Lectures 10-11  Effect of source inductance on phase controlled AC-DC converters

10.1  Single-phase converters

10.1.1  Overlap in single-phase, CT fully controlled converter

The presence of source inductance means that commutation of load current from one thyristor to the next, as they are triggered with a firing angle \( \alpha \), can not be instantaneous. This source inductance, \( L_s \), is invariably because of the inductance of the supply lines and the leakage inductance of the input transformer. For this circuit, the overlap of conduction for the duration \( \mu \) makes the output voltage zero (which is the mean of the overlapping input voltages) during this period.

![Figure 10.1 Single-phase C-T converter with source inductance](image)
Figure 10.2 Waveforms in the converter of figure 10.1, commutation overlap and commutation notches, for $\alpha = 45^\circ$. 
Figure 10.3 Waveforms in the converter of figure 10.1, commutation overlap and commutation notches, for $\alpha = 130^\circ$. 

$$i_1, \quad v_i, \quad i_2, \quad v_o$$

$$V_{\text{max}} \sin \omega t$$

$$R, \quad L$$

$$T_1, \quad T_2$$

$$L_s$$

$$v_s$$

$$i_p$$
The part of the input voltage waveform which is missing from the output voltage is given by

\[ V_{\text{max}} \sin \omega t = L_s \frac{di}{dt} \]  \hspace{1cm} 10.1

\[ \int_{\alpha}^{\alpha+\mu} V_{\text{max}} \sin(\omega t) \, d(\omega t) = \omega L_s \int_{0}^{I_d} \, di \]  \hspace{1cm} 10.2

where \( V_{\text{max}} \) is the peak of the input ac voltage and \( I_d \) is the dc level of the output (load) current.

Note that in each half cycle of the above waveform, the input current through the incoming thyristor rises from zero to \( I_d \) in time \( \mu/\omega \), starting from the instant of firing. Thus, integrating equation 10.2,

\[ -V_{\text{max}} \left[ \cos(\alpha + \mu) - \cos \alpha \right] = \omega L_s I_d \]

\[ \cos(\alpha + \mu) = \cos \alpha - \frac{\omega L_s}{V_{\text{max}}} I_d \]  \hspace{1cm} 10.3

The commutation or overlap angle \( \mu \) can be found from expression 10.3. It increases with load current \( I_d \) at any firing angle. The output dc voltage is then given by

\[ V_d = \frac{1}{\pi} \int_{\alpha}^{\pi+\alpha} V_{\text{max}} \sin(\omega t) \, d(\omega t) - \frac{1}{\pi} \int_{\alpha}^{\alpha+\mu} V_{\text{max}} \sin(\omega t) \, d(\omega t) \]

\[ = \frac{2V_{\text{max}}}{\pi} \cos \alpha - \frac{\omega L_s}{\pi} I_d \]  \hspace{1cm} 10.4

Figure 10. 4 Variation of \( V_d \) and commutation angle \( \mu \) with \( I_d \) and firing angle \( \alpha \).
10.1.2 Overlap in single-phase, fully controlled bridge converter

Without overlap, two thyristors conduct at all times, if turned on. With overlap, all four thyristors conduct during overlap (or commutation of current from the outgoing pair to the incoming pair of thyristors).

Figure 10.5 (a) Single-phase, F-C, bridge converter with source inductance and (b) its waveforms, for $\alpha = 45^\circ$. 
The part of the input voltage waveform which is missing from the output during overlap is given by

\[ V_{\text{max}} \sin \omega t = L_s \frac{di}{dt} \]  \hspace{1cm} 10.5

As before

\[ \int_{\alpha}^{\alpha+\mu} V_{\text{max}} \sin(\omega t) d(\omega t) = \omega L_s \int_{-I_d}^{I_d} di = 2\omega L_s I_d \]

\[ \cos(\alpha + \mu) = \cos \alpha - \frac{2\omega L_s}{V_{\text{max}}} I_d \]  \hspace{1cm} 10.6

\[ \angle \alpha = 145^\circ \]

Figure 10.5(c) Waveforms in the converter of figure 10.5(a) for \( \alpha = 145^\circ \)
Note that during overlap, the input current $i$, which flows through $L_s$, changes by $2I_d$. The overlap angle $\mu$ can be found from the above expression. The angle $\mu$ increases when larger load current is commutated.

