Review of Financial Economics 18 (2009) 10–22

Contents lists available at ScienceDirect

Review of Financial Economics

journal homepage: www.elsevier.com/locate/rfe

Monte Carlo valuation of natural gas investments

Luis M. Abadie [a,1], José M. Chamorro [b,*]

[a] Bilbao Bizkaia Kutxa, Gran Vía, 30, 48009 Bilbao, Spain
[b] University of the Basque Country, Av. Lehendakari Aguirre, 83, 48015 Bilbao, Spain

ARTICLE INFO

Article history:
Received 26 September 2007
Accepted 1 October 2008
Available online 17 October 2008

JEL codes:
C6
D8
G1

Keywords:
Real options
Power plants
Stochastic revenues and costs
CO_2 allowances
LNG

ABSTRACT

In this evaluation of energy assets related to natural gas, our particular focus is on a base load natural gas combined cycle power plant and a liquefied natural gas facility in a realistic setting. We also value several American-type investment options following the least squares Monte Carlo approach. We calibrate mean-reverting stochastic processes for gas and electricity prices by using data from NYMEX NG futures contracts and the Spanish wholesale electricity market, respectively. Additional sources of uncertainty concern the initial investment outlay, or the option's time to maturity, or the cost of CO_2 emission permits.

1. Introduction

The aim of this paper is to use the real options method to evaluate energy investments related to natural gas. Energy resource prices are now at or near record levels. From the demand side, strong economic growth in America, China, and India has placed world reserves under substantial strain. From the supply side, new discoveries and investments by the industry have not kept pace. In sum, energy markets are currently very tight, and energy security concerns are increasingly acute. Therefore, consumers can expect any external shock to translate into greater volatility of oil and gas prices.

However, we cannot ignore regulatory uncertainties. A decade ago, the United States decided to break up the vertically integrated electricity industry (generation, transmission, and distribution). Basically, the regulators' rationale was to accomplish higher levels of competition and efficiency at every stage. This process has brought about a number of deals concerning the buying and selling of assets. Meanwhile, the European Union has been pushing ever harder for the creation of a single market in traditionally fragmented industries, such as energy. As a consequence, several takeovers and mergers have taken place at the national level within the EU, and even cross-border mergers and acquisitions are being proposed. In addition, European power utilities now face a new carbon market (the EU Emissions Trading Scheme), which, regardless of whether it is seen as a threat or an opportunity, will no doubt influence firms' decision making.

In the case of energy investments, this uncertain environment is coupled with either irreversibility considerations, or a chance to defer investment, or to manage investment in a flexible way. Under these circumstances, valuation techniques based on the methods for pricing options (such as Contingent Claims Analysis or Dynamic Programming) are superior to the traditional approaches that are based on discounted cash flows [see, e.g., Dixit & Pindyck, 1994; Sick, 1995; Trigeorgis, 1996]. Our aim in this paper is to use the real options method to evaluate energy investments related to natural gas. We evaluate a base load natural gas combined cycle (NGCC) power plant and an ancillary installation, a liquefied natural gas (LNG) facility, in a realistic setting. These investments enjoy a long useful life but require some non-negligible time to build. Then we focus on the valuation of several investment options again in a realistic setting.

A variety of models have been proposed for representing the stochastic process followed by commodity prices. Differences among them have to do with the number of risk factors necessary to describe uncertainty and the way to specify the convenience yield. According to Schwartz (1997), three factors (spot prices, interest rates, and convenience yields) are necessary to capture the dynamics of futures prices. More recently, Casassus and Collin-Dufresne (2005) have developed a three-factor model of commodity futures prices which nests many frequent specifications. One of them is Schwartz and Smith (2000), who present a model with two factors, a (mean-reverting) short-term disturbance in prices and a long-term price level (which follows a Brownian motion). The two factors are not directly observable, but they may be conveniently estimated from spot

* Corresponding author. Tel.: +34 946013769; fax: +34 946013891.
E-mail addresses: imabadie@euskalnet.net (L.M. Abadie), jm.chamorro@ehu.es (J.M. Chamorro).
[1] Tel.: +34 607408748; fax: +34 944017996.

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doi:10.1016/j.rfe.2008.10.002

and futures prices. Näsäkkälä and Fleten (2005) adopt this model for the spark spread in their valuation of a gas-fired power plant.

Our basic model comprises three sources of risk. We consider uncertain gas prices for both the current level and the long-term equilibrium level. Regarding the revenues side, the current electricity price is also stochastic. These prices are assumed to show mean reversion, and to follow an Inhomogeneous Geometric Brownian Motion (IGBM). This is a relatively general stochastic process in that it reduces to standard Geometric Brownian Motion, or even accounts for wild swings in the state variable, depending on its parameter values.

In the applied part we combine several methods developed elsewhere in isolation. First, we calibrate our two-factor model for natural gas price by using data from NYMEX NG futures contracts. The specific procedure is based on Cortazar and Schwartz (2003). Also, we calibrate the one-factor model for electricity price with data from the Spanish wholesale electricity market. Then we use the estimated parameter values alongside physical parameters to value an actual gas-fired plant and a LNG plant. The LNG market was relatively small a few years ago, but is now growing rapidly. To our knowledge, valuation of a LNG plant has not been previously addressed. We also use the calibrated parameters in a Monte Carlo simulation framework to evaluate several American-type options to invest in these energy assets. These include the option to invest in the power plant when there is uncertainty concerning the initial disbursement, or the option's time to maturity, or the cost of CO_2 emission permits, or when there is a chance to double the plant size in the future. We adopt the Least Squares Monte Carlo (LSM) approach as developed by Longstaff and Schwartz (2001).

The paper is organized as follows. In Section 2 we briefly describe the technology for producing electricity in a gas-fired power plant. Then we introduce the technology for storing natural gas, which has the potential to connect previously fragmented markets by unlocking the intimate relation between consumers and local providers. In Section 3 we show the mean-reverting stochastic process for input and output prices. We then derive the risk-neutral version of the model for valuation purposes. Also, since we adopt a Monte Carlo simulation as a numerical technique, we must adapt the model to a discrete-time context. In Section 4 we derive the parameter values of the stochastic processes. Thus, we calibrate the models for the gas price and for electricity price separately, using actual data. In Section 5 we describe our case study, which includes the physical parameter values of the NGCC plant and the LNG plant. In Section 6 we derive the value of each plant in operation. In Section 7 we evaluate several options to invest in a NGCC power plant. Section 8 concludes.

2. Basic description

2.1. The NGCC technology

The NGCC technology is based on using of two turbines, one of natural gas and the other one of steam. The exhaust gases from the first turbine are used to generate the steam that is used in the second turbine, which produces approximately one-third of the total power output. Thus, the technology consists of a Gas–Air (Brayton) cycle and a Water–Steam (Rankine) cycle. This system allows for a higher net efficiency (i.e., the percentage of the heating value of the fuel that is transformed into electric energy) than that of coal-fired power plants.

The advantages of a NGCC power plant are lower emissions of CO_2, estimated about 350 g/kWh, which makes it easier to comply with the Kyoto Protocol; a higher net efficiency between 50% and 60%; a low cost of the investment, about 422.5 €/kW installed; less consumption of water and space requirements, which allows companies to build in a shorter period of time and closer to consumer sites (a NGCC power plant can be built in 30 months on a surface of 100 m^2/MW); a useful life of 25 years; lower operating costs, with typical values of 0.35 cents €/kWh [our reference is ELCOGAS, 2003]; and, depending on the design of the gas turbine, some facilities can use other combustibles as diesel oil and fuel.

In addition, a NGCC power plant can be designed either as a base load plant or as a peaking plant. In the latter case, it operates only when electricity prices are high enough, which usually happens during periods of strong growth in demand. A further advantage of the NGCC stations is that these plants make it easier for new electricity producers to obtain permission for construction.

The disadvantages of a NGCC Power Plant are the higher cost of the natural-gas-fired generation in relation to coal; the concern over gas supplies, since reserves are less evenly distributed over the world; and the strong rise in the demand for natural gas, which can cause a consolidation of prices at higher than historical levels. Nonetheless, the spread of LNG plants brings an improvement in ensuring fuel supplies and a stronger link among formerly geographically fragmented natural gas markets.

2.2. The LNG technology

Cooling natural gas to about −163 °C at normal pressure puts the gas in its liquid form. In addition to the gain in transportability (it reduces some 600-fold the volume needed for storing it), liquefaction has the merit of removing some impurities, resulting in a type of gas whose emissions are much lower if fired later in a power plant (Geman, 2005).

Capital costs of liquefaction plants and regasification terminal costs have fallen significantly in the last 10 years, as have ship construction costs. These falling costs of supplying LNG have eventually coincided with declining gas reserves in Europe, the U.S., and parts of Asia, and with strong growth in natural gas demand. In fact, LNG demand is rising at an even faster rate than is the overall demand for natural gas. (Most of the power plants that have been or will be commissioned within the EU are located in Spain (20.5 GW of additional gas-fired units). The limited gas pipeline import capacity, the poor domestic gas storage potential, and the rapid increase in demand are apparently not sufficient to cover peak demand if a major failure occurs on one of the main import connections [Kjärstad & Johnsson, 2007].) Further, as the average distance between market participants drops, a firm that is looking at gas for an ongoing project can profit from a growing number of potential trading partners. Also, from the point of view of a gas provider, a deeper market reduces the costs associated with the loss of any one customer [Brito & Hartley, 2007].

LNG can make economically profitable some of the stranded natural gas deposits for which the construction of pipelines is not an option, but liquefying plants are. It also allows isolated regions to receive gas without being integrated into a main domestic gas network. Another advantage over piped gas is that LNG is only subject to country risk in the producing country (i.e., there is no need for transport through a third country).

