DOES THE TERM SPREAD PLAY A ROLE IN THE FED’S REACTION FUNCTION? An empirical investigation∗

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Abstract
Using US data for the period 1967:5-2002:4, this paper empirically investigates the performance of a Fed’s reaction function (FRF) that (i) allows for the presence of switching regimes, (ii) considers the long-short term spread in addition to the typical variables, (iii) uses an alternative monthly indicator of general economic activity suggested by Stock and Watson (1999), and (iv) considers interest rate smoothing. The estimation results show the existence of three switching regimes, two characterized by low volatility and the remaining regime by high volatility. Moreover, the scale of the responses of the Federal funds rate to movements in the rate of inflation and the economic activity index depends on the regime. The estimation results also show robust empirical evidence that the importance of the term spread in the FRF has increased over the sample period and the FRF has been more stable during the term of office of Chairman Greenspan than in the pre-Greenspan period.

Key words: Fed funds rate, switching regimes, term spread
JEL classification numbers: C32, E43

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1 INTRODUCTION

Taylor (1993) shows that a simple rule (called the Taylor rule) based on inflation and the output gap characterizes the evolution of the US Federal funds rate well for the first five years (1987-1992) of the term of office of Fed Chairman Greenspan. Recently, a strand of literature (for instance, Svensson (1997), Clarida, Galí and Gertler (1999)) has shown that this rule can be obtained from the optimizing behavior of a central bank that seeks to minimize a loss function that includes expected deviations of the rate of inflation from a target level and the output gap. Moreover, Rotemberg and Woodford (1998a) derive a Taylor rule from optimizing individual behavior.

The aim of this paper is to study empirically the Fed’s reaction function (FRF) that generalizes the Taylor rule with four additional features in order to provide a better understanding of how US monetary policy has been reacting to aggregate variables over a long period of time. First, the FRF allows for the presence of switching regimes to capture asymmetries in the reaction function.1 Second, the FRF considers the long-short term spread in addition to inflation and a real activity index as a simple way of capturing market expectations of both future real activity and inflation. Among others, Fama (1990), Mishkin (1990), Estrella and Hardouvelis (1991) and Estrella and Mishkin (1997) have shown robust empirical evidence that the term spread contains useful information concerning market expectations of both future real activity and inflation and that the spread summarizes predictive information that is not captured by the variables entering into a standard Taylor rule. The idea of including the term spread (or some other element related to the term structure of interest rates) in the FRF is not new. For instance, Carey (2001) includes a 10 year bond yield. As pointed out by Carey (2001), identifying an independent role for the term spread as a variable to which the Fed reacts is difficult because according to the expectations hypothesis of the term structure the long-term rate and the spread should anticipate movements in the short-term rates. Therefore, any response of the funds rate to the spread may only reflect the ability of the long rate to anticipate future funds rate movements.

Third, I use an alternative definition of general economic activity. The economic activity index considered is the CFNAI-MA3. This index is the three-month moving average of the Chicago Fed National Activity index, which is computed using the methodology suggested by Stock and Watson.

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1There is recent empirical literature (for example, Dolado, María-Dolores and Naveira (2003), Dolado, María-Dolores and Ruge-Murcia (2002) and Bec, Ben Salem and Collard (2002)) that finds evidence of an asymmetric reaction function by the US Fed, among other central banks.
The CFNAI-MA3 is a monthly index, so it allows us to consider monthly data instead of quarterly data, as is the case when a GDP measure of economic activity is used. As shown by Stock and Watson (1999), this type of economic activity index is a good indicator of future inflation. Therefore, the FRF is able to capture the presence of forward-looking components by considering the CFNAI-MA3 and the term spread, which are not present in the standard Taylor rule.

Finally, following Rotemberg and Woodford (1998b) among others, interest rate smoothing is introduced into the FRF by considering the lagged interest rate. There is recent empirical literature (for instance, Carey (2001), Gerlach-Kristen (2002) Rudebusch (2002)) that studies whether the interest rate smoothing evidence is the result of monetary policy inertia or an unobserved variable problem.