The dc output voltage of the converter with overlap is given by

$$V_d = \frac{1}{\pi} \int_{\alpha}^{\alpha+\pi} V_{\text{max}} \sin(\omega t) dt - \frac{1}{\pi} \int_{\alpha}^{\alpha+\mu} V_{\text{max}} \sin(\omega t) dt$$

$$= \frac{2V_{\text{max}}}{\pi} \cos \alpha - \frac{2\omega L_s}{\pi} I_d$$

10.7

Figure 10.6(a) Variation of $V_d$ and commutation angle $\mu$ with load current and firing angle $\alpha$ for the converter of figure 10.5.
10.1.3 Overlap in single-phase, half-controlled converter

The commutation of the load current through the thyristors and the diodes occur in two stages. At first, the load current commutates to the freewheeling diode at the zero crossings of the input voltage. When the incoming thyristor is triggered, the freewheeling load current commutates to this thyristor. The part of the input voltage waveform missing from the output is thus dropped across the source inductance due to the load current rising to \( I_d \) during the commutation overlap angle \( \mu \) when a thyristor is triggered. Consequently, the overlap angle \( \mu \) and the dc output voltage \( V_d \) for this converter with source inductance \( L_s \) are given by,

\[
\frac{I}{\omega} \int_{\alpha}^{\alpha+\mu} V_{\text{max}} \sin \omega t d(\omega t) = L_s \int_0^{I_d} di
\]

\[\therefore \cos(\alpha + \mu) = \cos \alpha - \frac{\omega L_s}{V_{\text{max}}} I_d \quad 10.8\]

The dc output voltage is given by

\[
V_d = \frac{1}{\pi} \left[ \int_{\alpha}^{\pi} V_{\text{max}} \sin \omega t d(\omega t) - \int_{\alpha}^{\alpha+\mu} V_{\text{max}} \sin \omega t d(\omega t) \right]
\]

\[\therefore V_d = \frac{V_{\text{max}}}{\pi} \left[ 1 + \cos \alpha \right] - \frac{\omega L_s}{\pi} I_d \quad 10.9\]
For $\alpha = 45^\circ$

Figure 10.8. Waveforms in the H-C converter with source inductance; for $\alpha = 45^\circ$. 
For $\alpha = 145^\circ$

Figure 10.9. Waveforms in the H-C converter with source inductance; for $\alpha = 145^\circ$
10.2 Three-phase converters

10.2.1 Overlap in a three-phase, CT, fully-controlled converter,

In this circuit, the converter output voltage during overlap is half of the incoming and outgoing voltages. For instance, when T1 is triggered with angle $\alpha$, after the crossover of $v_{an}$ and $v_{cn}$, the output voltage $v_o$ is given by

$$v_o = \frac{v_{an} + v_{cn}}{2}$$  

10.10

The voltage pulse missing from the output voltage waveform is bounded by $v_a$ and $v_o$. This is given by

$$v_{OL} = v_{an} - \frac{v_{an} - v_{cn}}{2} = \frac{v_{cn} - v_{an}}{2} = \frac{V_{ACL-L}}{2}$$  

10.11

By expressing $v_{OL} = \frac{\sqrt{3} V_{\text{max}} \sin \omega t}{2}$ where $V_{\text{max}}$ is the peak of the line-neutral voltage.

$$\frac{1}{\omega} \int_{\alpha}^{\alpha+\mu} \sqrt{3} \frac{V_{\text{max}}}{2} \sin \omega t \, dt = L_s \int_0^{I_d} di$$

Hence $\cos(\alpha + \mu) = \cos \alpha - \frac{2 \omega L_s}{\sqrt{3} V_{\text{max}}} I_d = \cos \alpha - \frac{2 \omega L_s}{V_{\text{max}} \lambda - l} I_d$  

10.12

from which $\mu$ can be found. The dc output voltage is given by

$$V_d = \frac{1}{2\pi / 3} \left[ \int_{\pi / 6 + \alpha}^{5\pi / 6} V_{\text{max}} \sin \omega t \, (\omega t) - \frac{\sqrt{3} V_{\text{max}}}{2} \int_{\alpha}^{\alpha+\mu} \sin \omega t \, (\omega t) \right]$$
\[
\frac{3\sqrt{3}V_{max}}{2\pi} \cos \alpha - \frac{3\omega L_s}{2\pi} I_d
\]

