Notes on

MARKET POWER AND STRATEGY

3rd Year

Degree in Economics

2020-2021

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Chapter 1. **Game Theory and Competitive Strategy**

Introduction

The Theory of Non-Cooperative Games studies and models *conflict situations* among economic agents; that is, it studies situations where the profits (gains, utility or payoffs) of each economic agent depend not only on her own acts but also on the acts of the other agents.

We assume *rational players* so each player will try to maximize her profit function (utility or payoff) given her conjectures or beliefs on how the other players are going to play. The outcome of the game will depend on the acts of all the players.

A fundamental characteristic of non-cooperative games is that it is *not* possible to sign *contracts* between players. That is, there is no external institution (for example, courts of justice) capable of enforcing the agreements. In this context, co-operation among players only arises as an equilibrium or solution proposal if the players find it in their best interest.

For each game we try to propose a “solution”, which should be a reasonable prediction of *rational behavior* by players (OBJECTIVE).

We are interested in Non-Cooperative Game Theory because it is very useful in modeling and understanding *multi-personal* economic problems characterized by *strategic interdependency*. Consider, for instance, competition between firms in a market. Perfect competition and pure monopoly (not threatened by entry) are special non-realistic cases. It is more frequent in real life to find industries with not many firms (or with a lot of firms but with just a few of them producing a large part of the total production). With few firms, competence between them is characterized by strategic considerations: each firm takes its decisions (price, output, advertising, etc.) taking into account or conjecturing the behavior of the others. Therefore, competition in an oligopoly can be seen as a non-cooperative game where the firms are the players. Many predictions or solution proposals arising from Game Theory prove very useful in understanding competition between economic agents under strategic interaction.

Section 1 defines the main notions of Game Theory. We shall see that there are two ways of representing a game: the extensive form and the strategic form. In Section 2 we analyze the main solution concepts and their problems; in particular, we study the Nash equilibrium and its refinements. Section 3 analyzes repeated games and, finally, Section 4 offers concluding remarks.

1.1. **Basic notions**

There are two ways of representing a game: the extensive form and the strategic form. We start by analyzing the main elements of an extensive form game.

1.1.1. **Games in extensive form** (dynamic or sequential games)

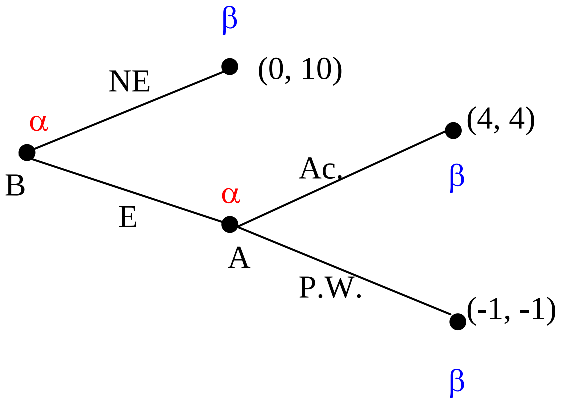
An extensive form game specifies:

1) The players.
2) The order of the game.
3) The choices available to each player at each turn of play (at each decision node).
4) The information held by each player at each turn of play (at each decision node).
5) The payoffs of each player as a function of the movements selected.
6) Probability distributions for movements made by nature.

An extensive form game is represented by a decision tree. A decision tree comprises nodes and branches. There are two types of node: decision nodes and terminal nodes. We have to assign each decision node to one player. When the decision node of a player is reached, the player chooses a move. When a terminal node is reached, the players obtain payoffs: an assignment of payoffs for each player.

EXAMPLE 1: Entry game

Consider a market where there are two firms: an incumbent firm, A, and a potential entrant, B. At the first stage, the potential entrant decides whether or not to enter the market. If it decides "not to enter" the game concludes and the players obtain payoffs (firm A obtains the monopoly profits) and if it decides "to enter" then the incumbent firm, A, has to decide whether to accommodate entry (that is, to share the market with the entrant) or to start a mutually injurious price war. The extensive form game can be represented as follows:



Players: B and A.

Actions: *E* (to enter), *NE* (not to enter), *Ac.* (to accommodate), *P.W.* (price war).

Decision nodes: α.

Terminal nodes: β.

(x, y): vector of payoffs. x: payoff of player B; y: payoff of player A.

At each terminal node we have to specify the payoffs of each player (even though some of them have not actually managed to play).

Assumptions:

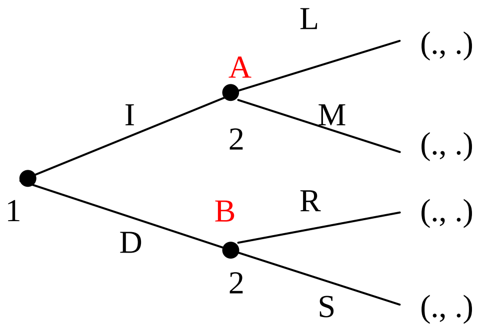
(i) All players have the same perception of how the game is like.

(ii) Complete information: each player knows the characteristics of the other players: preferences and strategy spaces.

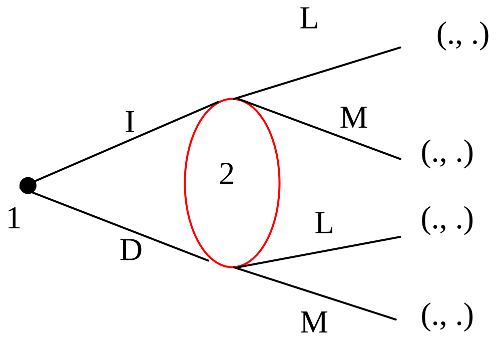
(iii) Perfect recall (perfect memory): each player remembers her previous behavior in the game.

Definition 1: **Information set**

“The information available to each player at each one of her decision nodes”.



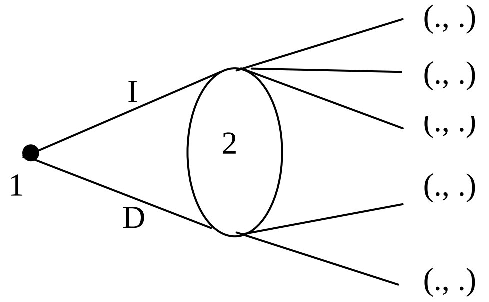
Game 1



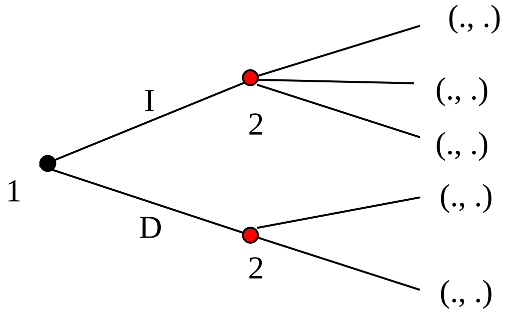
Game 2

In game 1, player 2 has different information at each one of her decision nodes. At node *A*, if she is called upon to play she knows that the player 1 has played *I* and at *B* she knows that player 1 has played *D*. We say that these information sets are singleton sets consisting of only one decision node. *Perfect information game*: a game where all the information sets are singleton sets or, in other words, a game where all the players know everything that has happened previously in the game. In game 2, the player 2 has the same information at both her decision nodes. That is, the information set is composed of two decision nodes. Put differently, player 2 does not know which of those nodes she is at. A game in which there are information sets with two or more decision nodes is called an *imperfect information game*: at least one player does not observe the behavior of the other(s) at one or more of her decision nodes.