I follow two approaches for studying the performance of the FRF. First, I carry out a multiple time series analysis by estimating a three-state four-variable Markov-switching vector auto-regressive model (MSVAR) that includes the term spread, the short-term rate, inflation and the CFNAI-MA3. This MSVAR builds upon the two-state bivariate MSVAR that includes the term spread and the short-term rate, suggested by Ang and Bekaert (2002) to replicate the non-linear dynamic patterns of short-term interest rates found by the non-parametric studies (e.g., Aït-Sahalia, 1996). This two-state bivariate MSVAR is also studied by Vázquez (2004) to analyze the performance of alternative rational expectations equilibria in the context

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2 More precisely, the Chicago Fed National Activity index is the first principal component of 85 existing, monthly real indicators of economic activity. These 85 monthly indicators can be classified in five groups: production and income (21 series), employment, unemployment and labor hours (24 series), personal consumption and housing (13 series), manufacturing and trade sales (11 series) and inventories and orders (16 series). Therefore, neither the term spread nor inflation is considered in building the CFNAI-MA3. For more details on this index and demonstrations of how well it works both in forecasting inflation and identifying recessions as defined by the NBER, see also Fisher (2000) and Evans, Liu and Pham-Kanter (2002).

3 A number of studies consider an industrial production index in order to avoid this shortcoming. However, the use of such an index can also be questioned on the grounds that the share of domestic output represented by industrial output has decreased steadily in all industrial countries, including the US, over the last 30 years.

4 Two recent related papers are Sims and Zha (2002) and Valente (2003). Sims and Zha analyze a 6-variable 3-state structural MSVAR using monthly US data that includes a commodity price index, M2, the Fed funds rate, interpolated monthly real GDP, the consumer price index and the unemployment rate. Valente (2003) analyzes a two-state three variable Markov switching VAR that includes a short-term interest rate, expected inflation and expected output using data from the four largest European economies, Japan and US.
of the term structure of interest rates.\footnote{Ang and Bekaert (2002) use monthly data on 3-month short rates and 5-year long rates of zero coupon bonds from the US, Germany and the UK whereas Vázquez (2004) uses monthly data on 1-month US Treasury bill rate and US Treasury 20-year yields.} As pointed out by Ang and Bekaert (2002, p.1244), considering term spreads in the short-term rate analysis helps identification both from an econometric perspective (term spreads Granger-cause short rates) and from a modeling perspective (spreads are closely related to short rates in most term structure models). Moreover, the inclusion of both the term spread and the short rate in the MSVAR may help to disentangle the predictive power of the term spread for funds rate changes from its independent role in the FRF.

Second, I estimate the single equation described by the FRF allowing also for the presence of a three-state Markov switching process characterizing the parameters of the Fed policy rule. The inclusion of a third state in the two approaches allows for a stringent test of the FRF stability results found below during the term of office of Chairman Greenspan.

A comparison of the results from the two approaches allows us to assess the robustness of the FRF empirical analysis. We believe that highlighting the similarities and differences of these two approaches is currently of great relevance after the heated debate on the role of VAR’s in analyzing monetary policy between Glenn Rudebusch and Christopher Sims (see Rudebusch (1998a,b) and Sims (1998)). Interestingly, we find that the main conclusions of the paper are upheld by both approaches.

The estimation results show the existence of three regimes displaying rather different features. Two of the regimes are characterized by low volatility and the remaining regime by high volatility. Moreover, the scale of the responses of the Federal funds rate to movements in the rate of inflation and the economic activity index is much smaller in the two low volatility regimes than in the high volatility regime. In particular, the estimation results show a strong connection between the periods characterized by major shocks/high inflation (as was the case with the two oil shocks in the mid-seventies and the first half of the eighties, and also Volcker’s monetary experiment in 1980-1983) and the high volatility regime where the Fed reacts strongly to economic aggregates. Furthermore, the estimation results show robust empirical evidence that the FRF, which includes a significant term spread, has remained stable in explaining the dynamics of the funds rate during the term of office of Chairman Greenspan, in sharp contrast with the pre-Greenspan period, when frequent switches between the three alternative regimes show up and the role of the term spread was rather weak.

The rest of the paper is structured as follows. Section 2 introduces and
estimates the three-state four-variable MSVAR model considered. Section 3 estimates the single equation FRF. The results from the two approaches are also compared. Section 4 concludes.

2 THE MARKOV-SWITCHING VAR

In this section, I estimate a three-state MSVAR model that includes the variables entering a standard Taylor rule (i.e., the short-term rate, the economic activity index and inflation) and the term spread. Formally,

\[ Z_t = \Upsilon(s_t) + B(s_t)Z_{t-1} + \Omega(s_t)^{1/2}\xi_t, \]

where \( Z_t = (sp_t, r_t, \pi_t, \bar{y}_t)' \) and \( \xi_t \sim N(0, I) \). \( sp_t = R_t - r_t \) is the long-short term spread, \( R_t \) is the long-term rate, \( r_t \) is the short-term rate, \( \pi_t \) is the annualized rate of inflation (\( \pi_t = \frac{1}{12} \sum_{i=0}^{11} \pi_{t-i}, \pi_t = 1200(\ln P_t - \ln P_{t-1}) \) and \( P_t \) is the consumer price index) and \( \bar{y}_t \) is an index of economic activity that aims to capture the cyclical component of output. The regime variable \( s_t \) is either 1, 2 or 3 and follows a first-order three-state Markov process with the transition matrix given by