10.13

Figure 10.11 Waveforms in the circuit of figure 10.10 for \(\alpha = 45^\circ\)
Figure 10.12 Waveforms in the circuit of figure 10.10 for $\alpha = 130^\circ$
10.2.2 Overlap in 3-φ, fully-controlled bridge converter

In this converter, overlap due to source inductance occur every 60°. When a thyristor connected with the positive voltage is triggered, the positive dc bus voltage becomes the average of the incoming and the outgoing phase voltages. The same happens when one of the thyristors connected with the negative dc bus is triggered.

During the commutation angle $\mu$, current in the outgoing line falls gradually to zero, while the current in the incoming line rises to $I_d$. The voltage pulse missing from the output voltage waveform is the difference between the incoming line voltage minus the average of the incoming and the outgoing line voltages. For instance, when thyristor $T_1$ is triggered with angle $\alpha$, this voltage is given by,

$$V_{OL} = v_{an} - \frac{v_{an} + v_{cn}}{2} = \frac{v_{an} - v_{cn}}{2} = \frac{v_{acl-l}}{2} \quad \text{from } \alpha \text{ to } \alpha + \mu \text{ on the } v_{ac} \text{ waveform.}$$

Using a similar analysis as in the previous section, it can be shown that for this converter,

$$\cos(\alpha + \mu) = \cos \alpha - \frac{2\omega L_s}{V_{max l-l}} I_d \quad 10.14$$

and

$$V_d = \frac{3V_{max l-l}}{\pi} \cos \alpha - \frac{3\omega L_s}{\pi} I_d \quad 10.15$$
Figure 10.14 Waveforms in the converter of figure 10.13; $\alpha = 45^\circ$
Figure 10.15 Waveforms in the converter of figure 10.13; $\alpha = 130^\circ$
10.2.3 Overlap in 3-φ, half-controlled bridge converter

It can be shown that

\[
V_d = \frac{3V_{\text{max}}}{2\pi} [I + \cos \alpha] - \frac{3\omega L_s}{2\pi} I_d \tag{10.16}
\]

The above equation assumes that load current transfers to the free-wheeling diode completely before the next thyristor is triggered, and that the load current transfers from the free-wheeling diode to the incoming thyristor when it is triggered.
10.3 Converter voltage regulation due to source inductance

The commutation overlap affects the performance of all converter circuits, including the rectifiers which rely for their operation on natural commutation. The overlap phenomenon affects the operation of the converter in many ways, such as introducing a voltage regulation characteristic. The voltage regulation characteristic of the converter is in general given by

\[ V_d = V_{dc\text{max}} \alpha \cos \alpha - X_L I_L \]  \hspace{1cm} \text{(for fully controlled converters)} \hspace{1cm} 10.17

and \[ V_d = V_{dc\text{max}} (1 + \cos \alpha) - X_L I_L \]  \hspace{1cm} \text{(for half controlled converters)} \hspace{1cm} 10.18

Here \( V_{dc\text{max}} \) is the maximum dc output voltage for the converter circuit and \( X_L \) is a parameter determined by the input source inductance \( L_s \) and the converter circuit.

\begin{figure}[h]
\centering
\includegraphics[width=0.8\textwidth]{voltage_regulation}
\caption{Voltage regulation characteristic of the fully-controlled converter}
\end{figure}
10.3.1 Other effects

A. Output voltage ripple

The output voltage waveform of the converter is modified by the loss of voltage pulses due to overlap. Analysis of the output voltage waveforms shows that the output ripple frequencies remain the same as for the converter without overlap. Only the output ripple voltage amplitudes are affected. The overlap actually reduces the amplitudes of the ripples. For a p-pulse converter the output ripple voltages, assuming continuous conduction, are given by

\[
v_n(t) = \frac{V_{d_{\text{max}}} \cos \left( \frac{n\pi}{P} \right)}{n^2 - 1} \left[ n \sin \alpha \times \sin n(\omega t + \mu) - \cos \alpha \cos n(\omega t + \mu) \right] \\
+ n \sin(\alpha + \mu) \sin n\omega t - \cos(\alpha + \mu) \cos n\omega t
\]

where \( \alpha = \) firing angle  \\
\( \mu = \) commutation angle  \\
\( n = \) order of the output voltage ripple

