Market Power and Strategy

Problems

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Degree in Economics

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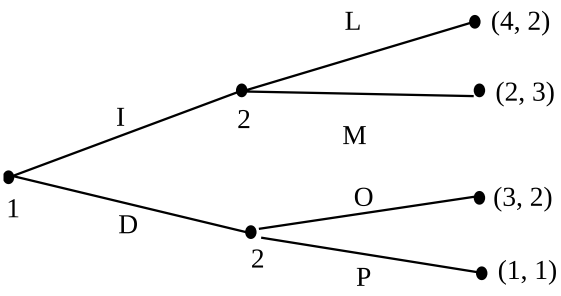
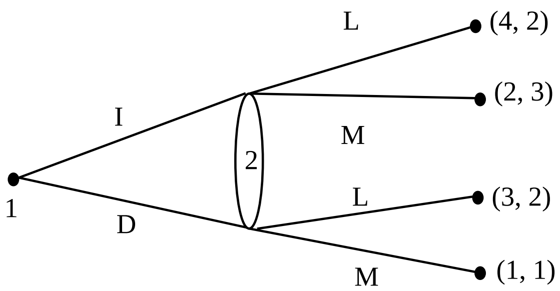
Chapter 1. Game Theory and Competitive Strategy

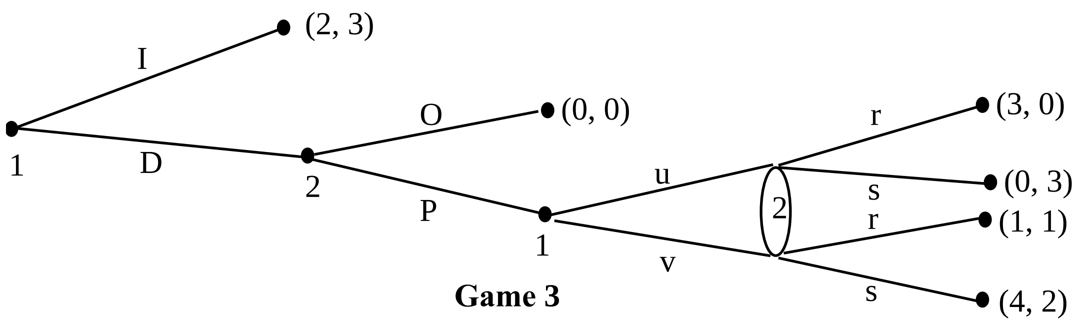
1. Consider the following games in extensive form:



