002
G6-G8	0.0001	0.0042
G6-G9	0.0033	0.0001
G6-G10	0.0001	0.0001
		Potential 3
G11-G12	0.0006	0.0038
G11-G13	0.0005	0.0002
G11-G14	0.0001	0.0001
G11-G15	0.0001	0.0001

Notes: The null hypotheses are $H_0 : \mu_1 = \mu_2$ where μ_1 and μ_2 correspond to the means of the distributions of the strategy choices for each of the two games considered (first column), respectively. For p-values lower than the significance level, the null hypothesis is rejected in favor of the alternative, $H_1 : \mu_1 \neq \mu_2$. Results are similar, although qualitatively weaker, when using paired two-sided t-test and Kolgomorov-Smirnof test.

Table A4: Estimation Results

Experiment 1: 11 Games

	All 11 Games			Harmonic			CSG			Potential	
	p_k	ε_k		p_k	ε_k		p_k	ε_k		p_k	ε_k
Rules	(1)	(2)	Rules	(3)	(4)	Rules	(5)	(6)	Rules	(7)	(8)
NE	0.45	0.30	$NE = L1$	0.00	–	$NE = P$	0.78	0.35	NE	0.53	0.28
MMS	0.06	0.80	$MMS = A$	0.19	0.25	MMS	0.00	–	MMS	0.06	0.64
A	0.09	0.35				A	–	–	A	0.08	0.38
$L1$	0.12	0.36				$L1$	0.17	0.40	$L1$	0.18	0.27
P	0.27	0.23	P	0.58	0.45				P	0.12	0.12
O	0.01	0.47	O	0.23	0.95	O	0.00	–	O	0.03	0.40
LL	3502.49			1191.55			603.33			1672.12	

Experiment 2: 15 Games

	All 15 Games	
	p_k	ε_k
Rules	(1)	(2)
NE	0.15	0.30
MMS	0.05	0.80
A	0.09	0.49
$L1$	0.55	0.20
P	0.08	0.46
O	0.13	0.66
LL	3930.14	

Notes: The first part of the table reports the estimation results for the uniform error specification for all 11 games, in columns 1 and 2, for the three harmonic games, in columns 3 and 4, for the two constant-sum games in columns 5 and 6 and for the 6 potential games, in columns 7 and 8. Columns 1, 3, 5 and 7 present the estimated frequencies of each behavioral model, while columns 2, 4, 6 and 8 show the estimated error for each of the behavioral models. All models are identifiable in all 11 games and in the 6 potential games. In the harmonic games, NE and $L1$ are confounded, as well as MMS and A. In the constant-sum games, NE and P are confounded. The second part of the table reports the estimation results for the uniform error specification for all 15 games in columns 1 and 2.

D Appendix: English Translation of Experimental Instructions

[The original experimental instructions were in Spanish.]
[These general instructions were read aloud and provided in paper only.]
[The experimental instructions were identical between experiments 1 and 2 with a few exceptions, which will be explained below in []. N is equal to 11 in experiment 1 and equal to 15 in experiment 2.]

THANK YOU FOR PARTICIPATING IN OUR EXPERIMENT!

Let's start the experiment. From now on, you are not allowed to talk, watch what other participants are doing or walk around the classroom. Please turn off and put away your mobile phone. If you have any questions or need help, raise your hand and one of the researchers will come and talk to you. Please do not write over these instructions. If you do not comply with these rules, YOU WILL BE ASKED TO LEAVE THE EXPERIMENT WITH NO PAYMENT. Thank you.

The University of the Basque Country UPV/EHU and the research projects have provided the funds for this experiment. **You will receive 3 Euros for coming on time.** Additionally, if you follow the instructions correctly you have the chance to win more money. This is a group experiment. The amount you can earn depends on your decisions, the decisions of other participants, as well as on chance. Different participants can earn different amounts.

No participant will be able to identify any other participant by his or her decisions or by his or her earnings in the experiment. We, the researchers, will be able to observe at the end of the experiment the earnings of each participant, but we will not associate the decisions you have made with the names of any participant.

EARNINGS:

[First Experiment] **During the experiment you will be able to earn experimental points. At the end, each experimental point will be exchanged for Euros, exactly 1 experimental point is worth 1 Euro.** In addition, we will round up decimals to the nearest tenth.

Everything you earn will be paid to you in cash in a strictly private manner at the end of the experimental session. **Your final earnings will be the sum of the 3 Euros you receive for participating plus whatever you earn during the experiment.**

If, for example, you get a total of 25.19 experimental points you will get a total of 28.20 Euros (3 Euros as payment for participating and 25.20 Euros from converting the 25.19 experimental points to 25.20 Euros).

If, for example, you get 0.20 experimental points you will get 3.20 Euros (3+0.20 = 3.20).

If, for example, you get 12.83 experimental points you will get 15.90 Euros (3+12.90 = 15.90).

[Second Experiment] **During the experiment you will be able to earn experimental points. At the end, each experimental point will be exchanged for Euros, exactly 1 experimental point is worth 0.25 Euro.** In addition, we will round up decimals to the nearest tenth.

Everything you earn will be paid to you in cash in a strictly private manner at the end of the experimental session. **Your final earnings will be the sum of the 3 Euros you receive for participating plus whatever you earn during the experiment.**

If, for example, you get a total of 60 experimental points you will get a total of 18 Euros (3 Euros as payment for participating and 15 Euros from converting the 60 experimental points to 15 Euros).