\[
P = \begin{pmatrix}
p_{11} & p_{21} & 1 - p_{33} - p_{32} \\
p_{12} & p_{22} & p_{32} \\
p_{12} & 1 - p_{11} - p_{12} & 1 - p_{21} - p_{22} & p_{33}
\end{pmatrix},
\]

where the row \( j \), column \( i \) element of \( P \) is the transition probability \( p_{ij} \) that gives the probability that regime \( i \) will be followed by regime \( j \).\(^6\) We estimate the Cholesky decomposition \( \Psi(s_t) \) of \( \Omega(s_t) \) where \( \Omega(s_t) = \Psi(s_t)\Psi(s_t)' \).

The short-term rate considered is the US Federal funds rate. The term spread is defined by the difference between the one-year Treasury constant maturity rate and the Federal funds rate.\(^7\) The interest rate and inflation

\(^6\)The three-regime MSVAR (1) considered may seem quite restrictive but it is the most the data can bear without extreme problems in estimation. Dealing with the three-regime MSVAR already implies the cumbersome task of estimating 96 coefficients. I also estimate the three-regime MSVAR including two lags (this exercise involves estimation of 144 parameters!). The estimation results of the MSVAR with two lags are qualitatively similar to those obtained estimating the MSVAR (1). Moreover, according to the Schwarz information criterion, the MSVAR (1) is selected instead of the MSVAR with two lags. The estimation results of the MSVAR with two lags are available from the author upon request.

\(^7\)Robustness exercises have been performed using other long-term rates to define the term spread. The long-term rates considered cover almost the whole maturity spectrum from the 3-month Treasury bill rate to the 10 year Treasury constant maturity rate. To save space the results using other definitions of the term spread are not shown unless they reveal relevant results.
data were collected from the websites of the Federal Reserve Bank of St. Louis and the Bureau of Labor Statistics, respectively. As mentioned in the Introduction, the economic activity index considered is the CFNAI-MA3. The period studied runs from May, 1967 to April, 2002. Figure 1 shows the plots of the main time series analyzed in this paper. A bird’s-eye view of the Funds fund rate and inflation time series shows evidence of the Fisher effect; the nominal interest rate (roughly) moves one to one with inflation. Moreover, the term spread and the economic activity index display a similar dynamic pattern. As shown in Figure 2, the term spread leads the economic activity index. Thus, the highest cross-correlation value is at the 7-lead (that is, \( \rho(y_{t+7}, sp_t) = 0.526 \)) whereas the contemporaneous cross-correlation is only 0.250.

The second equation of system (1) can be viewed as an FRF. We believe it is appropriate not to include contemporaneous variables in the Fed’s reaction function since monthly data is used in the estimation of the alternative specifications. Arguably, this allows for a closer match between the information set available to the researcher and real time data used by the Fed at the time of implementing monetary policy. Work by Orphanides (for instance, Orphanides et al. (2000)) has shown the importance of using the information available to the central bank at the time the policy decision was made.

The maximum likelihood estimation of the alternative Markov-switching models considered in this paper follows the procedures suggested by Hamilton (1994, ch. 22). The appendix briefly summarizes these procedures.
Figure 1: Times Series Plots
Figure 2: Estimated cross-correlations of the term spread with lags and leads of CFNAI_MA3
Table 1. Estimation results for the three-state four-variable MSVAR model (1).

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Estimate</th>
<th>Stand. error</th>
<th>Parameter</th>
<th>Estimate</th>
<th>Stand. error</th>
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<td>0.2131</td>
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<td>( b_{21}(2) )</td>
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Notes: \( \gamma_i(s_t) \) denotes a generic element of vector \( \Upsilon(s_t) \), \( b_{ij}(s_t) \) denotes a generic element of matrix \( B(s_t) \) and \( \psi_{ij}(s_t) \) denotes a generic element of matrix \( \Psi(s_t)' \). The symbol (*) denotes that the non-negative restriction is binding for the corresponding parameter.
We focus our attention on three aspects of the estimation results of the MSVAR (1) displayed in Table 1. First, the volatility of innovations in each regime. Second, the persistence of the system in each regime. Finally, the coeﬃcient associated with the log likelihood — 59.038

Table 1. (Continued)

