Problems of Chapter 1:

Bayesian Games in Normal Form

Uncertainty and Contracts

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1. A sheriff (player 2) faces an armed suspect (player 1) and they each must (simultaneously) decide whether to shoot (S) the other or not (N), and:

- the suspect is either a criminal (state bad) with probability p or not (state good) with probability $1 - p$.

- the sheriff would rather shoot if the suspect shoots, but not if the suspect does not.

- the criminal would rather shoot even if the sheriff does not, as the criminal will be caught if she does not shoot.

- the innocent suspect would rather not shoot even if the sheriff shoots.

Payoffs matrices as a function of actions and states are (being the suspect the row player and the Sheriff the column player):

	S	N
S	(-3, -1)	(-1, -2)
N	(-2, -1)	(0, 0)

$Good$

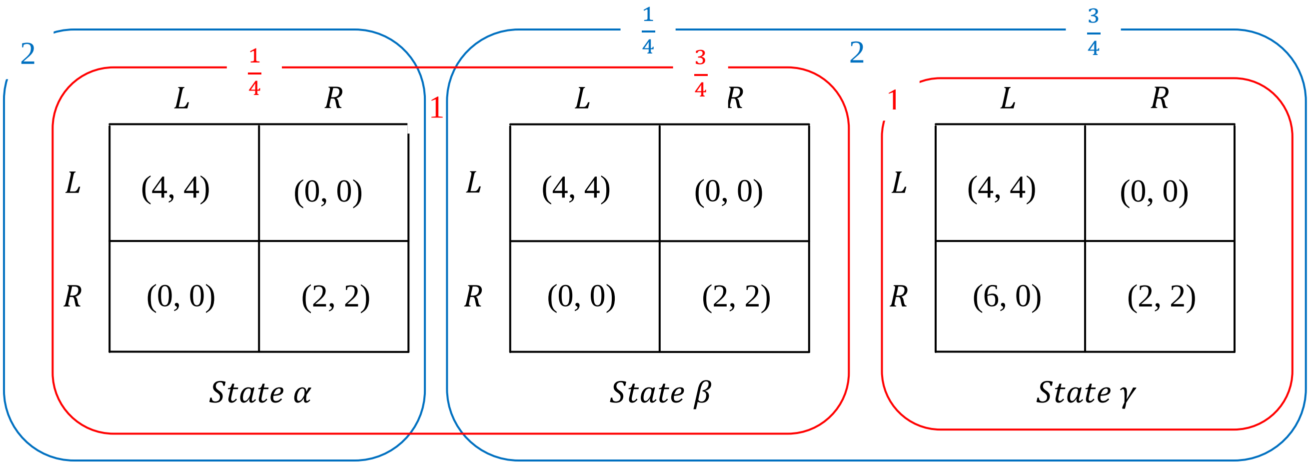
	S	N
S	(0, 0)	(2, -2)
N	(-2, -1)	(-1, 1)

Bad

Represent the Bayesian game and identify the main elements of the Bayesian game. Obtain the Nash equilibrium.

https://www.youtube.com/watch?v=-aeRHk55hmo

2. Consider a two-player game in a context of uncertainty, where states (of nature) are α, β and γ.



Player 2's preferences are the same in the three states. However, player 1's preferences are the same in the states α and β (in this case, we denote player 1 by type A), but differ from her preferences in state γ (in which case we denote player 1 by type B). On the other hand, player 2 can know or not whether player 1 is type A. Finally, if player 1 is type A, she thinks with probability ¼ that player 2 knows her type and with probability ¾ does not. Moreover, when player 2 does not know the type of player 1, with probability ¼ thinks that is type A and with probability ¾ thinks that is type B.

(i) Identify the main elements of the Bayesian game.

(ii) Obtain the Nash equilibria of the game.

3. Consider a market served by two firms, 1 and 2, that produce a homogenous product and compete on quantity (Cournot model). There are no fixed costs and marginal costs of firm 1 and firm 2 are $c_1 = 2$ and $c_2 = 1$, respectively. Assume that demand can be high or low: if it is high the inverse demand function is given by $p(Q) = 10 - Q$ and if it is low, the inverse demand is $p(Q) = 6 - Q$ where $Q = q_1 + q_2$. Firm 2 has private information on how the demand is, but firm 1 does not know it. Firm 1's beliefs are that with probability ¾ the demand is high and with probability ¼ is low.

(i) Identify the main elements of the Bayesian game.

(ii) Obtain the Cournot-Nash equilibrium.

4. Two firms, 1 and 2, compete in a market. Market demand can be *high*, *medium* or *low*. Firms decide simultaneously and independently whether to produce a high or a lower quantity.

- Firm 1, if demand is high, knows that demand is high, but in other case it does not know if demand is medium or low. If demand is not high, firm 1 assigns probability ¼ to medium demand and ¾ to low demand.