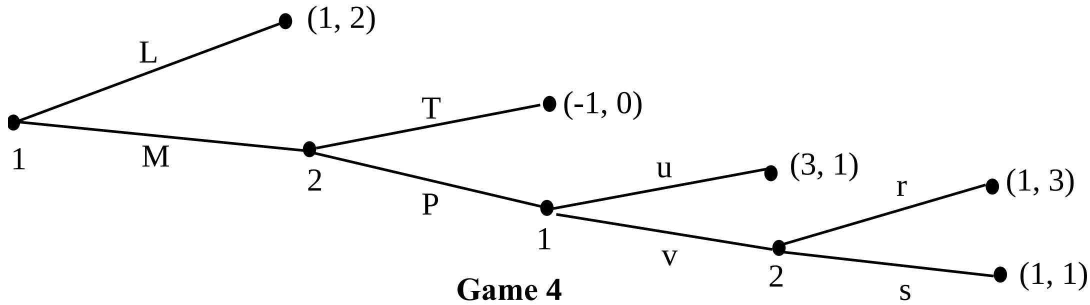
Game 1



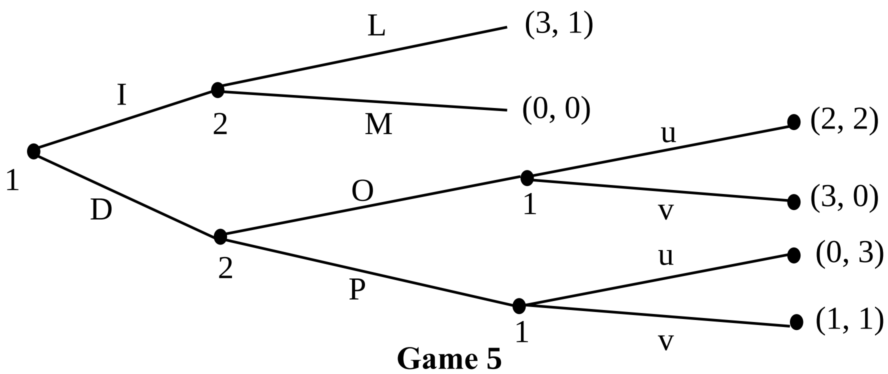
Game 2



Game 3



Game 4



Game 5

(i) For all the games: describe the strategies of each player and the subgames.

(ii) Represent games 1, 2, 3 and 5 in normal form.

(iii) Obtain the Nash equilibria of all the games. Considering the normal form representation of the games, what equilibria remain after the elimination of weakly dominated strategies?

(iv) Obtain the subgame perfect equilibria.

Answer

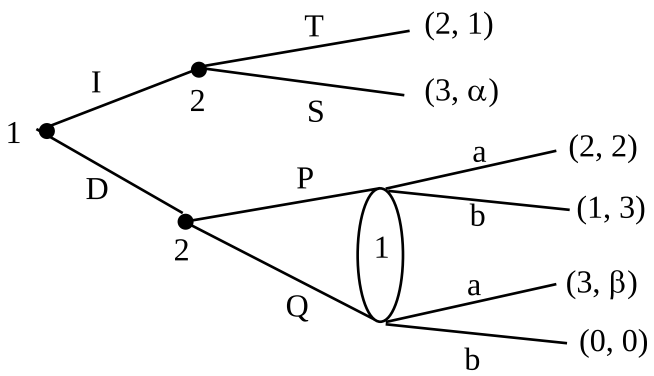
(i) Game 1$\Rightarrow$ 3 subgames. Player 1 strategies: I and D. Player 2 strategies: LO, LP, MO and MP. Game 2$\Rightarrow$ 1 subgame. Player 1 strategies: I and D. Player 2 strategies: L and M. Game 3$\Rightarrow$ 3 subgames. Player 1 strategies: Iu, Iv, Du and Dv. Player 2 strategies: Or, Os, Pr and Ps. Game 4$\Rightarrow$ 4 subgames. Player 1 strategies: Lu, Lv, Mu and Mv. Player 2 strategies: Tr, Ts, Pr and Ps. Game 5$\Rightarrow$ 5 subgames. Player 1 strategies: Iuu, Iuv, Ivu, Ivv, Duu, Duv, Dvu and Dvv. Player 2 strategies: LO, LP, MO and MP.

(ii) Follows immediately from question (i).

(iii) Game 1$\Rightarrow$ NE$\Rightarrow$ (I, MP) and (D, MO) (remains after IEWDS). Game 2 $\Rightarrow$NE$\Rightarrow$ (I, M) .

(iv) Game 1$\Rightarrow$ SPE$\Rightarrow$ (D, MO). Game 2$\Rightarrow$ SPE$\Rightarrow$ (I, M). Game 3$\Rightarrow$ SPE$\Rightarrow$ (Dv, PS). Game 4$\Rightarrow$ SPE$\Rightarrow$ (Mu, Pr). Game 5$\Rightarrow$ SPE$\Rightarrow$ (Ivv, LP).

2. Consider the following game in extensive form:



(i) Represent the game in normal form.

(ii) At what values of α and β does the combination of strategies **(Ia, SP)** constitute the unique subgame perfect equilibrium?