<table>
<thead>
<tr>
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<th>Stand. error</th>
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<td>*</td>
</tr>
<tr>
<td>$\psi_{23}(2)$</td>
<td>0.0307</td>
<td>0.0402</td>
<td>$p_{21}$</td>
<td>0.0442</td>
<td>0.0166</td>
</tr>
<tr>
<td>$\psi_{24}(2)$</td>
<td>0.1787</td>
<td>0.0413</td>
<td>$p_{22}$</td>
<td>0.9246</td>
<td>0.0162</td>
</tr>
<tr>
<td>$\psi_{33}(2)$</td>
<td>0.4403</td>
<td>0.0434</td>
<td>$p_{32}$</td>
<td>0.0187</td>
<td>0.0082</td>
</tr>
<tr>
<td>$\psi_{34}(2)$</td>
<td>$-0.1128$</td>
<td>0.0419</td>
<td>$p_{33}$</td>
<td>0.9449</td>
<td>0.0209</td>
</tr>
<tr>
<td>$\psi_{44}(2)$</td>
<td>0.3607</td>
<td>0.0365</td>
<td>log</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\psi_{11}(3)$</td>
<td>0.2131</td>
<td>0.0212</td>
<td>likelihood</td>
<td></td>
<td>$-59.038$</td>
</tr>
</tbody>
</table>

8 More precisely, the eigenvalues of $\Omega(1)$ are 0.05776, 0.04569, 0.02553 and 0.01249, the eigenvalues of $\Omega(2)$ are 1.69534, 0.40162, 0.21739 and 0.10510, and the eigenvalues of $\Omega(3)$ are 0.08712, 0.07217, 0.03670 and 0.01646.

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lagged interest rate is not significantly different from one in any regime at any standard significance level. This result is not surprising at all since the lower bound for the estimated value for the smoothing parameter is found to be around 0.73 using quarterly data (among others, see Rudenbusch (2002)) and this result implies a lower bound value of 0.9 (≈ 0.73^{1/3}) when using monthly data.\footnote{Notice that the point estimate of the smoothing parameter for state 1 is greater than one. Several authors have shown that a smoothing coefficient greater than unity has good properties. Thus, Rotemberg and Woodford (1998b) show that this feature guarantees the existence of a locally unique equilibrium. More recently, Benhabib, Schmitt-Grohé and Uribe (2003) show that a smoothing coefficient greater than unity ensures global stability.}

In order to compare easily the estimation results of the second equation of system (1) for the alternative regimes, we next display these results in regression format (standard errors in parentheses):

**Regime 1:**

\[
    r_t = -0.0772 + 0.1085 s_p - 0.0254 \pi_{t-1} + 0.1936 \tilde{y}_{t-1} + 1.0273 r_{t-1},
\]

\[
    \begin{align*}
        (2) & \\
        (0.0651) & (0.0375) & (0.0136) & (0.0234) & (0.0164)
    \end{align*}
\]

**Regime 2:**

\[
    r_t = -0.2609 + 0.3226 s_p + 0.1543 \pi_{t-1} + 0.5329 \tilde{y}_{t-1} + 0.9542 r_{t-1},
\]

\[
    \begin{align*}
        (3) & \\
        (0.3749) & (0.1411) & (0.0446) & (0.0234) & (0.0394)
    \end{align*}
\]

**Regime 3:**

\[
    r_t = -0.1993 + 0.0172 s_p + 0.0275 \pi_{t-1} + 0.1603 \tilde{y}_{t-1} + 0.9931 r_{t-1},
\]

\[
    \begin{align*}
        (4) & \\
        (0.1584) & (0.0460) & (0.0116) & (0.0871) & (0.0212)
    \end{align*}
\]

The estimation results show that the Fed funds rate significantly responds to all three variables in regime 2, whereas it only responds significantly to the spread and the economic activity index in regime 1. However, the funds rate’s responses to the three variables are small or not significant in regime 3. Moreover, the estimation results show that the responses of the funds rate to the rate of inflation and the economic activity index are significantly larger in the high volatility regime (regime 2) than in the other two.\footnote{Form now on, by saying that an estimated parameter is significantly larger than other I mean that the estimated value of the parameter is larger and that the 95% confidence intervals associated with the two estimated parameters do not overlap.} Furthermore, the low volatility regimes are significantly different in the way the funds rate responds to the term spread and inflation. Thus, the funds rate’s response is significant on the spread but not on inflation (the point estimate is actually negative) in the first regime whereas the opposite is true in the third regime.
The estimation results also show that the Fed funds rate’s response to the economic activity index is significantly larger than the response to inflation in regimes 1 and 2 (the point estimate response in regime 3 is larger, but not significantly so). Should we interpret this finding as evidence that the Fed cares more about output fluctuations than inflation? No. As shown by Stock and Watson (1999) the CFNAI anticipate inflation movements. Then, this finding only reflects that the Fed reacts more to anticipated than current inflation. Moreover, the standardized funds rate response to the economic activity index is larger than the standardized response to the term spread in all regimes.