An isolated LNG plant can purchase gas at the foreign price and sell it at the domestic price, thus exploiting the arbitrage opportunity (once transport costs between different sites have been accounted for). If instead the plant is attached to a power plant, then the same amount accrues to the LNG plant, since it represents the savings it provides in terms of extra expenses avoided In fact, should the power plant cease to operate for any reason, the LNG plant would still have a value in keeping with the price spread.

An LNG plant can be modelled as a spread option between using domestic gas or buying foreign gas, and allows the plant to freely choose the cheapest possibility at any time. The prices of LNG and piped gas can differ; see for instance the case of the U.S. (EIA, 2006). Similarly, according to EIA (2007), recent competition from buyers in Western Europe and Asia for LNG cargoes has resulted in LNG prices exceeding the corresponding natural gas market price in the United States.

3. The stochastic model for input and output prices

The stochastic behavior of energy prices shows both short- and long-term dynamics.[2] The short-term behavior displays mean reversion,

[2] Schwartz and Smith (2000) discuss these twin dynamics in a model which allows for mean reversion in the short term and uncertainty in the equilibrium price to which prices revert. See also Pilipovic (1998), and Baker, Mayfield and Parsons (1998).

seasonality, stochastic volatility, and in some instances, discrete jumps; but long-term behavior is determined by the equilibrium price's dynamics. Since one aim of our paper is to value an asset (a base load NGCC plant) with 25 years of useful life, we consider that all short-term features except mean reversion are less relevant.

3.1. A three-factor model

Our model is as follows:

$$dG_t = k_g(L_t - G_t)dt + \sigma_g G_t dW_t^G, \tag{1}$$

$$dL_t = \mu(L_g - L_t)dt + \xi L_t dW_t^L, \tag{2}$$

$$dE_t = k_e(L_e - E_t)dt + \sigma_e E_t dW_t^E, \tag{3}$$

where G_t denotes the time-t price of natural gas, and L_t is the natural gas equilibrium price level, which behaves according to Eq. (2). k_g stands for the speed of reversion of natural gas price towards its "normal" level; we can compute it as $k_g = \log 2/t_{1/2}$, where $t_{1/2}$ is the expected half-life, that is, the time for the gap between G_t and L_t to halve. σ_g is the instantaneous volatility of fuel price; μ is the speed of reversion of L_t towards its longer-term equilibrium value L_g; ξ denotes the instantaneous volatility of natural gas equilibrium price. E_t is the price of electricity at time t; k_e stands for the speed of reversion of electricity price towards its "normal" level over the long term L_e; and σ_e is the instantaneous volatility of electricity price. dW_t^G, dW_t^L, and dW_t^E are increments to standard Wiener processes. These increments are assumed to be normally distributed with mean zero and variance dt. We further assume that $\rho_{W^G W^L} = \rho_{W^L W^E} = 0$ and $\rho_{W^G W^E} = \rho$.

This model has some convenient implications: there is no chance for G_t or E_t to take on negative values; it allows the existence of an equilibrium level for both natural gas and electricity output, but for natural gas the equilibrium price is not constant; the expected values of long-term equilibrium prices remain finite, L_g and L_e. (In this model $E(L_t) = L_g + (L_0 - L_g)e^{-\mu t}$, which implies $E(L_\infty) = L_g$.) Further, the stochastic process for natural gas price is similar to Pilipovic (1998) model; yet they differ in that Eq. (2) above is of the IGBM type, as opposed to the standard GBM in Pilipovic. This kind of model seems preferable if the equilibrium price in the longer term is jointly determined by production cost and demand level. The model allows for market gyrations and wild swings in prices depending on parameter values. Admittedly, it does not account for discontinuous events that give rise to jumps. However, it also allows, as a particular case, that Eqs. (1) and (2) adopt a GBM format, again depending on the values of the parameters. Last, consistent with futures markets, volatilities do not grow without bound as $t \to \infty$; instead, they approach a finite value if reversion speed is high enough in relation to volatility.

In Appendix A we show that the expected value of natural gas price is

$$E(G_t) = L_g - \frac{k_g(L_0 - L_g)}{\mu - k_g}e^{-\mu t} + \left[G_0 - L_g + \frac{k_g(L_0 - L_g)}{\mu - k_g}\right]e^{-k_g t}, \tag{4}$$

and the expected value of electricity price is

$$E(E_t) = L_e + (E_0 - L_e)e^{-k_e t}. \tag{5}$$

3.2. The risk-neutral model

The model in a risk-neutral world is as follows,

$$d\hat{G}_t = \left[k_g\left(\hat{L}_t - \hat{G}_t\right) - \lambda_g \sigma_g \hat{G}_t\right]dt + \sigma_g \hat{G}_t dW_t^G, \tag{6}$$

$$d\hat{L}_t = \left[\mu\left(L_g - \hat{L}_t\right) - \lambda_l \xi \hat{L}_t\right]dt + \xi \hat{L}_t dW_t^L, \tag{7}$$

$$d\hat{E}_t = \left[k_e\left(L_e - \hat{E}_t\right) - \lambda_e \sigma_e \hat{E}_t\right]dt + \sigma_e \hat{E}_t dW_t^E, \tag{8}$$

where λ_g denotes the market price of risk stemming from current natural gas price (assumed to be constant); λ_l is the market price of equilibrium gas price risk; and λ_e is the market price of current electricity price risk.

In this risk-neutral setting, the expected value of natural gas price may be shown to be:

$$\begin{aligned} E\left(\hat{G}_t\right) = {} & \frac{\mu k_g L_g}{(\mu + \lambda_l \xi)(k_g + \lambda_g \sigma_g)}\left(1 - e^{-(k_g + \lambda_g \sigma_g)t}\right) \\ & + \left[\frac{\mu k_g L_g}{(\mu + \lambda_l \xi)(\mu + \lambda_l \xi - k_g - \lambda_g \sigma_g)} - \frac{k_g L_0}{(\mu + \lambda_l \xi - k_g - \lambda_g \sigma_g)}\right] \\ & \times \left(e^{-(\mu + \lambda_l \xi)t} - e^{-(k_g + \lambda_g \sigma_g)t}\right) + G_0 e^{-(k_g + \lambda_g \sigma_g)t}. \end{aligned} \tag{9}$$

This value $E(\hat{G}_t)$ equals the estimated futures price of natural gas $\hat{F}_t$ for maturity t. We note that in calibrating this model, we get a numerical estimate of $k_g L_0$ and the composites $\mu k_g L_g$, $\mu + \lambda_l \xi$, and $k_g + \lambda_g \sigma_g$. The last day in our sample price series allows to set the initial price of gas G_0. For an arbitrarily long maturity, the estimate for the futures price is

$$\hat{F}_\infty = \frac{\mu k_g L_g}{(\mu + \lambda_l \xi)(k_g + \lambda_g \sigma_g)}. \tag{10}$$

Now the expression for the forward risk premium is the difference between the values in Eqs. (4) and (9), $RP_{gt} = E(G_t) - E(\hat{G}_t)$. In principle, this difference could be either positive or negative.

The risk-neutral version for the electricity price is

$$E\left(\hat{E}_t\right) = E_0 e^{-(k_e + \lambda_e \sigma_e)t} + \frac{k_e L_e}{(k_e + \lambda_e \sigma_e)}\left(1 - e^{-(k_g + \lambda_e \sigma_e)t}\right). \tag{11}$$

3.3. The discrete-time version

In our Monte Carlo simulations, we use the following discretization of Eqs. (6)–(8):

$$\Delta \hat{G}_t = \left[k_g \hat{L}_t - \hat{G}_t(k_g + \lambda_g \sigma_g)\right]\Delta t + \hat{G}_t \sigma_g \sqrt{\Delta t}\epsilon_t^G, \tag{12}$$

$$\Delta \hat{L}_t = \left[\mu L_g - \hat{L}_t(\mu + \lambda_l \xi)\right]\Delta t + \hat{L}_t \xi \sqrt{\Delta t}\epsilon_t^L, \tag{13}$$

$$\Delta \hat{E}_t = k_e\left(L_e - \hat{E}_t\right)\Delta t + \sigma_e \hat{E}_t \sqrt{\Delta t}\epsilon_t^E, \tag{14}$$

where ϵ_t^G, ϵ_t^L and ϵ_t^E are standard normal variates, and Δt is measured in yearly terms. We assume that ϵ_t^G and ϵ_t^L, are independent, and also ϵ_t^L and ϵ_t^E, so $\rho_{G,L} = \rho_{L,E} =$ zero; the correlation coefficient between electricity and natural gas prices $\rho_{G,E}$ can be different from zero. Since there is no Spanish futures electricity market from which to infer a risk premium, we assume $\lambda_e = 0$.[3]

As we noted earlier, with regard to natural gas, Eqs. (12) and (13) show that generating a simulation path requires the state variable $k_g L_t$ on each day t, the three composites ($\mu k_g L_g$, $\mu + \lambda_l \xi$, $k_g + \lambda_g \sigma_g$), and the

[3] Diko, Lawford, and Limpens (2006) study the presence of risk premia in three of the most liquid continental European electricity markets, Germany (EEX), France (Powernext: PWN) and the Netherlands (APX). According to their results, for maturities close to a year, the risk premium usually is close to zero. Thus, our assumption of a zero risk premium may not be so stringent.