The fact that players know the game that they are playing and the perfect recall assumption restrict the situations where we can find information sets with two or more nodes.

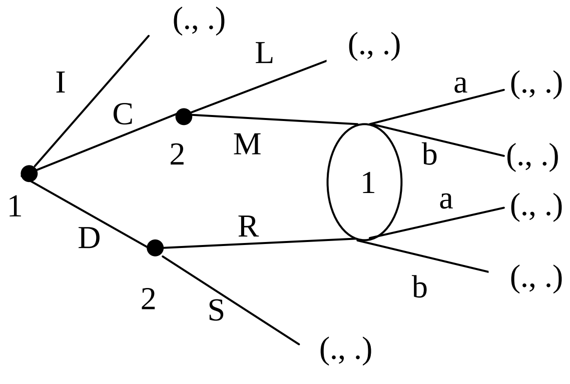


Game 3

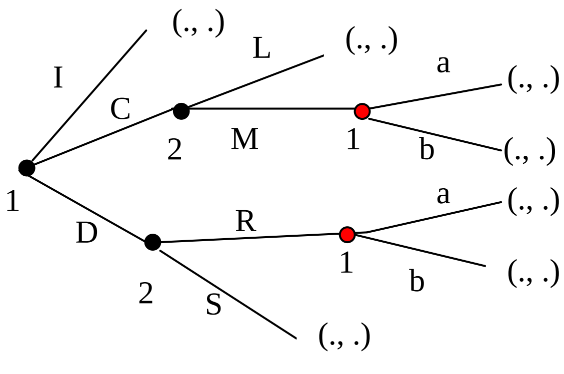


Game 4

Game 3 is poorly represented because it would not be an imperfect information game. Assuming that player 2 knows the game, if she is called on to move and faces three alternatives he/she would immediately deduce that the player 1 has played I. That is, the game should be represented like game 4. Therefore, *if an information set consists of two or more nodes the number of alternatives, actions or moves at each one should be the same*.



Game 5



Game 6

The assumption of perfect recall avoids situations like that in game 5. When player 1 is called on to play at her second decision node perfectly recalls her behavior at her first decision node. The extensive form should be like that of game 6.

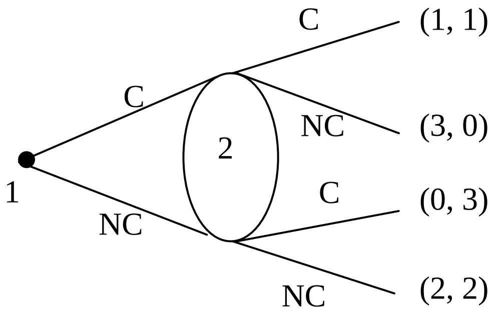
Definition 2: **Subgame**

"It is what remains to be played from a decision node with the condition that what remains to be played does not form part of an information set with two or more decision nodes. To build subgames we look at parts of the game tree that can be constructed without breaking any information sets. A subgame starts at a singleton information set and all the decision nodes of the same information set must belong to the same subgame."

EXAMPLE 2: The Prisoner's Dilemma

Two prisoners, 1 and 2, are being held by the police in separate cells. The police know that the two (together) committed a crime but lack sufficient evidence to convict them. So the police offer each of them separately the following deal: each is asked to implicate his partner. Each prisoner can "confess" (*C*) or "not confess" (*NC*). If neither confesses then each player goes to jail for one month. If both players confess each prisoner goes to jail for three months. If one prisoner confesses and the other does not confess, the first player goes free while the second goes to jail for six months.

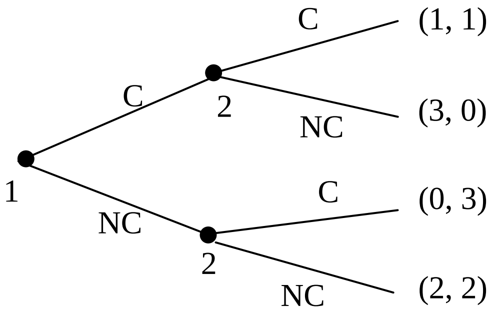
- **Simultaneous case**: each player takes her decision with no knowledge of the decision of the other.



PD1

There is an information set with two decision nodes. This is an imperfect information game. There is a subgame which coincides with the proper game.

- **Sequential game**: the second player observes the choice made by the first.



PD2

Game PD2 is a perfect information game and there are three subgames. “In perfect information games there are as many subgames as decision nodes”.

Definition 3: **Strategy**

"A player's strategy is a complete description of what she would do if she were called on to play at each one of her decision nodes. It needs to be specified even in those nodes not attainable by her given the current behavior of the other player(s). It is a *behavior plan* or *conduct plan*".

(Examples: consumer demand, supply from a competitive firm.). It is a player's function which assigns an action to each of her decision nodes (or to each of her information sets). A player's strategy has as many components as information sets the player has.

Definition 4: **Action**

"A choice (decision or move) at a decision node".

Actions are physical while strategies are conjectural.

Definition 5: **Combination of strategies or strategy profile**

"A specification of one strategy for each player". The result (the payoff vector) must be unequivocally determined.

EXAMPLE 1: The entry game

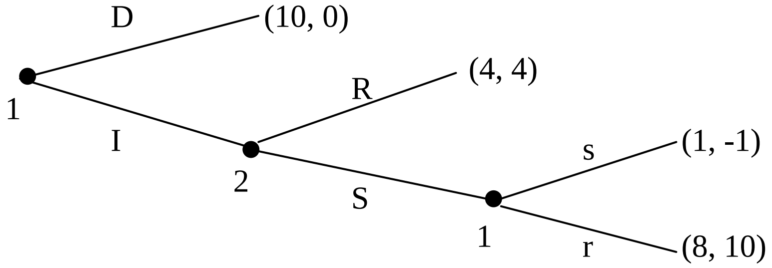
This is a perfect information game with two subgames. Each player has two strategies: $S_B = \{NE, E\}$ and $S_A = \{Ac., P.W.\}$Combinations of strategies: (*NE*, *Ac.*), (*NE*, *P.W.*), (*E*, *Ac.*) and (*E*, *P.W.*).

