Market Power and Strategy

Problems

3rd Year

Degree in Economics

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Chapter 1. Game Theory and Competitive Strategy

1. Consider the following games in extensive form:
(i) For all the games: describe the strategies of each player and the subgames.

(ii) Represent games 1, 2, 3 and 5 in normal form.

(iii) Obtain the Nash equilibria of all the games. Considering the normal form representation of the games, what equilibria remain after the elimination of weakly dominated strategies?

(iv) Obtain the subgame perfect equilibria.

**Answer**


(ii) Follows immediately from question (i).

(iii) Game 1 ⇒ NE ⇒ (I, MP) and (D, MO) (remains after IEWDS). Game 2 ⇒ NE ⇒ (I, M).

(iv) Game 1 ⇒ SPE ⇒ (D, MO). Game 2 ⇒ SPE ⇒ (I, M). Game 3 ⇒ SPE ⇒ (Dv, PS). Game 4 ⇒ SPE ⇒ (Mu, Pr). Game 5 ⇒ SPE ⇒ (Ivv, LP).
2. Consider the following game in extensive form:

(i) Represent the game in normal form.

(ii) At what values of $\alpha$ and $\beta$ does the combination of strategies $(Ia, SP)$ constitute the unique subgame perfect equilibrium?

(iii) Are there $\alpha$ and $\beta$ values such that the combination of strategies $(Db, TP)$ is a Nash equilibrium?

(iv) Suppose that $\alpha = 0$. Is there a $\beta$ value such that the combination of strategies $(Da, SQ)$ would constitute a subgame perfect equilibrium?

**Answer**

(ii) $\alpha > 1$ and $\beta < 2$. (iii) No. (iv) No.
3. We have the following information on the game in strategic form given below:

a) Strategy $B$ weakly dominates strategy $A$ of player 1.

b) The combination of strategies $(C, I)$ is not a Nash equilibrium.

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<tr>
<th></th>
<th>H</th>
<th>I</th>
<th>J</th>
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<tbody>
<tr>
<td><strong>A</strong></td>
<td>(4, 2)</td>
<td>(2, 0)</td>
<td>(0, 3)</td>
</tr>
<tr>
<td><strong>B</strong></td>
<td>(5, 1)</td>
<td>(3, 2)</td>
<td>(c, 4)</td>
</tr>
<tr>
<td><strong>C</strong></td>
<td>(5, 1)</td>
<td>(6, 2)</td>
<td>(a, b)</td>
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Are the following statements true or false? Discuss why:

(i) Player 2 has a dominant strategy.

If the combination of strategies $(C, J)$ constitutes a Nash equilibrium:

(ii) It is the unique Nash equilibrium.

(iii) Strategy $C$ strictly dominates strategy $A$.

(iv) $(C, J)$ is the only equilibrium not based on weakly dominated strategies.

**Answer**

(a) $\Rightarrow c \geq 0 \text{ and } b > 2.$

(i) **True**, $J$ is a dominant strategy.

(ii) **False**. $(a) \Rightarrow c \geq 0$ and if $(C, J)$ is a NE $\Rightarrow a \geq c \Rightarrow$ there may be more equilibria.

(iii) **False**.

(iv) **True**.
4. Consider the following three-player game in extensive form:

Are the following statements true or false? Why?:

(a) It is a perfect information game.

(b) Given the other players’ combination of strategies (M₁, L₃T₃), the best response of player 2 is S₂.

(c) (4, 2, 1) is a Nash equilibrium.

(d) (M₁, L₂T₂, L₃S₃) is a Nash equilibrium.

(e) There is only one subgame perfect equilibrium.

Answer

(a) No. (b) No, S₂ is not a strategy of player 2. (c) No. A Nash equilibrium will always be a combination of strategies. (d) No, player 1 has incentives to change strategy. (e) Yes. SPE⇒ (L₁, M₂S₂, L₃T₃).
5. Consider the following three-player game in extensive form:

(a) Define the notions of strategy and Nash equilibrium. Represent the game in normal form.

(b) Find the Nash equilibria.

(c) Find the subgame perfect equilibria.

**Answer**

(b) NE: (I, MP, w), (D, LP, u) and (D, MP, u).

(c) SPE: (D, MP, u).

6. Consider the following simultaneous three-player game:

Find the Nash equilibria.
7. (i) Define the notions of strictly dominated strategy and of Nash equilibrium (in pure strategies).

Consider the following game in normal form:


(iii) Find the pure-strategy Nash equilibria. Explain.

Answer

(iii) NE: (B, J, T).

8. Consider the following three-player game. In the first stage of the game player 1 has two possible actions, L and R. Once player 1 has decided her action, player 2, which does not observe what player 1 has decided, has to choose between O and P. Finally, it
is the turn for player 3 to play, which without observing what players 1 and 2 have decided, has to choose between h and s. Payments (from top to bottom in the decision tree) are (2,1,3) (4,2,1) (0,2,0) (1,0,1) (4,0,2) (3,1,1) (5,-1,3) (0,0,0).

(i) Represent the game in extensive form. Define strategy. Represent the game in normal form.

(ii) Define the notion of Nash equilibrium. Obtain the Nash equilibrium. **Answer:** (R, O, h) is the unique Nash equilibrium. Explain.

(iii) Define subgame and subgame perfect equilibrium. Obtain the subgame perfect equilibrium. **Answer:** In this game, there is only one subgame that coincides with the own game. Therefore, (R, O, h) is the subgame perfect equilibrium.

9. Given the following game in strategic form:

(i) What relation must exist between the parameters in order to have a prisoner’s dilemma?

(ii) Suppose that the game is repeated an infinite number of times. In order for cooperation to be sustained as an equilibrium, what would the discount factor have to be?
Answer

(i) \(c > b > a > d\).

(ii) \(\delta \geq \frac{c - b}{c - a}\).