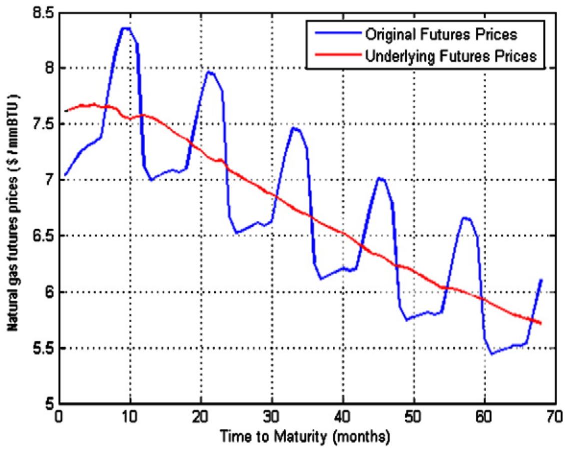

Fig. 1. NYMEX NG futures prices on 4/19/2005 and deseasonalized series.

two volatilities σ_g and ξ in the actual world. We note that Eq. (13) can be equivalently rewritten as

$$k_g \Delta \hat{L}_t = \left[\mu k_g L_g - k_g \hat{L}_t(\mu + \lambda_l \xi)\right] \Delta t + k_g \hat{L}_t \xi \sqrt{\Delta t} \epsilon_t^L. \quad (15)$$

4. Parameters of the stochastic processes

Certainly, several regional gas markets can be distinguished [see, e.g., Geman, 2005]. In this regard, the relation between Iberian gas prices and NYMEX gas prices is not obvious; there is no organized gas market in Spain. Instead, we can look at another European gas market, located at Zeebrugge (Belgium), which we expect to be related to NYMEX. The (log) daily prices (both in \$/mmBTU) from April 2004 to January 2008 show a correlation coefficient of 0.67, so there seems to be some ground for a relation and comovement between both markets.

4.1. Model calibration for natural gas price

Our data set consists of all NYMEX NG futures prices from January 5 2004 to April 29, 2005, a total of 330 days. The contract maturities range from 1 month up to 6 years. We deseasonalize these series for later use in all our computations.[4] Fig. 1 shows futures prices on a typical day and also the deseasonalized series on that day. The seasonal component displays a strong regularity. We note that NYMEX NG futures contracts refer to 10,000 mmBTU, but prices are quoted for one mmBTU (this figure amounts to 1055 GJ).

The sample consists of 23,571 futures contracts. The contracts trade for 72 consecutive months commencing with the nearest calendar month. Table 1 provides some basic statistics of the futures price series. For the period considered, the average futures curve decreases as the contract maturity increases. This fact implies a high degree of backwardation in natural gas prices. The average volatility term structure is also downward sloping, which implies mean reversion (the "Samuelson effect").

Fig. 2 shows deseasonalized prices for futures contracts with 1 month and 5 years to maturity. We note that volatility decreases as the time horizon increases; the price swings are much wider for futures contracts with 1 month to maturity than with 5 years to maturity. However, the positive drift that we observe in futures prices during trading sessions seems to suggest a structural change in the market, e.g., in the form of a higher equilibrium price level.

Table 1
Basic statistics of (deseasonalized) futures prices in the sample

Contract	No. contracts	Sample average	Sample S.D.
F-01	330	6.29	0.83
F-03	330	6.62	0.78
F-06	330	6.61	0.71
F-12	330	6.39	0.69
F-24	330	5.92	0.63
F-36	330	5.55	0.55
F-48	330	5.28	0.47
F-60	330	5.08	0.39
F-72	246	4.76	0.21

We base our calibration procedure on the approach developed by Cortazar and Schwartz (2003), who propose a method which minimizes a sum of square errors. These errors are the difference between actual futures prices and those predicted by the model as a function of global parameters and state variables for each day. We choose this method due to the non-linearity of Eq. (9) and the unequal number of futures contracts in which the market has not operated. In Appendix B1 we provide a complete description of the steps followed.

Fig. 3 shows the actual market data on 4/19/2005 compared to the results of the models when parameters are estimated using observations on futures prices over 330 days, 200, 100 and over 50 days. As we show below, in our case study we assume that the time to build natural gas plants ranges from 30 to 36 months. We find it interesting that for contract maturities of 30 months or more, the series of market futures prices and model (100 days) futures prices seem to agree most. In Appendix B1 we add results of calibration from samples consisting of 330, 200, 100, and 50 days. The coefficient of determination (R^2) in each case is 0.9116, 0.9652, 0.9859, and 0.9877, respectively. The highest values correspond to the two smallest samples; however, in principle the informational content in observations over 100 days is richer than with 50 days.

Besides, we compute several statistics of goodness of fit for regression model: mean squared error, normalized mean squared error, root mean squared error, normalized root mean squared error, mean absolute error, mean absolute relative error, and the coefficient of correlation. For parameter estimation we choose a number of days such that the resulting (predicted) futures curve matches as closely as possible the actual curve on the day chosen for the evaluation (i.e., April 29 2005, the last day of our sample series). Our results (available on request) show that there is not much difference in terms of the correlation coefficient. However, according to the other criteria, our choice seems more justified.

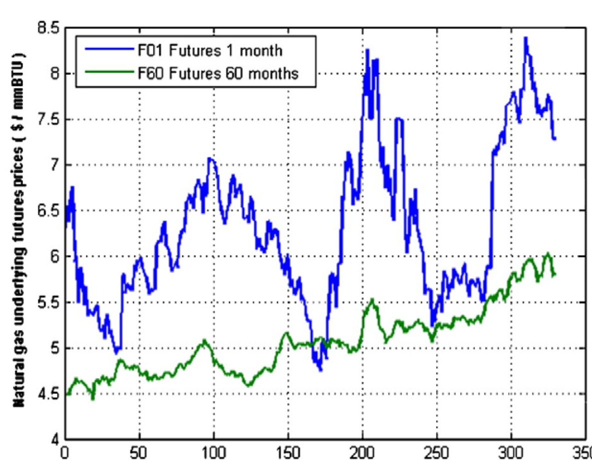

Fig. 2. NYMEX NG futures prices with 1 month (F01) and 5 years (F60) to expiry.

[4] As already mentioned, we assume that seasonality has a limited impact on investment valuations when long-term horizons (e.g., 25 years) are involved, since at the moment of discounting, most of seasonal variations would cancel each other. We deseasonalize by using the U.S. Census Bureau's X-12 Arima method.

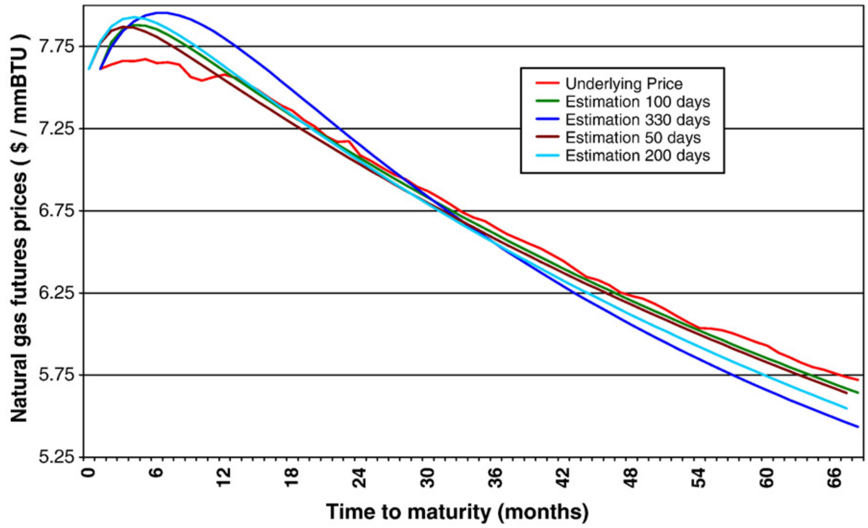

Fig. 3. Actual futures prices on 4/19/2005 against theoretical values from estimated models.

Fig. 4 shows for 4/19/2005 the comparison between actual futures prices and those predicted for the long term. We note the trend in the longer term, where $\hat{F}_\infty = 3.5025$. In sum, actual futures quotes from natural gas contracts seem rather consistent with the model adopted. However, we must stress the lower liquidity of futures contracts as the time to maturity increases, and the fact that there are no quotes for terms longer than 6 years.

Table 2 summarizes the results. The volatility of the variance's estimator allows us to get a consistent estimate of the variances σ_g^2 and ξ^2; this result does not hold for the drift parameter (sometimes even with an infinite number of observations). Therefore, we report only the estimates of the composite parameters that are strictly necessary for valuation, instead of individual values [see also Cartea & Williams, 2008]. We derive the first three values from calibrating the model with natural gas futures prices. For the initial values, which correspond to the last day in the sample series (April 29 2005), we get an initial price of gas $G_0 = 7.2822$ \$/mmBTU and $k_g L_0 = 4.2007$.

4.2. Model calibration for electricity price

The data set comprises 112 monthly average electricity prices from the Spanish wholesale spot market (OMEL). The time span covers the period from January 1998 to April 2007, as shown in Fig. 5. Table 3 displays some basic statistics from the mean price series.

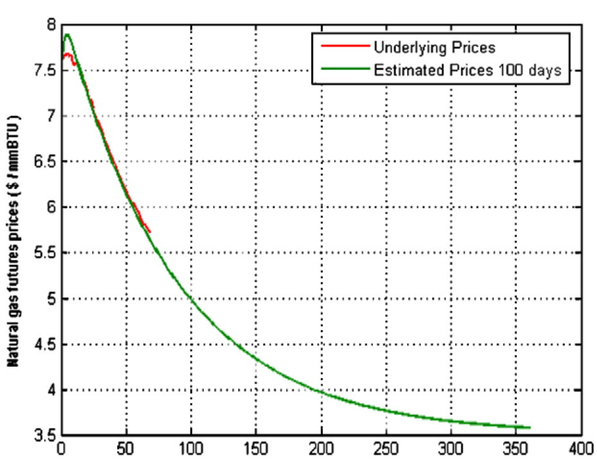

Fig. 4. Actual futures prices on 4/19/2005 and those predicted for the long term.

We estimate the following model:

$$dE_t = k_e(L_e - E_t)dt + \sigma_e E_t dW_t^E, \tag{16}$$

which corresponds to an autoregressive model of order 1, or AR(1). In fact, the partial autocorrelation function (not shown here) is consistent with an AR(1) model. As shown in Appendix B2, the parameter values in Table 4 result.