EXAMPLE 2: The Prisoner's Dilemma

PD1: This is an imperfect information game with one subgame. Each player has two strategies: $S_1 = \{C, NC\}$ and $S_2 = \{C, NC\}$. Combinations of strategies: (*C*, *C*), (*C*, *NC*), (*NC*, *C*) and (*NC*, *NC*).

PD2: This is a perfect information game with three subgames. Player 1 has two strategies $S_1 = \{C, NC\}$ but player 2 has four strategies $S_2 = \{CC, CNC, NCC, NCNC\}$. Combinations of strategies: (*C*, *CC*), (*C*, *CNC*), (*C*, *NCC*), (*C*, *NCNC*), (*NC*, *CC*), (*NC*,*CNC*), (*NC*, *NCC*) and (*NC*, *NCNC*).

EXAMPLE 3



Player 1 at his/her first node has two possible actions, *D* and *I*, and two actions also at her second: *s* and *r*. $S_1 = \{Ds, Dr, Is, Ir\}$ and $S_2 = \{R, S\}$.

1.1.2. **Games in normal or strategic form** (simultaneous or static games)

A game in normal or strategic form is described by:

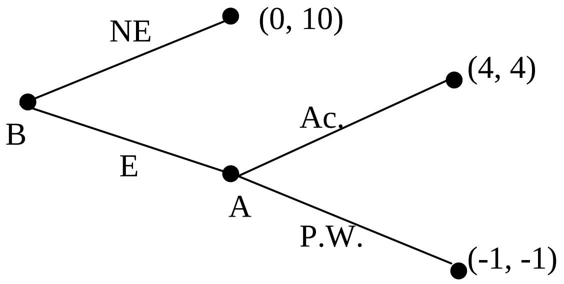
1) The players.

2) The set (or space) of strategies for each player.

3) A payoff function which assigns a payoff vector to each combination of strategies.

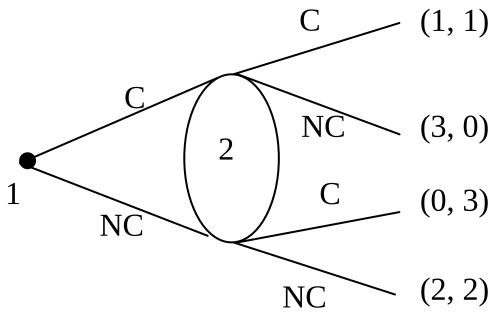
The key element of this way of representing a game is the description of the payoffs of the game as a function of the strategies of the players, without explaining the actions taken during the game. In the case of two players the usual representation is a bimatrix form game where each row corresponds to one of the strategies of one player and each column corresponds to one strategy of the other player.

EXAMPLE 1: The entry game

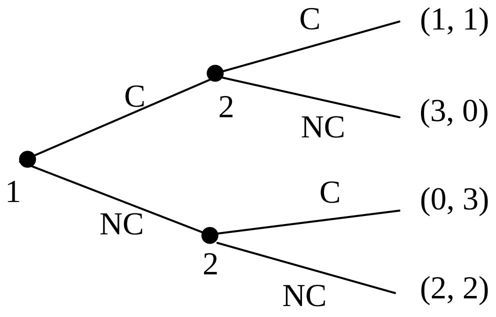


		A	
		Ac.	P.W.
B	NE	(0, 10)	(0, 10)
	E	(4, 4)	(-1, -1)

EXAMPLE 2: The Prisoner's Dilemma

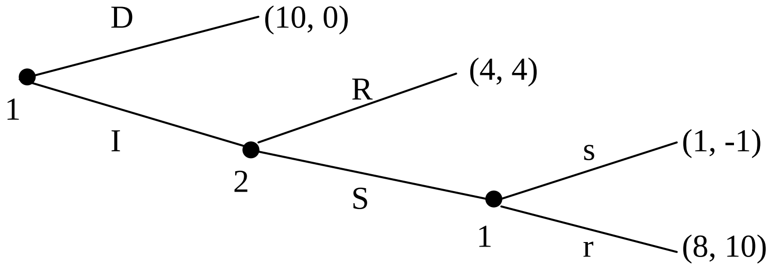


		2	
		C	NC
1	C	(1, 1)	(3, 0)
	NC	(0, 3)	(2, 2)



		2			
		CC	CNC	NCC	NCNC
1	C	(1, 1)	(1, 1)	(3, 0)	(3, 0)
	NC	(0, 3)	(2, 2)	(0, 3)	(2, 2)

EXAMPLE 3



		2	
		R	S
1	Ds	(10, 0)	(10, 0)
	Dr	(10, 0)	(10, 0)
	Is	(4, 4)	(1, -1)
	Ir	(4, 4)	(8, 10)

Link between games in normal form and games in extensive form

a) For any game in extensive form there exists a unique corresponding game in normal form. This is due to the game in normal form being described as a function of the strategies of the players.

b) (Problem) Different games in extensive form can have the same normal (or strategic) form. (Example: in the prisoner’s dilemma, PD1, if we change the order of the game then the game in extensive form also changes but the game in normal form does not change).

1.2. **Solution concepts (criteria) for non-cooperative games**

The general objective is to predict how players are going to behave when they face a particular game. NOTE: “A solution proposal is (not a payoff vector) a combination of strategies, one for each player, which leads to a payoff vector”. We are interested in predicting behavior, not gains.

Notation

i: Representative player, $i = 1,\ldots, n$

S_i: set or space of player *i*’s strategies.

$s_i \in S_i$: a strategy of player *i*.

$s_{-i} \in S_{-i}$: a strategy or combination of strategies of the other player(s).

$\Pi_i(s_i, s_{-i})$: the profit or payoff of player *i* corresponding to the combination of strategies $s \equiv (s_1, s_2, \ldots.., s_n) \equiv (s_i, s_{-i})$.

1.2.1. **Dominance criterion**

Definition 6: **Dominant strategy**

“A strategy is strictly dominant for a player if it leads to strictly better results (more payoff) than any other of her strategies no matter what combination of strategies is used by the other players”.

“If $\Pi_i(s_i^D, s_{-i}) > \Pi_i(s_i, s_{-i}), \forall s_i \in S_i, s_i \neq s_i^D; \forall s_{-i} \in S_{-i}$ then s_i^D is a strictly dominant strategy for player *i*”.

EXAMPLE 2: The Prisoner's Dilemma

In game PD1 "confess", *C*, is a (strictly) dominant strategy for each player. Independently of the behavior of the other player the best each player can do is "confess".

The presence of dominant strategies leads to a solution of the game. We should expect each player to use her dominant strategy. The solution proposal for game DP1 is the combination of strategies (*C*, *C*).