- Firm 2, if demand is low, knows that demand is low, but in other case it does not know if demand is high or medium. If demand is not low, firm 2 assigns probability ½ to high demand and ½ to medium demand.

Payoffs matrices for the three possible situations are given by:

	H	L
H	(32, 44)	(40, 26)
L	(20, 52)	(24, 30)

High demand

	H	L
H	(0, 12)	(8, 10)
L	(4, 20)	(8, 14)

Medium demand

	H	L
H	(-8, 4)	(0, 6)
L	(0, 12)	(4, 10)

Low demand

(i) Represent the Bayesian game and identify the main elements of the Bayesian game. Obtain the Nash equilibrium

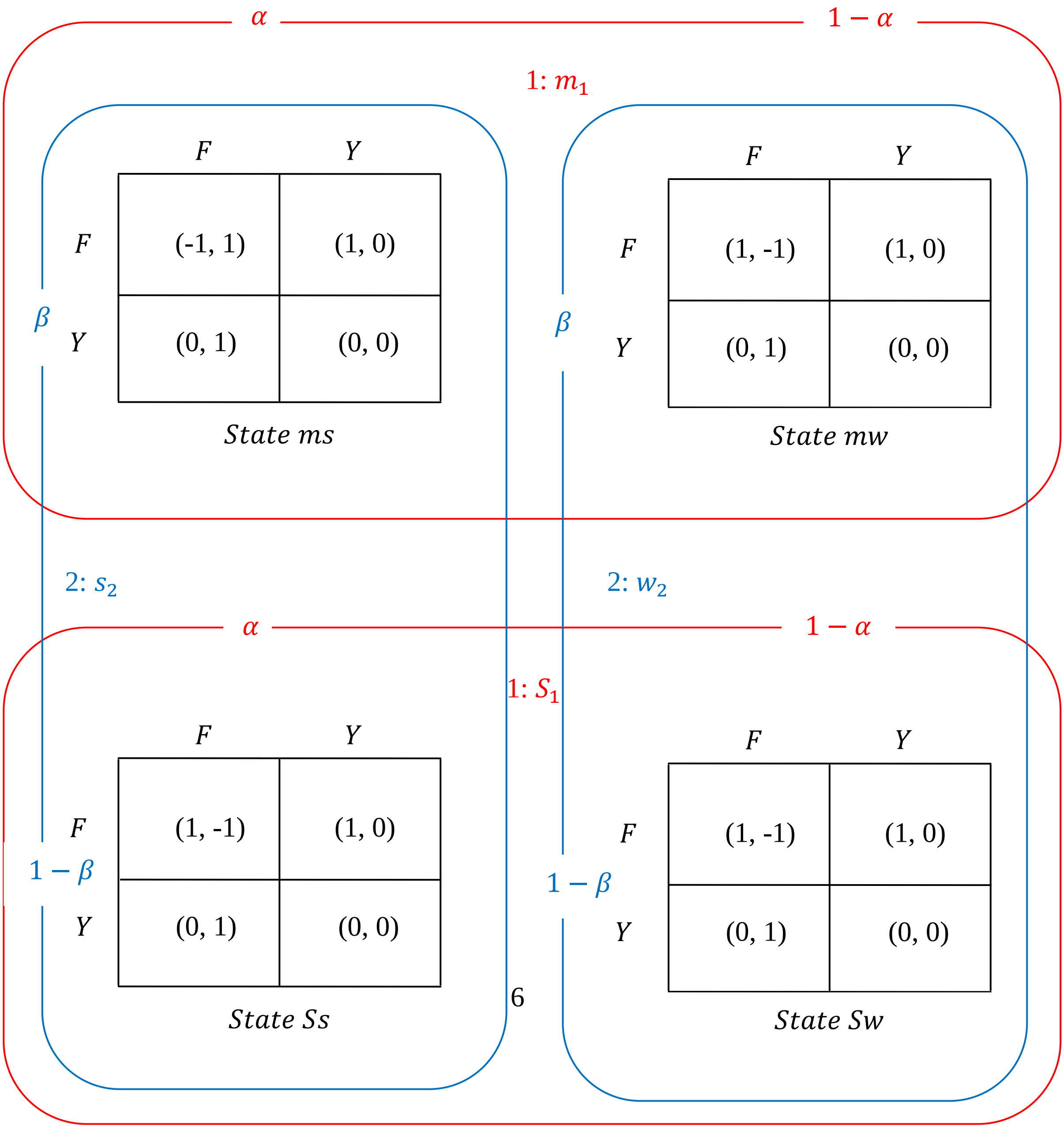
(ii) What are firms' profits (at equilibrium) is the demand is high?

(iii) Assume now that no firm can distinguish whether demand is high, medium or low. Firms believe that with probability ½ the demand is high, with probability ¼ is medium and with probability ¼ is low. Represent the game, identify the main elements of the Bayesian game and obtain the Nash equilibrium.