B. Input current harmonics

The input current waveforms of the converter have gentler rise and fall of current, rather than the abrupt changes in the converter without overlap. It is obvious that the harmonic amplitudes of the input current are reduced as a result of overlap. Assuming linear transitions, of duration m, the input current harmonics of a six-pulse converter is given by

\[
b_n = \frac{2 \sin \left( \frac{n\pi}{3} \right)}{n\pi} \times \frac{\sin \left( \frac{n\mu}{2} \right)}{\frac{n\mu}{2}}
\]

C. Commutation notches

During commutation overlap, simultaneous conduction of two thyristors makes the line to line voltage of the two input lines under commutation zero. Other line to line voltages are found by obtaining the difference of their potentials taking into account the commutation. If the converter input voltage terminals are shared with other loads, (these voltages are invariably used as signals which control the triggering of the thyristors), then adequate filter circuits must be used to reduce the commutation notches to acceptable levels. If these notches are not adequately filtered out, firing angles vary from thyristor to thyristor, leading to uneven output voltage ripples and thyristor currents.
10.4 Converter characteristics with discontinuous load current

In analyzing ac-dc converter circuits we have so far assumed that the load current was continuous, i.e., positive at all times. This allowed the output voltage to be described in terms of the firing angle $\alpha$ and the input ac voltage, $V_{\text{max}}$. When load current becomes discontinuous, the output voltage becomes either zero or equal to the voltage of the active dc source in the load. In general, it results in an increased output dc voltage (because of the removal of part of the negative voltage of the ac source in the load).

The dc output voltage of a fully-controlled converter with discontinuous conduction and input source inductance are thus of the form as in the figure below.

Consider the figure below in which a single-phase bridge supplies a load with a back emf of $E_b$.

![Figure 10.18 Single-phase bridge rectifier with discontinuous current.](image)
For this circuit,

\[ L \frac{di}{dt} + Ri = V_{\text{max}} \sin \omega t - E_b \]  

10.21

The solution for \( i \) is

\[ i = Ae^{-(R/L)t} + \frac{V_{\text{max}}}{|Z|} \sin(\omega t - \phi) - \frac{E_b}{R} \]  

10.22

where \(|Z| = \sqrt{R^2 + (\omega L)^2}\)

10.23

and \( \phi = \tan^{-1}\left(\frac{\omega L}{R}\right) \)  

10.24

The instantaneous load current \( i \) is obtained by noting that \( i = 0 \) at the effective firing angle \( \alpha \) which is either the actual firing angle (when \( E_b < V_{\text{max}} \sin \alpha \)) or the angle \( \sin^{-1}(E_b/V_{\text{max}}) \)

Thus

\[ i = \frac{V_{\text{max}}}{R} \left\{ \frac{R}{|Z|} \sin(\omega t - \phi) - \frac{E_b}{V_{\text{max}}} \right\} + \left\{ \frac{E_b}{V_{\text{max}}} - \frac{R}{|Z|} \sin(\alpha - \phi) \right\} e^{-\frac{R}{\omega L}(\omega t - \phi)} \]  

10.25

Current \( i \) falls to zero at angle \( \beta \) which is obtained by equating the above equation to zero. Thus

\[ e^{R\beta/\omega L} \times \frac{\cos \phi \sin(\beta - \phi) - \frac{E_b}{V_{\text{max}}}}{\cos \phi \sin(\alpha - \phi) - \frac{E_b}{V_{\text{max}}}} = e^{R\alpha/\omega L} \]  

10.26

The extinction angle \( \beta \) is found by solving the above transcendental equation. \( V_d \) can then be calculated.

By solving for \( \beta \) for a firing angle \( \alpha \), the \( V_d - I_L \) characteristic of the converter for different load parameter values (\( R, L \) and \( E_b \)) can be obtained. The boundary between continuous and discontinuous conduction is a semicircle. The \( V_d - I_L \)
characteristics of a half controlled and a fully controlled converter are shown in figures below. These also include the effect of commutation overlap.

Figure 10.19 Converter $V_d$ vs $\alpha$ and regulation characteristic with source inductance and discontinuous conduction.