Definition 7: **Strict dominance**

"One strategy strictly dominates another when it leads to strictly better results (more payoff) than the other no matter what combination of strategies is used by the other players".

"If $\Pi_i(s_i^d, s_{-i}) > \Pi_i(s_i^{dd}, s_{-i}), \forall s_{-i} \in S_{-i}$, then s_i^d strictly dominates s_i^{dd}".

Definition 7': **(Strictly) Dominated strategy**

"One strategy is strictly dominated for a player when there is another strategy which leads to strictly better results (more payoff) no matter what combination of strategies is used by the other players".

"s_i^{dd} is a strictly dominated strategy if $\exists s_i^d$ such that $\Pi_i(s_i^d, s_{-i}) > \Pi_i(s_i^{dd}, s_{-i})\ \forall s_{-i} \in S_{-i}$".

The dominance criterion consists of the iterated deletion of strictly dominated strategies.

EXAMPLE 4

$$\begin{array}{cc|c|c|c|} & & & 2 & \\ & & t_1 & t_2 & t_3 \\ \hline & s_1 & (4, 3) & (2, 7) & (0, 4) \\ \hline 1 & & & & \\ & s_2 & (5, 5) & (5, -1) & (-4, -2) \\ \hline \end{array}$$

In this game there are no dominant strategies. However, the existence of dominated strategies allows us to propose a solution. We next apply the dominance criterion. Strategy t_3 is strictly dominated by strategy t_2 so player 1 can conjecture (predict) that player 2 will never use that strategy. Given that conjecture, which assumes rationality on the part of player 2, strategy s_2 is better than strategy s_1 for player 1. Strategy s_1 would be only used in the event that player 2 used strategy t_3. If player 1 thinks player 2 is rational then she assigns zero probability to the event of player 2 playing t_3. In that case, player 1 should play s_2 and if player 2 is rational the best she can do is t_1. The criterion of iterated deletion of strictly dominated strategies (by eliminating dominated strategies and by computing the reduced games) allows us to solve the game.

EXAMPLE 5

$$\begin{array}{cc|c|c|} & & t_1 \quad 2 & t_2 \\ \hline & s_1 & (10, 0) & (5, 2) \\ \hline 1 & & & \\ & s_2 & (10, 1) & (2, 0) \\ \hline \end{array}$$

In this game there are neither dominant strategies nor (strictly) dominated strategies.

Definition 8: **Weak dominance**

"One strategy weakly dominates another for a player if the first leads to results at least as good as those of the second for any combination of strategies of the other players and to strictly better results for some combination of strategies of the other players".

"If $\Pi_i(s_i^{wd}, s_{-i}) \geq \Pi_i(s_i^{wdd}, s_{-i}), \forall s_{-i} \in S_{-i},$ and $\exists s_{-i}$ such that $\Pi_i(s_i^{wd}, s_{-i}) > \Pi_i(s_i^{wdd}, s_{-i}),$ then s_i^{wd} weakly dominates s_i^{wdd}".

Definition 8': **Weakly dominated strategy**

"One strategy is weakly dominated for a player if there is another strategy which leads to results at least as good as those of the first one for any combination of strategies of the other players and to strictly better results for some combination of strategies of the other players".

"s_i^{wdd} is a weakly dominated strategy if there is a strategy s_i^{wd} such that $\Pi_i(s_i^{wd}, s_{-i}) \geq \Pi_i(s_i^{wdd}, s_{-i}), \forall s_{-i} \in S_{-i},$ and $\exists s_{-i}$ such that $\Pi_i(s_i^{wd}, s_{-i}) > \Pi_i(s_i^{wdd}, s_{-i})$".

Thus, a strategy is weakly dominated if another strategy does at least as well for all s_{-i} and strictly better for some s_{-i}.

In example 5, strategy s_1 weakly dominates s_2. Player 2 can conjecture that player 1 will play s_1 and given this conjecture the best she can do would be to play t_2. By following the criterion of weak dominance (iterated deletion of weakly dominated strategies) the solution proposal would be (s_1, t_2).

However, the criterion of weak dominance may lead to problematic results, as occurs in example 6, or to no solution proposal as occurs in example 7 (because there are no dominant strategies, no dominated strategies and no weakly dominated strategies).

EXAMPLE 6

		2		
		t_1	t_2	t_3
1	s_1	(10, 0)	(5, 1)	(4, -200)
	s_2	(10, 100)	(5, 0)	(0, -100)

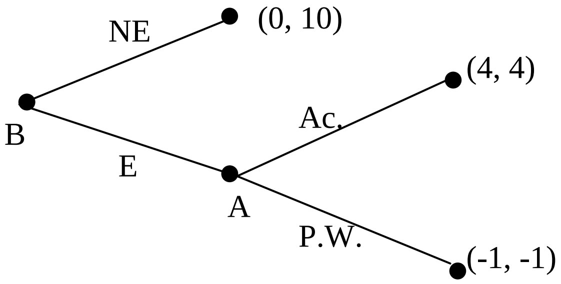
EXAMPLE 7

		2		
		t_1	t_2	t_3
1	s_1	(4, 10)	(3, 0)	(1, 3)
	s_2	(0, 0)	(2, 10)	(10, 3)

1.2.2. **Backward induction criterion**

We next use the dominance criterion to analyze the extensive form. Consider example 1.

EXAMPLE 1: The entry game



		A	
		Ac.	P.W.
B	NE	(0, 10)	(0, 10)
	E	(4, 4)	(-1, -1)

In the game in normal form, player A has a weakly dominated strategy: *P.W.*. Player B might conjecture that and play *E*. However, player B might also have chosen *NE* in order to obtain a certain payoff against the possibility of player A playing *P.W.*.

In the game in extensive form, the solution is obtained more naturally by applying backward induction. As she moves first, Player B may conjecture, correctly, that if she plays *E* then player A (if rational) is sure to choose *Ac*.. Price war is therefore an incredible threat and anticipating that player A will accommodate entry, the entrant decides to enter. By playing before A, player B may anticipate the rational behavior of player A.

In the extensive form of the game we have more information because when player A has to move she already knows the movement of player B.

The criterion of backward induction lies in applying the criterion of iterated dominance backwards starting from the last subgame(s). In example 1 in extensive form the criterion of backward induction proposes the combination of strategies (*E*, *Ac*.) as a solution.