(iv) Assume now that firm 1 cannot distinguish whether the demand is high, medium or low. It believes that with probability ½ the demand is high, with probability ¼ is medium and with probability ¼ is low. However firm 2, if the demand is not low then it believes with probability ½ that the demand is high and with probability ½ that is medium. If the demand is low, firm 2 knows that is low.

Represent the Bayesian game and identify the main elements of the Bayesian game. Obtain the Nash equilibrium.

5. Two people are involved in a dispute. Person 1 does not know whether person 2 is *strong* or *weak*; she assigns probability α to person 2's being strong. Assume that person 2 neither knows whether person 1 is *medium strength* or *super strong*; she assigns probability β to person 1's being medium strength. Each person can either fight or yield. If player 1 is medium strength payoffs are (-1, 1) when both fight and person 2 is strong and (1, -1) when both fight and person 2 is weak. If player 1 is super strong, payoff are (1, -1) when both fight independently whether person 2 is strong or weak. In the rest of cases payoffs are as those in the previous game. That is, if one person fights, and other does not, then she obtains a payoff 1. If one person does not fight, independently of the rival's behavior, then she obtains 0.



Represent the Bayesian game and identify the main elements of the Bayesian game. Obtain the Nash equilibria to any $\alpha, \beta \in [0,1]$.

6. Consider the following payoffs matrix (being player 1 the row player and player 2 the column player):

	L	*R*
U	(2, 1)	(0, 0)
D	(1, *x*)	(3, -*x*)

Parameter x can take two possible values: 1 and -1. Player 2 knows this parameter. Player 1 believes that x = 1 with probability ¼ and x = -1 with probability ¾. Represent the Bayesian game and identify the main elements of the Bayesian game. Obtain the Nash equilibrium.

7. Consider a Cournot duopoly in which firms 1 and 2 produce a homogenous product and compete on quantity. The inverse demand function is given by:

$$p(q) = \begin{cases} 100 - q & if\ \ 100 \geq q \\ 0 & if\ 100 \leq q \end{cases}$$

where q is the total output of the industry. Assume that there are no fixed costs and each firm's marginal cost is constant. Marginal cost of each firm is known only by the own firm and can be high, $c = 4$, or low, $c = 0$. Firm 1 believes that with probability 2/3 firm 2's marginal cost is high and with probability 1/3 firm 2's marginal cost is low. Firm 2 believes that with probability ½ firm 1's marginal cost is high and with probability ½ firm 1's marginal cost is low. Represent the Bayesian game and identify the main elements of the Bayesian game. Obtain the Cournot-Nash equilibrium.

8. Consider a market in which two firms, 1 and 2, compete being firm 1's productive capacity known by both firms. Firm 2 knows its own capacity, but firm 1 does not know whether firm 2 has a high capacity or a low capacity. Firm 1 believes that with probability ½ firm 2's capacity is high and with probability ½ firm 2's capacity is low. Both firms have to choose whether taking an aggressive market behavior, A, or not, N. Payoffs matrices as a function of actions and states are (being firm 1 the row player and firm 2 the column player):

	A	N
A	(16, 15)	(48, 7)
N	(24, 35)	(40, 11)

High capacity

	A	N
A	(16, -25)	(48, -1)
N	(24, 5)	(40, 3)

Low capacity

Represent the Bayesian game and identify the main elements of the Bayesian game. Obtain the Nash equilibrium.

9. Consider a Cournot duopoly in which firms 1 and 2 produce a homogenous product and compete on quantity. The inverse demand function is given by:

$$p(q) = \begin{cases} 25 - q & if \;\; 100 \geq q \\ 0 & if \;\; 100 \leq q \end{cases}$$

where q is the total output of the industry. Assume that there are no fixed costs and each firm's marginal cost is constant. Firm 1's marginal cost is known by both firms and equal to $c_1 = 5$. However, firm 2's marginal cost (that is is known by firm 2) is unknown by firm 1: it believes that with probability ¼ is high, $c_H = 12$, and with probability ¾ is low, $c_L = 4$

Represent the Bayesian game and identify the main elements of the Bayesian game. Obtain the Cournot-Nash equilibrium.

10. Two players have to choose between S and C. The payoffs for both players (being player 1 the row player and player 2 the column one), as a function of the selected actions are

	S	C
S	(2, 2)	(-1, x_c)
C	(x_r, -1)	(0, 0)

Where player 1 knows the value of x_r (and knows that can be 0 or 3) and player 2 knows the value of x_c (and knows that can be 0 or 3). However, player 1 does not know x_c with certainty and believes that $x_c = 0$ with probability p and that $x_c = 3$ with probability $1 - p$. Player 2 neither knows x_r with certainty and believes that $x_r = 0$ with probability q and that $x_r = 3$ with probability $1 - q$.

Represent the Bayesian game and identify the main elements of the Bayesian game. Obtain the Nash equilibria to any $p > \frac{2}{3}$ and $\frac{1}{3} < q < \frac{2}{3}$.