5. Case study

5.1. The NGCC plant

Table 5 shows the representative values. The production factor is the percentage of the total capacity used on average over the year. Using these data, the heat rate, the plant's consumption of energy, and the total production of electricity can be computed. Heat rate: $HR = 3600/RDTO/1{,}000{,}000$, in GJ/kWh. (1 kWh amounts to 3600 kilojoules, kJ, and one gigajoules, GJ, is 1 million kJ.) Since power is measured in MW, the investment cost $I = 1000 \times i \times P$, in euros. Total annual production: $A = 1000 \times P \times 365 \times 24 \times FP$, in kWh. Fuel energy needs: $B = 1000 \times P \times 365 \times 24 \times FP \times HR$, in GJ/year. Yearly CO_2 emissions: $EM = 350 \times A/1{,}000{,}000$, in tonnes per year. Using these formulae we can estimate the parameters in Table 6.

We consider a firm that is in its initial stages. It has no prior emission allowances, so the firm will have to purchase as much carbon permits as tonnes emitted at a price. Initially, we assume a fixed permit price, then we adopt a stochastic price process typical of financial assets. Similarly, the firm has no prior contractual links to any gas supplier. Thus, the gas consumed by the power plant may in principle be taken either from the local pipeline or brought from abroad and then processed at the LNG plant. (We note that firms already in operation may be subject to contracts that bind them to local suppliers. However, as these contracts approach their

Table 2
Underlying parameter estimates for gas price

$\mu k_g L_g$	2.9469
$k_g + \lambda_g \sigma_g$	0.1393
$\mu + \lambda_l \xi$	6.0412
σ_g	43.44%
ξ	43.66%

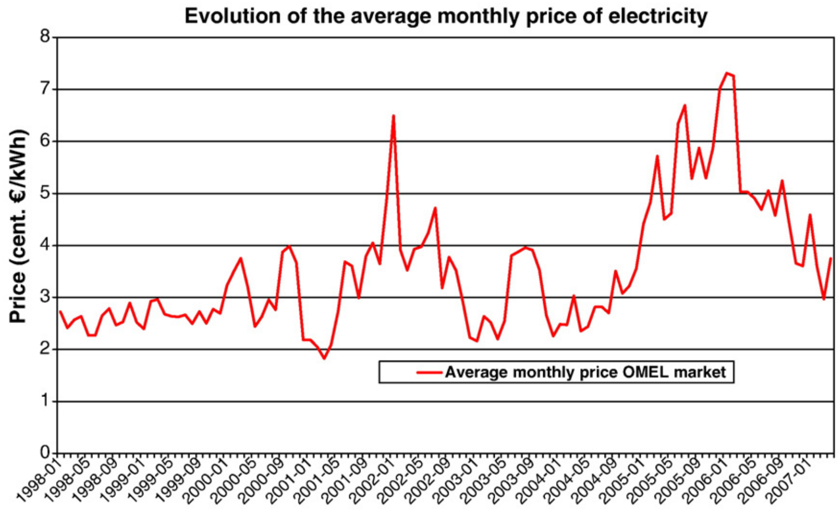

Fig. 5. Monthly average electricity price from Spanish OMEL market (January 1998–April 2007).

expiration, the firms might consider the possibility of adopting the LNG technology.)

5.2. The LNG plant

We consider a LNG plant with total capacity of 200,000 Nm^3/h which is equivalent to 1752 million Nm^3/year. The units Nm^3 refer to cubic metres measured at normal conditions of 0 °C temperature and 1 atm pressure. Assuming that 1 Nm^3 of natural gas has a calorific content of 9500 kcal, this amount is equivalent to:

$$1,752,000,000 \times 9,500 \times 0.000004186 = 69,671,784\,GJ. \qquad (17)$$

This quantity amounts to 5,805,982 GJ for each month in which the plant operates. The representative values used are shown in Table 7. Total investment cost is €318 million. Fixed costs per month amount to €982,938.

6. Valuation of natural gas investments

6.1. The operating NGCC plant

First we compute the value of an immediate investment in a NGCC power plant, which is designed to operate 80% of the time. Revenues come from the electricity produced; we assume that they are stochastic. Costs include the initial outlay, which for the time being is assumed constant; the variable costs, among them those due to the consumption of fuel (in our case natural gas), assumed stochastic; and the costs related to the emission of CO_2 (tonnes), which we initially assume are deterministic.

Taking into account that there will be no production of electricity until the construction phase is completed, and therefore, it will only be in operation from time τ_1 to τ_2, the present value of revenues, for a finite number of periods is (see Eq. (45) in Appendix C1):

$$PVR = A\left[\frac{L_e}{r}\left(e^{-r\tau_1} - e^{-r\tau_2}\right) + \frac{E_0 - L_e}{k_e + r}\left(e^{-(k_e+r)\tau_1} - e^{-(k_e+r)\tau_2}\right)\right], \qquad (18)$$

where A stands for annual production: 3,504 million kWh; the current wholesale electricity price E_0 = 0.05014991 €/kWh (as of April-07, deseasonalized); the long term equilibrium price L_e = 0.037852 €/kWh; the speed of reversion for electricity price k_e = 0.9604; time to build τ_1 = 2.5 years; useful life of the plant $\tau_2 - \tau_1$ = 25 years; riskless interest rate r = 5%. With these values, we get PVR = €1,673.7 million.

We compute the present value of variable costs (other than the gas fired and the emission allowances) as follows:

$$PVC_{var} = A.C_{var} \cdot \frac{e^{-r\tau_1} - e^{-r\tau_2}}{r}, \qquad (19)$$

where C_{var} denotes unit variable costs of 0.0032 €/kWh. Using these data, we get PVC_{var} = €141.20 million.

We assume that the cost of CO_2 emissions is C = 18.01 €/tonne. This figure times EM = 1,226,400 tonnes emitted per year, and applying a formula similar to Eq. (19) allow us to compute a present value of carbon costs PVC = €278.15 million. In calculating this cost, we assume that initially, the plant has no emission permit (unlike incumbent plants under their National Allocation Plans in the EU).

We compute the present value of natural gas by using the estimates from model II, taking as a base date the last day of the series (4/29/2005): G_0 = 7.2822 \$/million BTU and $k_g L_0$ = 4.2007. We determine the value of a unit of natural gas consumed from year 2.5 to year 27.5 by using Eq. (46) in Appendix C2: 58.4867 \$/mm BTU fired per year. This figure amounts to 55.4376 \$/GJ, which translates into 42.7859 €/GJ for an exchange rate (assumed constant) of 1.2957 \$/€. Given that 22,935,273 GJ/year are needed, the present value of total fuel costs is PVG = €981.31 million.

Table 3
Statistics from OMEL market price

Average	3.5504
Median	3.2165
Minimum	1.8250
Maximum	7.3140
Standard deviation	1.2351
Coeff. variation	0.3478
Skewness	1.1447
Excess Kurtosis	0.8417

Table 4
Underlying parameter estimates for electricity price

k_e	0.9604
L_e	3.7852
σ_e	0.4968
ρ	0.5311

Table 5
Basic NGCC parameters (source: ELCOGAS, 2003)

Output MW (*P*)	500
Production factor (% Capacity) (*FP*)	80
Net efficiency (%) (*RDTO*)	55
Investment cost (€/kW) (*i*)	422.5
Useful life (years)	25
Time to build (months)	30
Necessary surface (m^2/MW)	100
Annual maintenance (weeks)	3.5
Time to start (minutes)	15–60
O & M (cts.€/kWh) (C_{var})	0.32
Average CO_2 (emissions (g/kWh)	350

In sum, the present value of an operating NGCC plant is a function of G_0, L_0 and E_0 *ceteris paribus*:

$$V(G_0, L_0, E_0) = PVR - PVC_{var} - PVG - PVC = 273.02 \text{ million €}. \tag{20}$$

This value takes into account the positive influence from the expected fall in natural gas price over the construction period of 30 months.

Since the initial disbursement amounts to I=€211.25 million, if the alternatives were to invest at that precise moment or not to invest, we would invest and get a net present value of $V(G_0, L_0, E_0)-I$= €61.77 million.

6.2. The operating LNG plant

We use the above results to value an operating LNG plant. However, our implicit assumption that its value depends on spot prices needs some explanation. First, according to Brito and Hartley (2007) the structure of the LNG market is changing. Although the market is dominated by long-term contracts, the contracts are becoming more flexible. In addition, more producers and consumers are leaving a larger proportion of anticipated supplies or demands to be traded in short-term markets. For example, major expansions in liquefaction capacity in Australia, Nigeria, Trinidad, and Norway are being planned, with part of the expected production being available for spot transactions.

New LNG tankers also have been built without firm LNG contracts. For instance, China LNG Shipping Ltd. has announced that it intends to utilize the full capacity of its LNG ships for spot trading rather than long-term charters. Korea Gas Corporation has also announced formation of a consortium with four other Korean shipping companies to form a LNG ship pooling system to facilitate LNG spot market trading. As a consequence, short-term trading has been growing more rapidly than the market as a whole. As recently as 1997, short-term LNG transactions accounted for only 1.5% of international LNG trade. By 2002, that proportion had risen to 8%. These changes should produce a more integrated world market for natural gas.

Second, long-term contract prices themselves can be linked to spot prices. Following EIA,[5] in the United States, the competing fuel is pipeline natural gas, and the benchmark price is either a specified market in long-term contracts or the Henry Hub15 price for short-term sales. Given the high degree of price volatility in U.S. natural gas markets, importers and exporters involved in U.S. LNG transactions are exposed to a significant level of risk. In Europe, LNG prices are related to competing fuel prices, such as low-sulfur residual fuel oil. However, LNG is now starting to be linked to natural gas spot and futures market prices.