Result: *In perfect information games with no ties, the criterion of backward induction leads to a unique solution proposal.*

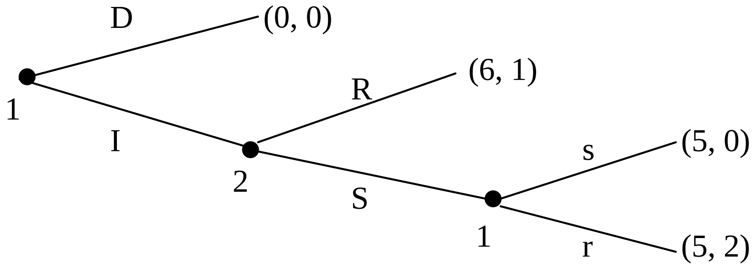
Problems

(i) *Ties*.

(ii) *Imperfect information*. Existence of information sets with two or more nodes.

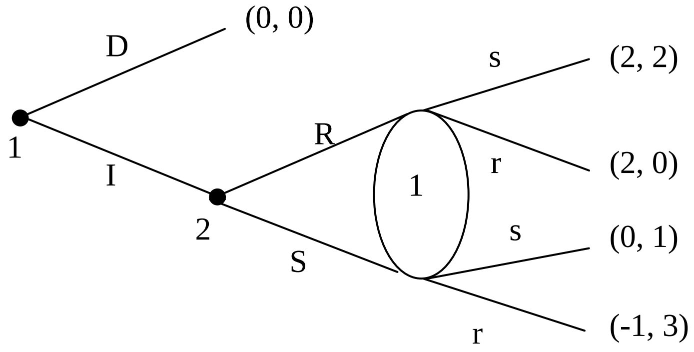
(iii) The success of backward induction is based on all conjectures about the rationality of agents checking out exactly with independence of how long the backward path is. (It may require *unbounded rationality*).

EXAMPLE 8



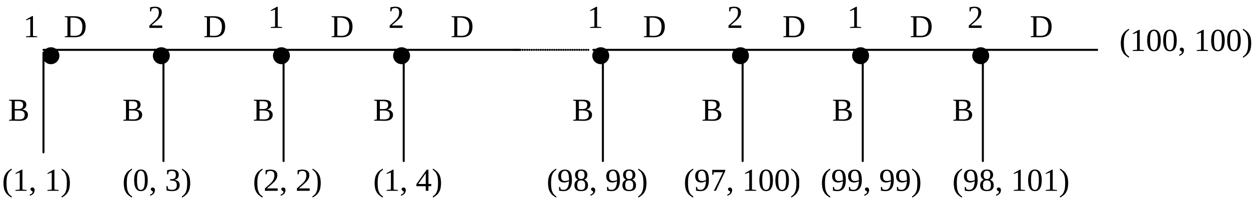
Backward induction does not propose a solution because in the last subgame player 1 is indifferent between *s* and *r*. In the previous subgame, player 2 would not have a dominated action (because she is unable to predict the behavior of player 1 in the last subgame).

EXAMPLE 9



We cannot apply the criterion of backward induction.

EXAMPLE 10: Rosenthal's (1981) centipede game



In the backward induction solution the payoffs are (1, 1). Is another rationality possible?

1.2.3. **Nash equilibrium**

Player *i, i* = 1,…, *n*, is characterized by:

(i) A set of strategies: S_i.

(ii) A profit function, $\Pi_i(s_i, s_{-i})$ where $s_i \in S_i$ and $s_{-i} \in S_{-i}$.

Each player will try to maximize her profit (utility or payoff) function by choosing an appropriate strategy with knowledge of the strategy space and profit functions of the other players but with no information concerning the current strategy used by rivals. Therefore, each player must conjecture the strategy(ies) used by her rival(s).

*Definition 9***: Nash equilibrium**

"A combination of strategies or strategy profile $s^* \equiv (s_1^*, \ldots, s_n^*)$ constitutes a Nash equilibrium if the result for each player is better than or equal to the result which would be obtained by playing another strategy, with the behavior of the other players remaining constant.

$s^* \equiv (s_1^*, \ldots, s_n^*)$ is a Nash equilibrium if: $\Pi_i(s_i^*, s_{-i}^*) \geq \Pi_i(s_i, s_{-i}^*)$ $\forall s_i \in S_i, \forall i, i = 1, \ldots, n$."

At equilibrium two conditions must be satisfied:

(i) The *conjectures* of players concerning how their rivals are going to play must be *correct*.

(ii) No player has incentives to change her strategy given the strategies of the other players. This is an element of *individual rationality*: do it as well as possible given what the rivals do. Put differently, no player increases her profits by *unilateral deviation*.

Being Nash equilibrium is a necessary condition or minimum requisite for a solution proposal to be a reasonable prediction of rational behavior by players. However, as we shall see it is not a sufficient condition. That is, being Nash equilibrium is not in itself sufficient for a combination of strategies to be a prediction of the outcome for a game.

Definition10: **Nash equilibrium**

"A combination of strategies or strategy profile $s^* \equiv (s_1^*, \ldots, s_n^*)$ constitutes a Nash equilibrium if each player's strategy is a best response to the strategies actually played by her rivals.

That is, $s^* \equiv (s_1^*, \ldots, s_n^*)$ is a Nash equilibrium if:

$$s_i^* \in BR_i(s_{-i}^*) \quad \forall i, i = 1, \ldots, n$$

where $BR_i(s_{-i}^*) = \left\{ s_i^{'} \in S_i : \Pi_i(s_i^{'}, s_{-i}^*) \geq \Pi_i(s_i, s_{-i}^*), \ \forall s_i \in S_i, s_i \neq s_i^{'} \right\}$".

A simple way of obtaining the Nash equilibria for a game is to build the best response sets of each player to the strategies (or combinations of strategies) of the other(s) player(s) and then look for those combinations of strategies being mutually best responses.

EXAMPLE 11

			2	
		h	i	j
	a	(5, 3)	$(5, \underline{11})$	$(\underline{20}, 5)$
1	b	$(\underline{9}, \underline{11})$	(2, 8)	(15, 6)
	c	$(3, \underline{10})$	$(\underline{10}, 2)$	(0, 5)

s_1	BR_2	s_2	BR_1
a	i	h	b
b	h	i	c
c	h	j	a

The strategy profile (*b*, *h*) constitutes the unique Nash equilibrium.

EXAMPLE 7

			2	
		t_1	t_2	t_3
1	s_1	($\underline{4}$, $\underline{10}$)	($\underline{3}$, 0)	(1, 3)
	s_2	(0, 0)	(2, $\underline{10}$)	($\underline{10}$, 3)

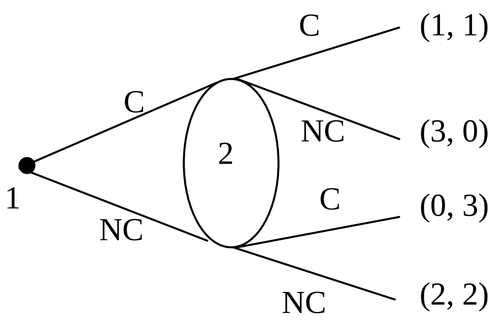
Note that the dominance criterion did not propose any solution for this game. However, the combination of strategies (s_1, t_1) constitutes the unique Nash equilibrium.