(iii) Are there α and β values such that the combination of strategies **(Db, TP)** is a Nash equilibrium?

(iv) Suppose that $\alpha = 0$. Is there a β value such that the combination of strategies **(Da, SQ)** would constitute a subgame perfect equilibrium?

Answer

(ii) $\alpha > 1$ and $\beta < 2$. (iii) **No**. (iv) **No**.

3. We have the following information on the game in strategic form given below:

a) Strategy **B** weakly dominates strategy **A** of player 1.

b) The combination of strategies **(C, I)** is not a Nash equilibrium.

1 \ 2	**H**	**I**	**J**
A	(4, 2)	(2, 0)	(0, 3)
B	(5, 1)	(3, 2)	(c, 4)
C	(5, 1)	(6, 2)	(a, b)

Are the following statements true or false? Discuss why:

(i) Player 2 has a dominant strategy.

If the combination of strategies **(C, J)** constitutes a Nash equilibrium:

(ii) It is the unique Nash equilibrium.

(iii) Strategy **C** strictly dominates strategy **A**.

(iv) **(C, J)** is the only equilibrium not based on weakly dominated strategies.

Answer

(a) $\Rightarrow c \geq 0$ and (b) $\Rightarrow b > 2$.

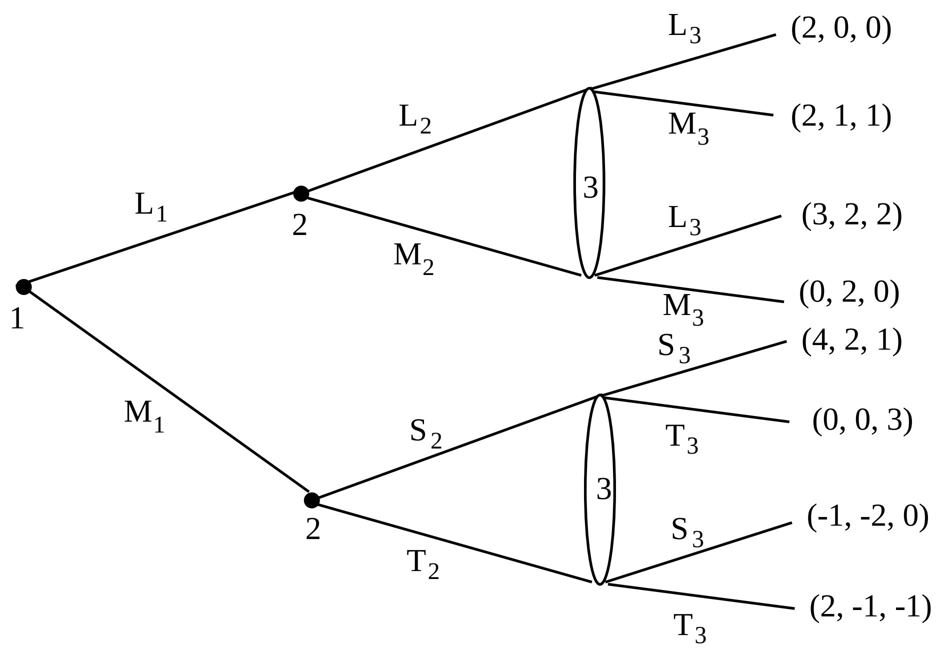
(i) **True**, J is a dominant strategy.

(ii) **False**. (a)$\Rightarrow c \geq 0$ and if (C, J) is a NE$\Rightarrow a \geq c \Rightarrow$there may be more equilibria.

(iii) **False**.

(iv) **True**.

4. Consider the following three-player game in extensive form:



Are the following statements true or false? Why?:

(a) It is a perfect information game.

(b) Given the other players' combination of strategies (M_1, L_3T_3), the best response of player 2 is S_2.

(c) (4, 2, 1) is a Nash equilibrium.

(d) (M_1, L_2T_2,L_3S_3) is a Nash equilibrium.