We consider an initial situation in which the spread between domestic and foreign gas prices is 0.70 \$/mmBTU, and that this difference reflects exactly the costs of transporting the natural gas to the consuming NGCC plant. In other words, initially, the import price plus transport costs just equal the domestic price (=7.2822 \$/mmBTU). Therefore, from the point of view of the fuel input, it makes no difference in the choice of whether to use the LNG plant or not, but it would not be used because of variable costs. From then on, both resources evolve on their own. We run 30,000 simulations for each gas price. First we assume a correlation coefficient of 75% between domestic and foreign gas prices. Also, the decision to burn one kind of gas or the other is taken on a monthly basis.

[5] http://www.eia.doe.gov/oiaf/analysispaper/global/lngmarket.html.

Table 6
Resulting NGCC parameters

Heat rate (GJ/kWh) (*HR*)	0.006545
Total investment (million €) (*I*)	211.25
Annual production (mill. kWh) (*A*)	3504
Fuel energy (GJ/year) (*B*)	22,935,273
CO_2 emissions (tonne/year) (*EM*)	1,226,400

The risk-neutral model is:

$$d\hat{G}_t^d = \left[k\left(\hat{L}_t^d - \hat{G}_t^d\right) - \lambda_g \sigma \hat{G}_t^d\right] dt + \sigma \hat{G}_t^d dW_t^{dG}, \tag{21}$$

$$d\hat{L}_t^d = \left[\mu\left(L_g^d - \hat{L}_t^d\right) - \lambda_l \xi \hat{L}_t^d\right] dt + \xi \hat{L}_t^d dW_t^{dL}, \tag{22}$$

$$d\hat{G}_t^f = \left[k\left(\hat{L}_t^f - \hat{G}_t^f\right) - \lambda_g \sigma \hat{G}_t^f\right] dt + \sigma \hat{G}_t^f dW_t^{fG}, \tag{23}$$

$$d\hat{L}_t^f = \left[\mu\left(L_g^f - \hat{L}_t^f\right) - \lambda_l \xi \hat{L}_t^f\right] dt + \xi \hat{L}_t^f dW_t^{fL}, \tag{24}$$

$$\rho_{W^{dG}W^{fG}} = 0.75, \tag{25}$$

with the remaining correlation coefficients assumed to equal zero.

Each month, the LNG plant provides a net cash flow that depends on the sign and size of the price gap between both fuel resources. The plant is set to operate whenever there is a profit to be gained. At the very least, this requires that domestic gas be more expensive than foreign gas; otherwise there is no point in switching to foreign gas. However, any positive gap will not do; since there are variable costs to operating the plant, they must be subtracted. We measure these gas prices and variable costs in euros per unit of fuel energy (€/GJ). Since we are computing monthly cash flows, we must multiply the net price margin by the amount of energy consumed in 1 month (i.e., annual consumption divided by 12 months). Obviously, the plant owner has no obligation to run this infrastructure, so we must consider the maximum between this figure and zero. Then, irrespective of the plant operation, the owner faces (monthly) fixed costs. Last, the net revenue can spread out over the whole time period. If it were obtained at the end of the month, discounting back to the beginning of the period would imply multiplying it by $exp(-r/12)$. Instead, we assign the net revenue to the middle of the month, and consequently, the discount factor is $exp(-r/24)$. Thus, we have:

$$CF = e^{-\frac{r}{24}}\left[\max\left(\left(\frac{B}{12}\left(\hat{S}_t^d - \hat{S}_t^f - 0.2195\right), 0\right) - 982{,}938\right], \tag{26}$$

where B=69,671,784 GJ is the amount of energy processed at the plant in a year if it operates every month. We assume a variable cost of

Table 7
Basic LNG parameters (source: Basque Govt. EVE; U.S. EIA)

Output (Nm^3/h)	200,000
Investment cost (million €)	318
Useful life (years)	30
Time to build (months)	36
Necessary surface (m^2)	150,000
Unit variable cost (\$/mm BTU)	0.30
Fixed costs (million €/year)	11.8

Table 8
Gross and net present values (million €) of a LNG plant as a function of the correlation between domestic and foreign gas prices

$\rho_{W^{dG}W^{fG}}$	GPV	NPV
0.65	948.54	630.54
0.70	867.02	549.02
0.75	776.80	458.80
0.80	675.24	357.24
0.85	557.98	239.98

0.30 \$/mmBTU, which is equivalent to 0.2195 €/GJ. (Again, we use the equivalence 1 mmBTU = 1055 GJ and an exchange rate 1.2957 \$/€.) Variable costs are only incurred if the LNG plant operates in that period. Yearly fixed costs are spread over 12 months and amount to €982,938 per month. Under the assumption of a useful life of 30 years, there are 360 monthly periods in which to compute the cash flows. Discounted at the risk-free interest rate $r = 5\%$, these flows will give us the present value of the plant. For this facility to be profitable, the value must be higher than the building costs, which are disbursed 36 months before starting operation.

Table 8 presents the LNG plant's average gross and net present values (in million euros) as a function of the correlation coefficient. When the correlation between domestic and foreign natural gas prices decreases, the LNG plant becomes more valuable.

7. Options on a base load NGCC plant

7.1. Value of a finite-lived option to double installed capacity

As we show in Section 6.1, the NPV of a 500 MW gas-fired plant amounts to €61.77 million. This is the value of the (single plant) project in a "now or never" investment context.

Here, we consider a compound project. The firm can build such a power plant immediately and also holds an option to build a twin (equal-sized) plant over the next 5 years. As before, if the firm decides to invest, building the new plant takes time, namely 2.5 years. Thus, the value of the compound project consists of two items: that of the immediate investment in the first plant, and that of the option to invest in the second module. With zero years to maturity, the value of this option is given by $\max(V_0 - I, 0) = €61.77$ M. Then the total project is worth 61.77 + 61.77 = €123.54 million. For longer maturities, the value of the option is less obvious.

To value this opportunity we use Matlab and run $N = 30{,}000$ Monte Carlo simulations. Also, the investment option has a finite life of 5 years, each of which is decomposed into 100 periods (i.e., $\Delta t = 1/100$). We know that the rate of improvement of the quality of MC estimates (or the rate of decrease of error) is on the order of $1/\sqrt{N}$. In this respect, we are relatively confident on the accuracy of the results.

However, as a robustness check, we look at the results of the 30,000 runs for the gas prices at the option's maturity (of course, the distribution of prices changes from one period to another). According to Eq. (9), the expected price of gas in 5 years time would be $E(\hat{G}_t) =$ 5.70 \$/mmBTU with the parameter values adopted. The average value resulting from the 30,000 simulations is 5.74 \$/mmBTU. Thus, the difference between the theoretical value and that derived by simulation amounts to a 0.7%. We also check the convergence of the average price for a longer term, say $\hat{G}_{27,5}$, towards the expected futures price for that specific date. We can compute this expectation analytically. The result is 3.5983 €/GJ.

Any simulation run fits the discretized Eqs. (12)–(14). Depending on the specific values of the correlation coefficients, the Monte Carlo simulation technique may require the generation of two or more correlated Normal variates. In our case, we assume $\rho_{G,L} = \rho_{L,E} = 0$, but $\rho_{G,E} = 0.5311$. The series obtained for $\hat{G}_t$, $\hat{L}_t$ and $\hat{E}_t$ allow us to compute at any time the value V_t of an investment at that time, taking into account the evolution of electricity and gas prices, as well as the behavior of the equilibrium gas price in the short term ($\hat{L}_t$). Apart from the initial outlay, which is disbursed at the time of the investment, all other costs and revenues are due starting 2.5 years later.

Given the values of V_t at any moment and in each path, we use the Least Squares Monte-Carlo (LSM) approach. At the last moment, the value of the investment in each path is:

$$\max(V(G_T, L_T, E_T) - I, 0). \tag{27}$$

At earlier moments, the method requires the computation of a series of parameters that allow us to construct a linear combination of basic functions. This combination allows us to estimate at each step the

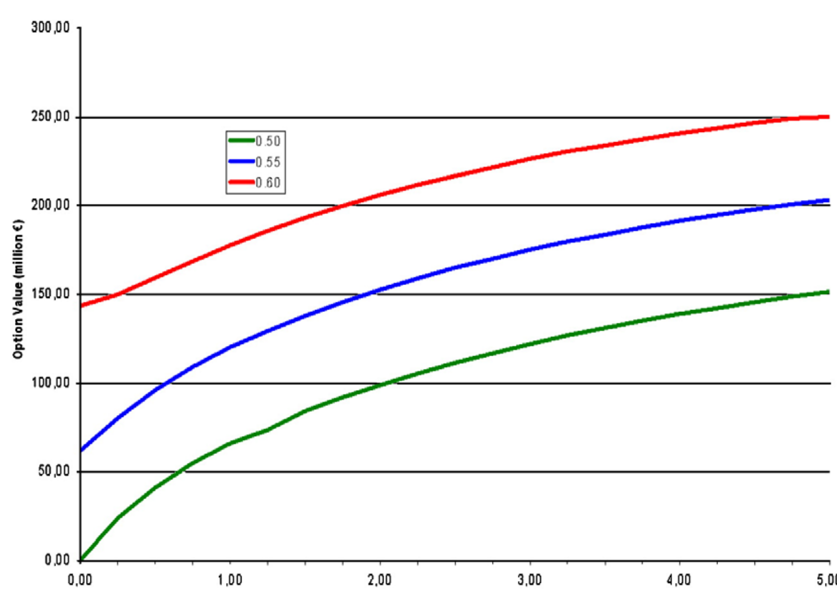

Fig. 6. Value of the option to invest as a function of the option's maturity for different net efficiency rates.