1.2.4. **Problems and refinements of the Nash equilibrium**

1.2.4.1. **The possibility of inefficiency**

It is usual to find games where Nash equilibria are not Pareto optimal (efficient).

EXAMPLE 2: The Prisoner's Dilemma



		2	
		C	NC
1	C	(1, 1)	(3, 0)
	NC	(0, 3)	(2, 2)

(*C*, *C*) is a Nash equilibrium based on dominant strategies. However, that strategy profile is the only profile which is not Pareto optimal. In particular, there is another combination of strategies, (*NC*, *NC*), where both players obtain greater payoffs.

1.2.4.2. **Inexistence of Nash equilibrium (in pure strategies)**

EXAMPLE 12

		2	
		t_1	t_2
1	s_1	$(\underline{1}, 0)$	$(0, \underline{1})$
	s_2	$(0, \underline{1})$	$(\underline{1}, 0)$

This game does not have Nash equilibria in pure strategies. However, if we allow players to use mixed strategies (probability distributions on the space of pure strategies) the result obtained is that "for any finite game there is always at least one mixed strategy Nash equilibrium".

1.2.4.3. **Multiplicity of Nash equilibria**

We distinguish two types of games.

1.2.4.3.1. ***With no possibility of refinement or selection***

EXAMPLE 13: The Battle of the Sexes

		Gf	
		M	P
Bf	M	$(\underline{3}, \underline{2})$	(1, 1)
	P	(1, 1)	$(\underline{2}, \underline{3})$

This game has two Nash equilibria: (*M*, *M*) and (*P*, *P*). There is a *pure coordination problem*.

1.2.4.3.2. ***With possibility of refinement or selection***

a) *Efficiency criterion*

This criterion consists of choosing the Nash equilibrium which maximizes the payoff of players. In general this is not a good criterion for selection.

b) *Weak dominance criterion*

This criterion consists of eliminating Nash equilibria based on weakly dominated strategies. Although as a solution concept it is not good, the weak dominance criterion allows us to select among the Nash equilibria.

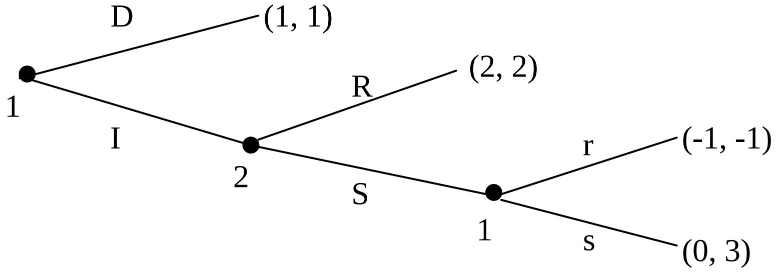
EXAMPLE 14

		2	
		D	I
1	D	($\underline{1}$, $\underline{1}$)	(0, 0)
	I	(0, 0)	($\underline{0}$, $\underline{0}$)

Nash equilibria: (*D*, *D*) and (*I*, *I*). Strategy *I* is a weakly dominated strategy for each player. By playing strategy *D* each player guarantees a payoff at least as high (and sometimes a higher) than that obtained by playing *I*. So we eliminate equilibrium (*I*, *I*) because it is based on weakly dominated strategies. So we propose the strategy profile (*D*, *D*) as the outcome of the game.

c) *Backward induction criterion and subgame perfect equilibrium*

EXAMPLE 15



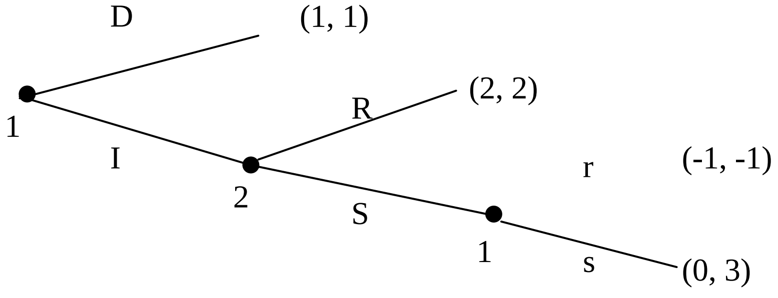
s_1	BR_2	s_2	BR_1
Dr	R, S	R	Ir, Is
Ds	R, S	S	Dr, Ds
Ir	R		
Is	S		

There are three Nash equilibria: (*Dr*, *S*), (*Ds*, *S*) and (*Ir*, *R*). We start by looking at the efficient profile: (*Ir*, *R*). This Nash equilibrium has a problem: at her second decision node, although it is an unattainable given the behavior of the other player, player 1 announces that she would play *r*. By threatening her with *r* player 1 tries to make player 2 play *R* and so obtain more profits. However, that equilibrium is based on a non credible threat: if player 1 were called on to play at his/her second node she would not choose *r* because it is an action (a non credible threat) dominated by *s*. The refinement we are going to use consists of eliminating those equilibria based on non credible threats (that is, based on actions dominated in one subgame). From the joint use of the notion of Nash equilibrium and the backward induction criterion the following notion arises:

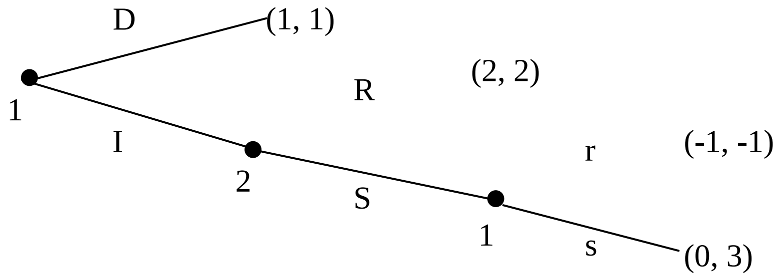
Definition 11: **Subgame perfect equilibrium**

"A combination of strategies or strategy profile $s^* \equiv (s_1^*, \ldots, s_n^*)$, which is a Nash equilibrium, constitutes a *subgame perfect equilibrium* if the relevant parts of the equilibrium strategies of each player are also an equilibrium at each of the subgames".