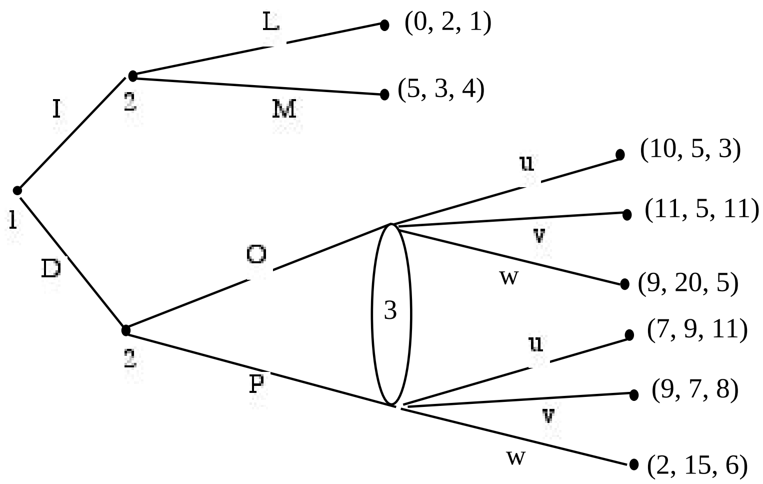
(e) There is only one subgame perfect equilibrium.

Answer

(a) **No**. (b) **No**, S_2 is not a strategy of player 2. (c) **No**. A Nash equilibrium will always be a combination of strategies. (d) **No**, player 1 has incentives to change strategy. (e) **Yes**. SPE$\Rightarrow$ **(L_1, M_2S_2, L_3T_3)**.

5. Consider the following three-player game in extensive form:

(a) Define the notions of strategy and Nash equilibrium. Represent the game in normal form.



(b) Find the Nash equilibria.

(c) Find the subgame perfect equilibria.

Answer

(b) NE: (I, MP, w), (D, LP, u) and (D, MP, u).

(c) SPE: (D, MP, u).

6. Consider the following simultaneous three-player game:

Player 3: **R**

Player 1 \ Player 2	**H**	**I**
A	(5, 3, 2)	(4, 11, 1)
B	(3, 11, 0)	(0, 8, 2)

Player 3: **T**

Player 1 \ Player 2	**H**	**I**
A	(1, 3, 6)	(5, 11, 0)
B	(9, 0, 3)	(4, 8, 2)

Find the Nash equilibria.

Answer

Player 3: R

Player 1 \ Player 2	H	I
A	(5, 3, 2)	(4, 11, 1)
B	(3, 11, 0)	(0, 8, 2)

Player 3: T

Player 1 \ Player 2	H	I
A	(1, 3, 6)	(5, 11, 0)
B	(9, 0, 3)	(4, 8, 2)

7. (i) Define the notions of strictly dominated strategy and of Nash equilibrium (in pure strategies).

Consider the following game in normal form:

Player 3: R

1 \ 2	H	I	J
A	(2, 0, 1)	(4, 1, 3)	(0, 3, 1)
B	(3, 2, 4)	(3, 3, 2)	(5, 1, 2)
C	(2, 1, 2)	(2, 2, 3)	(2, 0, 2)

Player 3: S

1 \ 2	H	I	J
A	(1, 2, 3)	(4, 4, 1)	(5, 1, 0)
B	(1, 0, 1)	(3, 1, 0)	(6, 3, 1)
C	(3, 0, 1)	(4, 1, 1)	(1, 0, 0)

Player 3: T

1 \ 2	H	I	J
A	(1, 2, 0)	(4, 3, 2)	(3, 3, 1)
B	(1, 1, 3)	(3, 2, 1)	(4, 4, 3)
C	(0, 0, 2)	(1, 1, 2)	(0, 2, 1)

(ii) What strategies remain after iterative elimination of **strictly** dominated strategies? Explain in detail.