Table 9
Option to double installed capacity sequentially

Maturity (years)	Option to twin plant	Compound project value
0	61.77	123.54
0.50	96.03	157.80
1.00	120.20	181.97
1.50	137.84	199.61
2.00	152.61	214.38
2.50	165.03	226.80
3.00	175.31	237.08
3.50	183.54	245.31
4.00	191.50	253.27
4.50	197.89	259.66
5.00	203.20	264.97

(Base plant's NPV=€61.77 mill.)

continuation value. The specification adopted consists of a second-order expected continuation value function with ten regressors (since there are three sources of risk), namely:

$$E_i^Q\left[e^{-r\Delta t}V_{i+1}\left(\hat{G}_{i+1},\hat{L}_{i+1},\hat{E}_{i+1}\right)\right]\approx a_1+a_2\,\hat{G}_i+a_3\,\hat{G}_i^2+a_4\,\hat{L}_i+a_5\,\hat{L}_i^2+a_6\,\hat{E}_i+a_7\,\hat{E}_i^2+a_8\,\hat{G}_i\,\hat{L}_i+a_9\,\hat{G}_i\,\hat{E}_i+a_{10}\,\hat{L}_i\,\hat{E}_i. \quad (28)$$

At any time, considering the paths that are in-the-money and by applying ordinary least squares, we can get the value of the ten coefficients. The optimal exercise frontier is given by a surface formed with those values of $\hat{G}_i$, $\hat{L}_i$ and $\hat{E}_i$ for which the present value of investing at that time i equals the continuation value.

The valuation results are shown in Fig. 6 for three different rates of plant efficiency. We note that the value of this option increases with the maturity of the option. Table 9 displays the most significant values for the base case in which the plant has a net efficiency rate of 55%. With 5 years to decide building the second module, the value of the option increases to €203.20 million. This value benefits from the expected decline in gas prices and from a lower present value of the initial disbursement. Project's total value, i.e., the net value of the initial investment in the first operating plant plus the option to double capacity, results from adding €61.77 million to the above series. With 5 years to plan, project's total value is €264.97 million, some €140 million above the value of building two lesser-sized plants at the initial moment (=2×61.77=123.54).

Alternatively, perhaps the firm might opt for a large 1,000 MW gas-fired plant from the outset. We assume that this plant enjoys a 10% saving in building costs and reaches a higher efficiency of 57%. Following the same steps as in Section 6.1, the NPV of this large plant turns out to be €234.66 million. Previously, we have obtained the NPV of investing now in two half-sized plants: €123.54 million. Hence, the value of investing initially in the larger plant is higher than the value of investing initially in two smaller plants.

However, we note the figures in Table 9. Here, our underlying assumption is that the firm can increase generation capacity step by step. With 3 years to maturity, the option based on the modularity strategy is worth €237.08 million, thus surpassing the value of €234.66 million attached to the mega-size strategy. For longer option maturities the advantage provided by this flexibility grows steadily.

7.2. Value of the option to invest when the initial investment cost I is stochastic

We again consider the investment in a base plant, which now requires a stochastic initial disbursement I according to the following equation:

$$dI_t=\alpha_I I_t dt+\sigma_I I_t dW_t^I, \quad (29)$$

with the traditional meaning for each variable and $\rho_{W^IW^G}=\rho_{W^IW^L}=\rho_{W^IW^E}=0$. The risk-adjusted version is:

$$d\hat{I}_t=(\alpha_I-\lambda_I)\,\hat{I}_t dt+\sigma_I\,\hat{I}_t dW_t^I. \quad (30)$$

After discretization

$$\Delta\hat{I}_t=(\alpha_I-\lambda_I)\,\hat{I}_t\Delta t+\sigma_I\,\hat{I}_t\sqrt{\Delta t}\epsilon_t^I, \quad (31)$$

where $\epsilon_t^I \sim N(0,1)$.

We analyze the case in which $\alpha_I-\lambda_I=0$, $\Delta t=0.01$ and $\sigma_I=0.30$. With this volatility, 1 year later the initial investment will range between -30% and +30% times the current amount with a probability of 68.27% (approximately 2/3). For each of the 30,000 paths of $\hat{G}_t$, $\hat{E}_t$, and $\hat{L}_t$ we get a path for $\hat{I}_t$. (At every time the average of the 30,000 simulations must be I=€211.25 million.) This procedure makes it possible for us to estimate, at each moment, the value of an immediate investment and hence what paths are in-the-money at that precise time. For the option with 5 years to maturity, we compute a value of €212.11 million, a figure which is slightly above the €203.20 million estimated above in Section 7.1. Table 10 shows the value of the option to invest as a function of initial investment's volatility. Fig. 7 plots these results.

7.3. Value of the option to invest when the cost of CO_2 emissions is stochastic

Cap-and-trade systems in the world have attracted a fair amount of attention. Abadie and Chamorro (2008) list some features of emission markets with special reference to the European carbon market (EU ETS). One key aspect is that the allowance price is at least partially determined by public regulation. Hence, uncertainty surrounding the allowance price is due to uncertainty surrounding public regulation. Thus, EU decisions on CO_2 emission restrictions from 2013 onwards will have a decisive influence on the continued growth in EU gas demand after 2010 (Kjärstad & Johnsson, 2007). Multilateral negotiations are already taking place, but uncertainty remains.

On the other hand, there is an important difference between standard commodities (such as oil) and carbon allowances. A priori, there is no need for allowances on a daily or hourly basis, but industrial facilities depend on a sustained flow of energy to work. This difference may result in a carbon market that is structurally less liquid and deep than the oil market, for instance (Reinaud, 2007). In such a context, temporary mismatches between buying and selling orders give rise to wide fluctuations in price. Thus, volatility in emissions markets may well be above standard levels in financial markets. For example, America's market for trading sulphur-dioxide permits has been in operation since the mid-1990s. The price of these permits has varied, on average, by more than 40% a year (see also Nordhaus, 2007, Table 4). This high volatility may imply a high value of the option to invest. Firms know it and regulators should be aware of it.

We consider that, in addition to the initial outlay, the cost of CO_2 emissions is also stochastic. Following Insley (2003), we assume that

Table 10
Value of the option to invest in 5 years with stochastic building costs

Volatility (σ_I)	Option value (€ million)
0	203.20
0.10	203.66
0.20	205.84
0.30	212.11
0.40	222.01
0.50	234.24
0.60	247.55
0.70	260.82
0.80	273.69
0.90	286.04
1.00	297.39

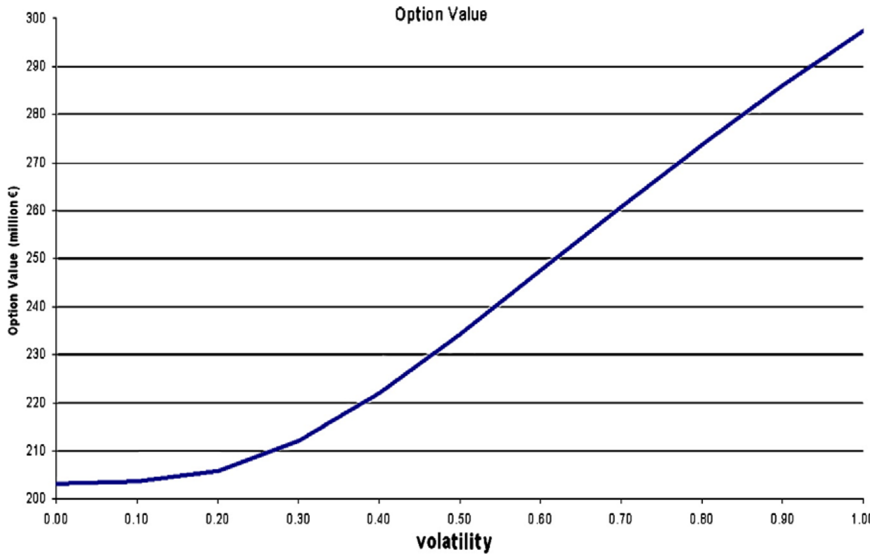

Fig. 7. Value of the option to invest in 5 years with stochastic building costs.

the price of an allowance is governed by a geometric Brownian motion:

$$dC_t = \alpha_C C_t dt + \sigma_C C_t dW_t^C \tag{32}$$

with the usual meaning for each variable and $\rho_{W^C W^G} = \rho_{W^C W^L} = \rho_{W^C W^E} = \rho_{W^C W^I} = 0$. The risk-neutral version is

$$d\hat{C}_t = (\alpha_C - \lambda_C)\,\hat{C}_t dt + \sigma_C\,\hat{C}_t dW_t^C. \tag{33}$$

After discretization,

$$\Delta\hat{C}_t = (\alpha_C - \lambda_C)\,\hat{C}_t \Delta t + \sigma_C\,\hat{C}_t \sqrt{\Delta t}\epsilon_t^C. \tag{34}$$

where $\epsilon_t^C \sim N(0,1)$.

Next we compute the value of the option to invest up to 5 years ahead for different values of emission permits' volatility and two possible values for $(\alpha_C - \lambda_C)$, namely zero and 0.0308. In their (2008) study, Abadie and Chamorro obtain this value after analyzing carbon prices on the EU ETS.

Given an initial level C_0, the expected value at time t under the risk-neutral probability measure is $E(\hat{C}_t) = C_0 e^{(\alpha_C - \lambda_C)t}$. The value of an annuity (or 1 tonne of emissions per year) between time τ_1 and time τ_2 is determined by

$$V_{\tau_1,\tau_2}^c = \int_{\tau_1}^{\tau_2} E\left(\hat{C}_t\right) e^{-rt} dt \tag{35}$$

which gives

$$V_{\tau_1,\tau_2}^c = \frac{C_0}{\alpha_C - \lambda_C - r}\left[e^{(\alpha_C - \lambda_C - r)\tau_2} - e^{(\alpha_C - \lambda_C - r)\tau_1}\right]. \tag{36}$$

The total cost comes from multiplying this amount times 1,226,400 tonnes of CO_2 per year.