In example 15 (*Dr*, *S*) and (*Ir*, *R*) are not subgame perfect equilibria. Subgame perfect equilibria may be obtained by backward induction. We start at the last subgame. In this subgame *r* is a dominated action (a non credible threat); therefore, it cannot form part of player 1's strategy in the subgame perfect equilibrium, so we eliminate it and compute the reduced game

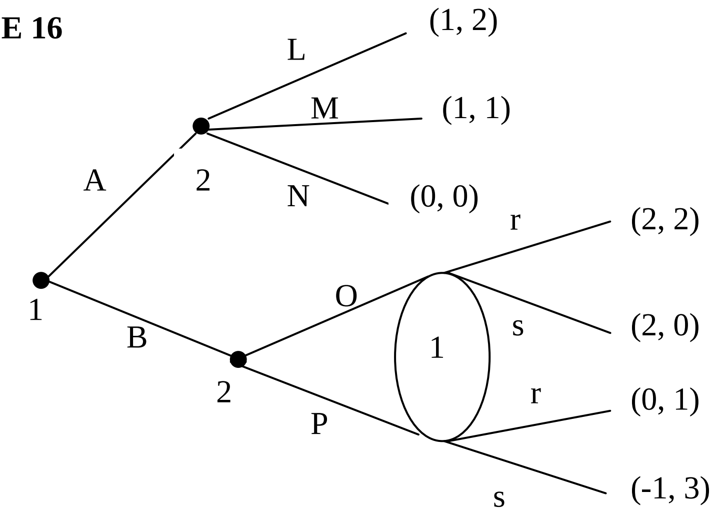


In the second stage of the backward induction we go to the previous subgame which starts at the decision node of player 2. In this subgame *R* is a dominated action for player 2. Given that player 2 anticipates that player 1 is not going to play *r* then *R* is a dominated action or non credible threat. We therefore eliminate it and compute the reduced game

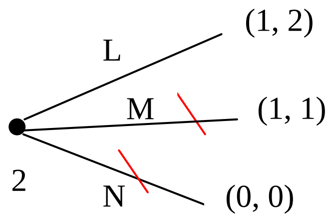


At her first node player 1 has *I* as a dominated action (in the reduced game) and, therefore, she will play *D*. Then the subgame perfect equilibrium is (*Ds*, *S*). We can interpret the logic of backward induction in the following way. When player 2 has to choose she should conjecture that if she plays *S* player 1 is sure to play *s*. Player 2 is able to predict the rational behavior of player 1 given that player 1 observes the action chosen by her. If player 1 is equally rational she should anticipate the behavior (and the reasoning) of player 2 and play *D*.

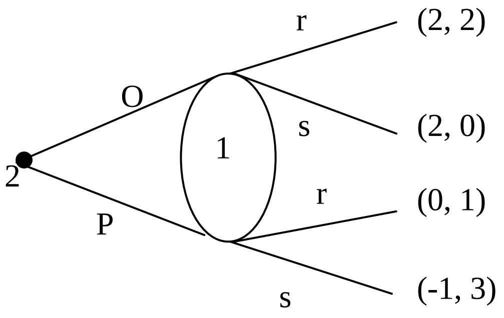
EXAMPLE 16



In this game there is a multiplicity of Nash equilibria and we cannot apply backward induction because there is a subgame with imperfect information. We shall use the definition of subgame perfect equilibrium and we shall require that the relevant part of the equilibrium strategies to be an equilibrium at the subgames.

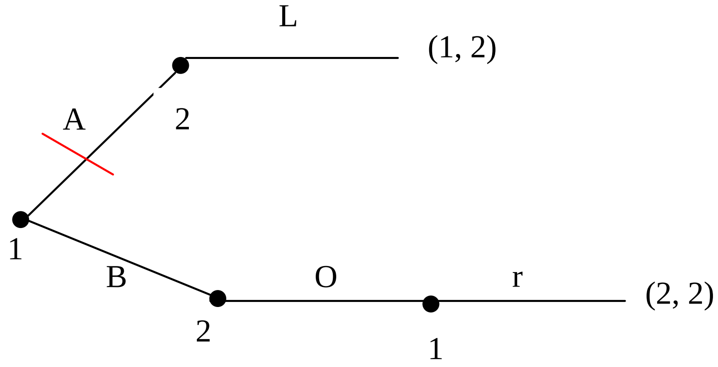


At the upper subgame (the perfect information subgame) the only credible threat by player 2 is *L*.



At the lower subgame (the imperfect information subgame) (which starts at the lower decision node of player 2), it is straightforward to check that the Nash equilibrium is *O*, *r*.

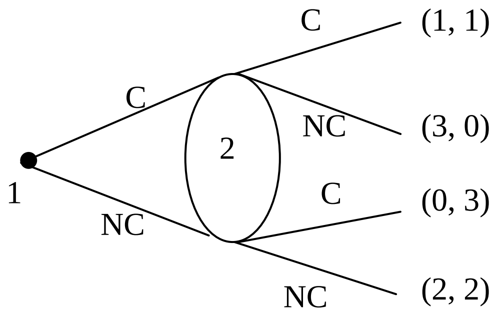
At her first decision node player 1 therefore has to choose between *A* and *B* anticipating that if she chooses *A* then player 2 will play *L* and if she chooses *B*, then they will both play the Nash equilibrium (of the subgame) *O*, *r*. Therefore, player 1 chooses *B*.



Therefore, the subgame perfect equilibrium is (*Br*, *LO*): the relevant part of the equilibrium strategies are also equilibrium at each of the subgames.

1.3. **Repeated games**

EXAMPLE 2: The Prisoner's Dilemma



		2	
		C	NC
1	C	(1, 1)	(3, 0)
	NC	(0, 3)	(2, 2)

When the game is played once the strategy profile (*C*, *C*) is the Nash equilibrium in dominant strategies and cooperation or collusion between players cannot hold as equilibrium. Even though both players obtain more profits in the combination of strategies (*NC*, *NC*), both players would have incentives to deviate by using the dominant strategy. In this section, we are going to study the possibilities of cooperation or collusion when the players play the game repeatedly.

1.3.1. **Finite temporal horizon**

Assume that the game (the Prisoner's Dilemma) is repeated a finite number of times **T** (known by both players). We know that if T = 1 the unique Nash equilibrium is (*C*, *C*).

The first point to note is that when the game is repeated T times, a player's strategy for the repeated game should indicate what the player would do at each stage of the game, contingent upon past history.

We shall use a *backward induction* argument to show that in the unique subgame perfect equilibrium of this repeated game each player (independently of past history) will choose "confess" at each stage of the game. Consider T, t = 1, 2,….., T, iterations of the Prisoner's Dilemma.

We start by looking at the last period **T**: in this last stage of the game, what has happened earlier (the past history) is irrelevant (because there is now no future) and all that remains is to play the Prisoner's Dilemma once. Therefore, as each player has "confess" as her dominant strategy (when the game is played once), in the last period each player will choose "confess". The only reason for playing "not confess" in any stage of the game would be to try to improve in the future, given that such behavior might be interpreted as a sign of goodwill by the other player so as to gain her cooperation. However, at the last stage of the game there is no future so (*C*, *C*) is unavoidable.