(iii) Find the pure-strategy Nash equilibria. Explain.

Answer

(iii) NE: (B, J, T).

8. Consider the following three-player game. In the first stage of the game player 1 has two possible actions, L and R. Once player 1 has decided her action, player 2, which does not observe what player 1 has decided, has to choose between O and P. Finally, it is the turn for player 3 to play, which without observing what players 1 and 2 have decided, has to choose between h and s. Payments (from top to bottom in the decision tree) are (2,1,3) (4,2,1) (0,2,0) (1,0,1) (4,0,2) (3,1,1) (5,-1,3) (0,0,0).

(i) Represent the game in extensive form. Define strategy. Represent the game in normal form.

(ii) Define the notion of Nash equilibrium. Obtain the Nash equilibrium. **Answer:** (R, O, h) is the unique Nash equilibrium. Explain.

(iii) Define subgame and subgame perfect equilibrium. Obtain the subgame perfect equilibrium. **Answer:** In this game, there is only one subgame that coincides with the own game. Therefore, (R, O, h) is the subgame perfect equilibrium.

9. Given the following game in strategic form:

		B: C	B: NC
A	C	(a, a)	(c, d)
	NC	(d, c)	(b, b)

(i) What relation must exist between the parameters in order to have a prisoner's dilemma?

(ii) Suppose that the game is repeated an infinite number of times. In order for cooperation to be sustained as equilibrium, what would the discount factor have to be?

Answer

(i) $c > b > a > d$. (ii) $\delta \geq \dfrac{c-b}{c-a}$.

10. We have the following information concerning the three-player game in normal form given below.

a) H is a strictly dominated strategy for player 2.

b) The best response of player 3 against (B, I) is not S.

R (player 3); rows: player 1; columns: player 2

1 \ 2	H	I	J
A	(2, 0, 1)	(4, 1, 3)	(0, 3, 2)
B	(3, 2, 4)	(**a**, 3, 2)	(5, 1, 3)
C	(2, 1, 2)	(2, 2, 3)	(2, 0, 2)

S (player 3)

1 \ 2	H	I	J
A	(1, 2, 3)	(4, 4, 1)	(5, 1, 0)
B	(1, 0, 1)	(3, 1, **c**)	(6, 3, 1)
C	(3, 0, 1)	(4, 1, 1)	(1, 0, 0)

T (player 3)

1 \ 2	H	I	J
A	(1, 2, 0)	(4, **b**, 2)	(3, 3, 1)
B	(1, 1, 3)	(3, 2, 1)	(4, 4, 2)
C	(0, 0, 2)	(1, 1, 2)	(0, 0, 1)

Are the following statements true or false? Discuss why:

(i) R is a (strictly) dominant strategy for player 3.

(ii) (A, I, R) survives the iterative elimination of strictly dominated strategies.

(iii) If (B, I, R) is a Nash equilibrium, then it is the unique Nash equilibrium.

Answer

(a) ⇒ $\boldsymbol{b > 2}$ and (b) ⇒ $\boldsymbol{c < 2}$.

(i) **False**. Against (A, H) is better R than S, and against (C, H) there is a tie between T and R.

(ii) **False**. First stage: Given that $\mathbf{b > 2}$, then H is a strictly dominated strategy (by I), and, therefore, we eliminate it. Second stage: In the reduced game, S and T are dominated strategies (by R). Therefore, we eliminate S and T. Note that if $\boldsymbol{a} > \mathbf{4}$ then B dominates A, and therefore (A, I, R) does not survive the IESDS (in fact, only (B, I, R) survives).

(iii) **True**. Note that ((B, I, R) is the only candidate to be Nash equilibrium. If $\boldsymbol{a} \geq \mathbf{4}$ then (B, I, R) is the unique Nash equilibrium. If $\boldsymbol{a} < \mathbf{4}$ then there is no pure strategy Nash equilibrium.