We consider two situations. In the first case, when $(\alpha_C - \lambda_C \sigma_C) = 0$ we have

$$V_{\tau_1,\tau_2}^c = \frac{C_0}{-r}\left[e^{-r\tau_2} - e^{-r\tau_1}\right]. \tag{37}$$

With $C_0 = 18.01$ €/tonne, $\tau_1 = 2.5$ and $\tau_2 = 27.5$ this annuity amounts to a cost of 226.80 €/tonne, which corresponds to a present value of total emission cost of €278.15 million, the same figure as in the base case (Section 6.1.). In the second situation, when $(\alpha_C - \lambda_C \sigma_C) = 0.0308$ we have an annuity value $V_{\tau_1,\tau_2}^C = 340.83$ €/tonne, which corresponds to a present value of total emission cost of €417.99 million.

The difference between the first value and the second relates to the fact that carbon allowance prices show different trends. In both cases, the current value at each moment is derived from a Monte Carlo simulation. However, in the first case they evolve around an average value that remains constant, but in the second case this average increases with the passage of time. The value of the option to invest (in million euros) is shown in Table 11. This value for other maturities is displayed in Fig. 8. When the expected growth rate of emission costs is zero, the option values obtained are slightly higher than in the base case, but with an expected growth rate of 3.08%, they are significantly lower.

7.4. Value of the option to invest when the option's maturity is stochastic

Here, we consider the case in which the option to invest has a stochastic time to maturity. There is a probability λ that it will disappear in a given year; thus the probability to disappear during a period dt is λdt. We assume that the opportunity to invest vanishes anyway at the end of the fifth year. Also, as soon as the option expires, the value of the project becomes zero.

We draw random samples from a Poisson distribution with parameter λ to get the moment at which the right to invest ceases. Given the value of the project at each simulation run the LSM approach is used. These values will be zero whenever the option to invest has disappeared. In other words, the continuation value takes into account the chance that while waiting for one more period, the option to invest may vanish.

Table 11
Value of the option to invest in 5 years with stochastic CO_2 emission costs

Maturity (years)	Option val. (drift=0)	Option val. (drift=3.08%)
0	61.77	0
0.50	105.50	40.45
1.00	135.44	69.73
1.50	157.62	92.11
2.00	176.06	109.81
2.50	191.99	125.61
3.00	206.00	139.48
3.50	218.26	152.13
4.00	228.05	161.98
4.50	236.96	171.32
5.00	244.95	179.41

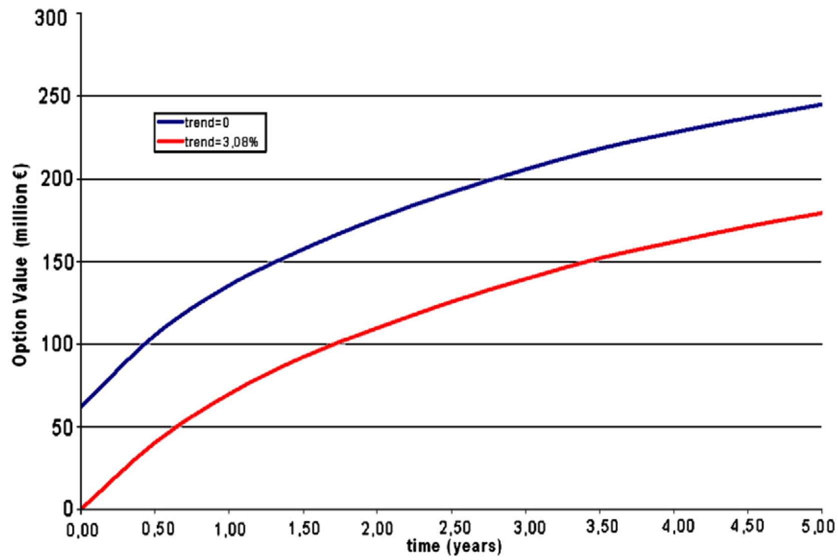

Fig. 8. Value of the option to invest in 5 years when CO_2 emission costs are stochastic with zero and positive expected growth rates.

Fig. 9 shows the value of the option to invest as a function of the probability λ and the option's maturity. For low probabilities, the values of the option are slightly lower than in the base case. For very high values of λ, the value approaches €61.77 million, which happens to be the option's value when the only decision is whether to invest at the initial moment or not.

8. Conclusion

In this paper we evaluate investments in natural gas plants. Our first example is a plant that burns gas to produce electricity by means of a combined cycle. Our second example is a plant for storing liquefied gas, be it for economic reasons and/or for energy security concerns. We note that the recent operation of the EU Emissions Trading Scheme has brought a new commodity, carbon, that will have an impact on energy investments, at least for European utilities.

We assume that both natural gas and electricity prices follow mean-reverting stochastic processes, namely, an Inhomogeneous Geometric Brownian Motion. We calibrate a model for each price by using actual market data. The first set consists of NYMEX Natural Gas futures contracts; the second one refers to the Spanish wholesale electricity market (OMEL). Once we have described the basic features of our case study, we value these operating plants under the assumption of a finite useful life and non-negligible time to build. Then, we use a Least Squares Monte Carlo simulation to value several American-type options to invest in a natural gas power plant.

At this point, we note several qualifications. It is well known that energy prices display very complex empirical distributions. Patterns of high volatility, skewness, or heavy tails are the usual characteristics of asset returns. Therefore, it may be necessary to adopt more sophisticated stochastic models to deal with these features properly. An alternative approach would be to incorporate models of time-varying conditional volatility. Similarly, the rate of mean reversion may not be constant, but time-varying. Failure to account for this might result in significantly mispricing long maturity options.

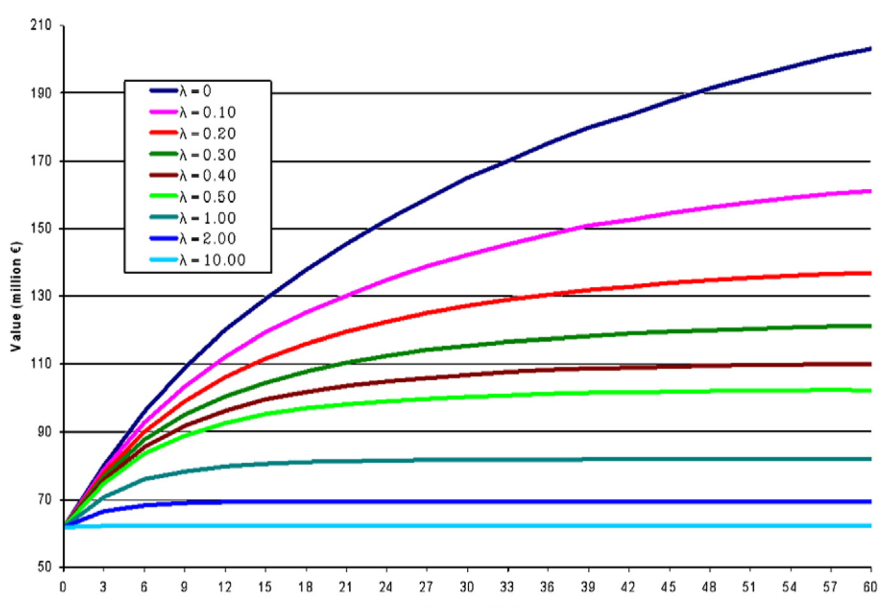

Fig. 9. Value of the option to invest as a function of its maturity under several expiration probabilities.

Our paper can be extended in several ways. Researchers could add a third factor to better fit natural gas futures prices. A researcher could also develop a richer characterization of the electricity price dynamics. A further improvement could come from exploiting the information that the carbon market is gathering step by step. Last, we note that at best, we are dealing with the perceived financial risks. Moreover, these risks are based on short-term (compared with the life of the investment) financial data. From the perspective of a long-term investor, short-term financially driven risks may not be the main source of concern. Technology or regulatory changes may have a deeper effect. Thus, whatever the numerical results, they should be taken with caution. They are only part of the entire picture.

Acknowledgements

We thank staff members of NYMEX (London) for providing us with the data on natural gas futures contracts, and seminar participants at the 10th Real Options Conference (Columbia Business School, NY), the 2nd Atlantic Workshop on Energy and Environmental Economics (A Toxa, Spain), and the XIV Foro de Finanzas (Castellón, Spain) for their helpful suggestions. Earlier versions of this paper have benefitted also from comments by Prof. Richard de Neufville (MIT) and Prof. Margaret Armstrong (Ecole des Mines, Paris). We also thank two anonymous referees for their valuable advices and remarks. Financial support from Subvención General al Grupo de Investigación GIU 05/39 UPV/EHU is gratefully acknowledged. Any errors, though, are our own.

Appendix A. Expectation of natural gas price

This model consists of Eqs. (1) and (2). The expected values must satisfy the following differential equations:

$$E(dG_t) = k_g[E(L_t) - E(G_t)]dt, \tag{38}$$

$$E(dL_t) = \mu\left[L_g - E(L_t)\right]dt. \tag{39}$$

We start by multiplying the second equation by the factor $e^{\mu t}$ to get

$$e^{\mu t}E(dL_t) + \mu e^{\mu t}E(L_t)dt = \mu e^{\mu t}L_g dt.$$

Integrating this expression yields $E(L_t)e^{\mu t} = e^{\mu t}L_g + c_1$, where c_1 is a constant of integration. Then, for $t=0$ we have $E(L_t)=L_0$. Therefore ($e^{0t}=1$), $E(L_t)=L_0=L_g+c_1$, which implies $c_1=L_0-L_g$. Thus,

$$E(L_t)e^{\mu t} = e^{\mu t}L_g + (L_0 - L_g) \Rightarrow E(L_t) = L_g + (L_0 - L_g)e^{-\mu t}.$$

Substituting this expression into the first equation yields

$$E(dG_t) = k_g\left[L_g + (L_0 - L_g)e^{-\mu t} - E(G_t)\right]dt.$$

Using a factor of integration $e^{k_g t}$ and integrating,

$$e^{k_g t}E(dG_t) + k_g e^{k_g t}E(G_t)dt = \left[k_g L_g e^{k_g t} + k_g(L_0 - L_g)e^{-(\mu - k_g)t}\right]dt,$$

$$e^{k_g t}E(G_t) = L_g e^{k_g t} - \frac{k_g(L_0 - L_g)}{\mu - k_g}e^{-(\mu - k_g)t} + c_2.$$

For $t=0$ we have $E(G_t)=G_0$. Hence:

$$c_2 = G_0 - L_g + \frac{k_g(L_0 - L_g)}{\mu - k_g}.$$

Then, solving for the expected value

$$E(G_t) = L_g - \frac{k_g(L_0 - L_g)}{\mu - k_g}e^{-\mu t} + \left[G_0 - L_g + \frac{k_g(L_0 - L_g)}{\mu - k_g}\right]e^{-k_g t}. \tag{40}$$

In the particular case in which Eq. (2) reduces to a GBM type, the solution of Pilipovic (1998) results.