Now consider period T-1. Given that players anticipate that in the last period they are not going to cooperate, the best they can do in period T-1 is follow the short term dominant strategy, that is, "confess". The only reason for playing "not confess" in this stage of the game would be to try to improve in the future, but in period T the players will choose (*C*, *C*). The same reasoning applies from periods T-2, T-3,….to period 1. Therefore, the unique subgame perfect equilibrium of the finitely repeated Prisoner's Dilemma simply involves T repetitions of the short term (static) Nash equilibrium.

Therefore, if the game is repeated a finite (and known) number of times, in the subgame perfect equilibrium each player would choose her short term dominant strategy at each stage of the game. As a consequence, cooperation between players cannot hold as equilibrium when the temporal horizon is finite.

1.3.2. **Infinite temporal horizon**

There are two ways of interpreting an infinite temporal horizon:

(i) *Literal interpretation*: the game is repeated an infinite number of times. In this context, to compare two alternative strategies, a player must compare the discounted present value of the respective gains. Let δ be the discount factor, 0 < δ < 1, and let r be the discount rate ($0 < r < \infty$) where $\delta = \frac{1}{1+r}$.

(ii) *Informational interpretation*: the game is repeated a finite but unknown number of times. At each stage there is a probability 0 < δ < 1 of the game continuing. In this setting, each player must compare the expected value (which might be also discounted) of the different strategies.

In this repeated game, a player’s strategy specifies her behavior in each period *t* as a function of the game’s past history. Let $H_{t-1} = \{s_{1\tau}, s_{2\tau}\}_{\tau=1}^{t-1}$, where $s_{i\tau} \in \{C, NC\}$, represents the past history (of the game).

Note first that there is a subgame perfect equilibrium of the infinitely repeated game where each player plays C (her short term dominant strategy) in each period. The strategy of each player would be "confess in each period independently of past history".

We now determine under what conditions there is also a subgame perfect equilibrium where the two players cooperate. Consider the following combination of long term strategies (called "trigger strategies"):

"Cooperate in each period playing NC if previously they have cooperated or $t=1$. Do not cooperate in each period by playing C if any player previously has deviated from cooperation."

($s_i^c \equiv \{s_{it}(H_{t-1})\}_{t=1}^{\infty}$, i =1,2.

where

$$s_{it}(H_{t-1}) = \begin{cases} NC & \text{if all elements of } H_{t-1} \text{ equal } (NC, NC) \text{ or } t=1 \\ C & \text{otherwise} \end{cases})$$

Note that these long term strategies incorporate "implicit punishment threats" in the case of breach of the (implicit) cooperation agreement. The threat is credible because "confess" in each period (independently of the past history) is a Nash equilibrium of the repeated game.

To check whether it is possible to maintain cooperation as equilibrium in this context, we have to check that players have no incentives to deviate; that is, we have to check that the combination of strategies (s_1^c, s_2^c) constitutes a Nash equilibrium of the repeated game. The discounted present value of gains of player *i* in the strategy profile (s_1^c, s_2^c) is given by:

$$\pi_i(s_i^c, s_j^c) = 2 + 2\delta + 2\delta^2 + ... = 2(1 + \delta + \delta^2 + ...) = \frac{2}{1-\delta}$$

Assume that player *i* deviates, and does so from the first period. Given that the other player punishes her (if the other player follows her strategy) for the rest of the game, the best that player *i* can do is also "confess" for the rest of the game. The discounted present value of deviating is:

$$\pi_i(s_i, s_j^c) = 3 + 1\delta + 1\delta^2 + ... = 3 + \delta(1 + \delta + \delta^2 + ...) = 3 + \delta\frac{1}{1-\delta}$$

Cooperation will be supported as a Nash equilibrium if no player has incentives to deviate; that is, when $\pi_i(s_i^c, s_j^c) \geq \pi_i(s_i, s_j^c)$. It is straightforward to check that if $\delta \geq \frac{1}{2}$ no player has any incentive to break the (implicit) collusion agreement.

We next see how that Nash equilibrium is also subgame perfect: that is, threats are credible. Consider a subgame arising after a deviation has occurred. The strategy of each player requires "confess" for the rest of the game independently of the rival's behavior. This pair of strategies is a Nash equilibrium of an infinitely repeated Prisoner's Dilemma because each player would obtain

$$\delta^{T-1}(1 + \delta + \delta^2 + ...) = \frac{\delta^{T-1}}{1-\delta}$$

if she does not deviate, while she would obtain 0 in each period in which she deviates from the cooperative strategy.

The above analysis serves as an example of a general principle arising in repeated games with an infinite temporal horizon. In these games it is possible to support as equilibrium behaviors that are not equilibrium in the short term. This occurs because of the "implicit punishment threat" that in the case of breach of the agreement one will be punished for the rest of the game. So the increase in profits (from a breach of the agreement) does not offset the loss of profits for the rest of the game.

1.4. **Conclusions**

We have analyzed different ways of solving games, although none of them is exempt from problems. The dominance criterion (elimination of dominated strategies) is useful in solving some games but does not serve in others because it provides no solution proposal. The weak version of this criterion (elimination of weakly dominated strategies) is highly useful in selecting among Nash equilibria, especially in games in normal or strategic form. The backward induction criterion allows solution proposals to be drawn up for games in extensive form. This criterion has the important property that in perfect information games without ties it leads to a unique outcome. But it also presents problems: the possibility of ties, imperfect information and unbounded rationality. This criterion is highly useful in selecting among Nash equilibria in games in extensive form. The joint use of the notion of Nash equilibrium and backward induction gives rise to the concept of subgame perfect equilibrium, which is a very useful criterion for proposing solutions in many games. Although it also presents problems (inefficiency, nonexistence and multiplicity) the notion of the Nash equilibrium is the most general and most widely used solution criterion for solving games. Being Nash equilibrium is considered a necessary (but not sufficient) condition for a solution proposal to be a reasonable prediction of rational behavior by players. If, for instance, we propose as the solution for a game a combination of strategies which is not a Nash equilibrium, that prediction would be contradicted by the development of the game itself. At least one player would have incentives to change her predicted strategy. In conclusion, although it presents problems, there is quasi-unanimity that all solution proposals must at least be Nash equilibrium.

Basic Bibliography

Varian, H. R., 1992, *Microeconomic Analysis*, 3th edition, Norton. Chapter 15, sections: introduction, 15.1, 15.2, 15.3, 15.4, 15.6, 15.7, 15.9, 15.10 and 15.11.

Complementary Bibliography

Kreps, D. M., 1994, *A course in microeconomic theory*, Harvester Wheatsheaf.

Tirole, J., 1990, *The Theory of Industrial Organization*, MIT Press.

Varian, H. R., 1998, *Intermediate Microeconomics: A Modern Approach*, Norton.