If, for example, you get 70 experimental points you will get 20.50 Euros (3+17.50 = 20.50).

If, for example, you get 30 experimental points you will get 10.50 Euros (3+7.50 = 10.50).

Before starting the experiment, we will explain in detail what kind of decisions you can make and how you can get experimental points.

[From now on, the instructions were read aloud and they were only provided on the computer screen.]

DETAILED INSTRUCTIONS OF THE EXPERIMENT:

This experiment consists of several rounds of decisions. In each of the rounds, you will be paired with a randomly chosen participant from this session. From now on, we will refer to you as "You" (in red) and the other participant as "Other Participant" (in blue) in these instructions.

In each round you will see a table and you will have to make a decision, choosing from three possible options. Each decision will be presented in the form of a table similar to the one below (but each time with different values). You will see the corresponding table each time you have to choose an option. Each row of the table corresponds to an option you can choose and the red numbers are the possible experimental points you can earn.

The other participant will also have to choose, independently from you, between her options, which correspond to the columns of the table and the blue numbers are the possible experimental points that the other participant can earn. That is, you choose rows, while the other participant chooses columns. However, to simplify things, the experiment is programmed in such a way that all participants - including the person you are paired with - see their decisions just as in our example. That is, each of you will be presented with your possible actions in the rows of the table.

When choosing, you will not know the option chosen by the other participant, and when the other participant is choosing among her options she will not know the option you have chosen either.

The amount of experimental points you can get in each of the rounds depends on the option you have chosen and the option the other participant has chosen.

The experimental points table you see is an example of what you will see in each of the rounds.

[Experiment 1:]

	Other Participant can choose:		
	○	○	○
○	10.47 ; 8.69	13.14 ; 12.03	8.06 ; 13.03
You can choose: ○	8.14 ; 0.28	14.81 ; 7.61	12.72 ; 11.61
○	15.14 ; 9.86	11.81 ; 7.19	4.72 ; 6.19

Example 1: if this round is chosen at random and you take the first choice (row) and the other participant takes the second choice (column), you will get 13.14 experimental points and the other participant 12.03 experimental points.

Example 2: if this round is chosen at random and you take the third option (row) and the other participant takes the first option (column), you will get 15.14 experimental points and the other participant 9.86 experimental points.

[Experiment 2:]

	Other Participant can choose:		
	○	○	○
○	20 ; 16	26 ; 24	16 ; 26
You can choose: ○	19 ; 2	28 ; 14	24 ; 22
○	30 ; 18	22 ; 14	8 ; 12

Example 1: if this round is chosen at random and you take the first choice (row) and the other participant takes the second choice (column), you will get 26 experimental points and the other participant 24 experimental points.

Example 2: if this round is chosen at random and you take the third option (row) and the other participant takes the first option (column), you will get 30 experimental points and the other participant 18 experimental points.

These are just two examples to better understand how to read the table, as well as to better understand how decisions affect the experimental points you can earn, but are not intended to suggest which decisions you should make.

To make your decision, click on the white button next to the option you want to make. The button will then turn red to indicate which option you have selected. Once you have chosen an option, the choice is not final and you can change it as many times as you like by clicking on another button, until you click on the "OK" button that will appear in the lower right corner of each screen. Once you have clicked "OK" your choice will be final and you will move on to the next round. You will not be able to move on to the next round until you have chosen an option and clicked "OK". You will not have any time restrictions. Take as much time as you need in each round.

Summary:

– Your experimental points will be in red and the other participant's experimental points will be in blue.

– You will participate in several different rounds. In each round you will be paired with a random participant and the experimental points table will be different.

– In each round, you can choose between three different options (rows) and the experimental points you earn depend on which option you have chosen, which option the other participant has chosen, as well as whether that round is randomly chosen at the end of the experiment.

We will start the experiment in a few moments. Before we begin, you will see an example again and you will have to answer several questions. If you have any questions or need help at any point during the experiment, please raise your hand and one of the researchers will come and talk to you.

[From now on, the instructions were not read aloud and they were provided on the computer screen.]

UNDERSTANDING TEST:

To make sure you understand the game, on the next screen we will ask you to answer some questions about the game.

[The table displayed on the screen was the same as the one shown above.]

- Write here your points earned in this round if you choose your second choice and the other participant chooses her third choice, if this round is randomly selected for your payment. **[Correct answer: 12.72 in the first experiment and 24 in the second experiment]**
- Write here the points earned by the other participant if you choose your third choice and the other participant chooses her second choice, if this round is chosen for payment. **[Correct answer: 7.19 in the first experiment and 14 in the second experiment.]**

DECISION SCREEN:

We will now show you N tables, one at a time, and will ask you to make a choice from each table.

At the end of the experiment, we will choose one of the N tables at random and pay you for that table.

Click OK to start viewing the tables.

[Once each participant made choices for the first N tables in the second experiment, they were shown the following instructions on the screen.]

DECISION SCREEN:

We will now show you other N tables, one at a time, and will ask you to make a choice on each table. After these N tables the experiment ends.

At the end of the experiment, we will choose one of these N tables at random and pay you for that table.

Click OK to start viewing the tables.