Appendix B. Calibration of price processes

B.1. Natural gas

We have seven parameters in the model, k_g, λ_g, λ_l, σ_g, ξ, μ, and L_g. However, given the optimization procedure that we choose, the observation of Eq. (9) leads us to conclude that only the following combinations may be estimated: (The particular values of the seven original parameters of the model should also take into account the behavior of the original time series.) $V_1 \equiv k_g + \lambda_g\sigma_g$, $V_2 \equiv \mu + \lambda_l\xi$, $V_3 \equiv \mu k_g L_g$. On the other hand, L_0 is a state variable for each trading day i, but only the product $k_g L_0$ on each day can be estimated by the procedure adopted, i.e.,$U_i \equiv k_g L_{0_i}$.

If we have price quotes in N days (dated t_i, with $i=1,2,\ldots,N$; in our case, initially the whole sample of 330 days would be used up), and on each day there are M_i different contracts, with no reason why there must be the same number of price quotes every day, we want to minimize the function

$$\sum_{i=1}^{N}\sum_{j=1}^{M_i}\left(\hat{F}_{ij}(V_1, V_2, V_3, U_i, T_j - t_i, F_{i1}) - F_{ij}\right)^2, \tag{41}$$

where F_{ij} denotes the futures quote on day i with maturity in (month) j. $\hat{F}_{ij}$ stands for the contract price when this price is estimated by means of Eq. (9) as a function of the parameters used (V_1, V_2, V_3), the state variables U_i for each day i , and time $T_j - t_i$, which is the difference (in years) between the futures contract maturity T_j and the date of the first futures contract on day i, which we assume is for 1 month (this is F_{i1} futures contract, which is used as the spot price).

The estimation procedure involves two steps that are repeated until the sum of square errors converges:

a) We choose a set of initial parameter values $\Omega=\{V_1, V_2, V_3\}$. Then it is possible to estimate the state variables on each day in a single step,

$$\{U_1, U_2, \ldots, U_N\} \in \underset{U_1, U_2, \ldots, U_N}{\operatorname{argmin}} \sum_{i=1}^{N}\sum_{j=1}^{M_i}\left(\hat{F}_{ij}(V_1, V_2, V_3, U_i, T_j - t_i, F_{i1}; \Omega) - F_{ij}\right)^2. \tag{42}$$

b) Our next step is to estimate the values of the model parameters

$$\{V_1, V_2, V_3\} \in \underset{V_1, V_2, V_3}{\operatorname{argmin}} \sum_{i=1}^{N}\sum_{j=1}^{M_i}\left(\hat{F}_{ij}(V_1, V_2, V_3, U_i, T_j - t_i, F_{i1}; \Omega) - F_{ij}\right)^2. \tag{43}$$

Table 12 shows the results obtained using the whole sample, the last 200 days, the last 100 days, and the last 50 days.

To estimate σ_g and ξ, which are necessary for Monte Carlo simulations, the series obtained for the values of G_t and $U_t \equiv k_g L_{0_t}$ will be used up, according to the discretization $\frac{G_{t+\Delta t} - G_t}{G_t} = \frac{k_g(L_t - G_t)\Delta t}{G_t} + \sigma_g \epsilon_t^G$, where ϵ_t^G is an $iid \sim N(0,1)$ variate. This procedure allows us to compute σ_g from the residuals' standard deviation.

Similarly, we can use $\frac{k_g L_{t+\Delta t} - k_g L_t}{k_g L_t} = \frac{(\mu k_g L_g - \mu k_g L_t)\Delta t}{k_g L_t} + \xi\epsilon_t^L$, where ϵ_t^L is an $iid \sim N(0,1)$ variate. We compute the volatility ξ with this formula.

Table 12
Risk-neutral parameter composites

Concept	Panel I	Panel II	Panel III	Panel IV
No. days	330	200	100	50
No. observations	23,571	14,199	6989	3436
Sum of square errors	405.47	171.44	32.02	15.82
Sum square errors/days	1.2287	0.8572	0.3202	0.3164
V_1	0.2122	0.1489	0.1393	0.1283
V_2	2.5338	4.6875	6.0412	6.1852
V_3	2.1346	2.3829	2.9469	2.5870
$\hat{F}_\infty$	3.9704	3.4164	3.5025	3.2604

Table 13
OLS estimates of the (transformed) electricity price process

Coefficient	Estimate	Standard dev.	*t*-statistic	*p*-value
$\hat{\beta}_1$	−0.0769185	0.0535881	−1.435	0.15405
$\hat{\beta}_2$	0.291154	0.169468	1.718	0.08863

B.2. Electricity

We note that $E(E_{t+1})=L_e+(E_t-L_e)e^{-k_e\Delta t}$. After discretization and rearranging, this equation becomes

$$\frac{E_{t+1}-E_t}{E_t}=\left(e^{-k_e\Delta t}-1\right)+L_e\left(1-e^{-k_e\Delta t}\right)\frac{1}{E_t}+\sigma_e\sqrt{\Delta t}\epsilon_t^E, \tag{44}$$

where $\epsilon_t^E \sim N(0,\ 1)$. Expressed as $Y_t=\beta_1+\beta_2X_{2t}+u_t$, we get the OLS estimates (adjusted for heteroskedasticity) in Table 13.

$$\hat{\beta}_1=-0.0769185=e^{-k_e\Delta t}-1,\quad k_e=-\frac{1}{\Delta t}\ln\left(\hat{\beta}_1+1\right)$$
$$\hat{\beta}_2=0.291154=L_e\left(1-e^{-k_e\Delta t}\right)=-\hat{\beta}_1L_e.$$

The standard deviation of the residuals is 0.143416. Hence $\sigma_e = 0.143416\sqrt{\Delta t}=0.4968=49.68\%$.

Appendix C. Valuation of annuities for an IGBM process

C.1. Annuity of electricity

The value of an annuity between time τ_1 and time τ_2 is determined by the integral $V_{\tau_1,\tau_2}=\int_{\tau_1}^{\tau_2}E(\hat{E}_t)e^{-rt}dt$. It can be shown that the resulting value for the annuity is

$$V_{\tau_1,\tau_2}=\frac{L_e}{r}\left(e^{-r\tau_1}-e^{-r\tau_2}\right)+\left(\frac{E_0}{k_e+r}-\frac{L_e}{k_e+r}\right)\left(e^{-(k_e+r)\tau_1}-e^{-(k_e+r)\tau_2}\right). \tag{45}$$

The first term in the right-hand side accounts for the equilibrium price. The second term adds the spread between current price and the equilibrium price.

C.2. Annuity of natural gas

Again, the value of an annuity between time τ_1 and time τ_2 is determined by the equation $V_{\tau_1,\tau_2}=\int_{\tau_1}^{\tau_2}E(\hat{G}_t)e^{-rt}dt$. In this case, the resulting value for the annuity is

$$\begin{aligned}V_{\tau_1,\tau_2}={}&\frac{\mu k_gL_g}{r(\mu+\lambda_l\xi)(k_g+\lambda_g\sigma_g)}\left(e^{-r\tau_1}-e^{-r\tau_2}\right)\\&+\frac{\mu k_gL_g}{(\mu+\lambda_l\xi)(k_g+\lambda_g\sigma_g)(k_g+\lambda_g\sigma_g+r)}\left(e^{-(k_g+\lambda_g\sigma_g+r)\tau_2}-e^{-(k_g+\lambda_g\sigma_g+r)\tau_1}\right)\\&+\frac{k_gL_0}{(\mu+\lambda_l\xi-k_g-\lambda_g\sigma_g)(\mu+r+\lambda_l\xi)}\left(e^{-(\mu+\lambda_l\xi+r)\tau_2}-e^{-(\mu+\lambda_l\xi+r)\tau_1}\right)\\&+\frac{k_gL_0}{(\mu+\lambda_l\xi-k_g-\lambda_g\sigma_g)(k_g+r+\lambda_g\sigma)}\left(e^{-(k_g+\lambda_g\sigma_g+r)\tau_1}-e^{-(k_g+\lambda_g\sigma_g+r)\tau_2}\right)\\&+\frac{\mu k_gL_g}{(\mu+\lambda_l\xi)(\mu+\lambda_l\xi-k_g-\lambda_g\sigma_g)(\mu+r+\lambda_l\xi)}\left(e^{-(\mu+\lambda_l\xi+r)\tau_1}-e^{-(\mu+\lambda_l\xi+r)\tau_2}\right)\\&+\frac{\mu k_gL_g}{(\mu+\lambda_l\xi)(\mu+\lambda_l\xi-k_g-\lambda_g\sigma_g)(k_g+r+\lambda_g\sigma_g)}\left(e^{-(k_g+\lambda_g\sigma_g+r)\tau_2}-e^{-(k_g+\lambda_g\sigma_g+r)\tau_1}\right)\\&+\frac{G_0}{k_g+r+\lambda_g\sigma_g}\left(e^{-(k_g+\lambda_g\sigma_g+r)\tau_1}-e^{-(k_g+\lambda_g\sigma_g+r)\tau_2}\right).\end{aligned} \tag{46}$$

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