In short, the estimation results provide empirical evidence of a different policy reaction function depending on the state of the economy. Thus, one can conclude that the funds rate behaves in a smooth manner during periods characterized by economic stability as in the first and third regimes where the scale of the responses of the funds rate is small. By contrast, the funds rate responses are rather large in the second regime characterized by high volatility.

Apart from the size of volatility and the scale of responses of the funds rate, what are the other features characterizing the alternative regimes? In order to answer this question, let us start by looking at Figure 3. This figure shows three plots with the allocation of time periods for the three alternative regimes based on the smoothed probabilities using the information over the whole sample of size $T$ (i.e., $\text{prob}[s_t = 1|I_T]$) together with the annualized inflation plot.11 Figure 3 clearly indicates that the funds rate dynamics are characterized by frequent switches between regimes during the pre-Greenspan period (1967:5 to 1987:8) whereas the term of office of Chairman Greenspan can be entirely attributed to a single regime (regime 1), characterized by low, stable inflation.12 Moreover, a strong connection between the periods

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11Interestingly, the estimated smoothed probabilities displayed in Figure 3 are similar to those found by Sims and Zha (2002) in their Figure 1 using a three-state six-variable structural MSVAR (notice that the high volatility state is called regime 3 in Sims and Zha, 2002). Moreover, the high volatility state estimated probabilities are similar to those found by Ang and Bekaert (2002) and Vázquez (2004) in their Figures 2, respectively. This finding suggests that the allocation of time periods between regimes is fairly robust to alternative specifications of the MSVAR. Since the latent variable $s_t$ is an exogenous variable, one would be concerned if the estimated smoothed probabilities were very different under alternative MSVAR specifications (that is, $s_t$ would be exhibiting some sort of endogeneity if the smoothed probabilities depended on the MSVAR specification).

12As stated above, moving from two to three states and then splitting the low volatility regime in two different low volatility regimes (regimes 1 and 3) allows for a more stringent test of the stability results found in the two-state Markov switching analysis of Ang and Bekaert (2002) and Vázquez (2004) during the term of office of Chairman Greenspan.
Figure 3: Smoothed Probabilities of (1) and Inflation
characterized by major shocks/high inflation and recession (as was the case with the two oil shocks in the mid-seventies and the first half of the eighties, and also Volcker’s monetary experiment in the period 1980-1983) and the high volatility regime. These findings are further confirmed by looking at the sample correlations of the smoothed probabilities of the three regimes with inflation and the economic activity index. Thus, the correlation between annualized inflation and the smoothed probabilities of regime 1, 2 and 3 are $-0.5022$, $0.5822$, and $-0.0616$, respectively. The correlation between the economic activity index and the smoothed probabilities of regime 1, 2 and 3 are $0.0396$, $-0.2895$, and $0.3162$, respectively.

Roughly speaking, we can conclude that, on the one hand, regime 1 and 3 are characterized by low volatility, but regime 1 can be also viewed as a state related to low inflation whereas regime 3 is more related to expansions. On the other hand, regime 2 characterized by high volatility is related to high inflation and recession. Thus the nineties, being a stable and low inflationary period, belong to the first regime whereas the periods 1973-1976 (first oil crisis) and 1979-1983 (second oil crisis and the Fed’s monetary experiment) are robustly attributed to the second, where monetary authorities are more likely to switch to an active policy regime where the scale of the responses is larger than in the low volatility regimes. These conclusions are similar to those drawn by Ang and Bekaert (2002, p.1257) and Vázquez (2004) using a two-state bivariate MSVAR.

As mentioned above, a large body of literature developed in the late 1980’s and early 1990’s found evidence that the term structure helps to predict future inflation and real economic activity as well. Interestingly, this academic work took place just before the period when regime 1, characterized by a significant response of the Fed funds rate to the spread, became the most stable and plausible regime ever. However, establishing causality between these two events is difficult since no severe recession has occurred since the early 1980’s. The significant Fed funds rate response to the term spread and the stability of regime 1 in describing the term of office Chairman Greenspan suggest that the Fed has paid more attention to the term spread in the last fifteen years than before, but this is also due to the fact that the Fed moved to target the Fed funds rate directly after the October 1987 stock market turmoil.

The estimated correlation matrix for innovations is

$$
\begin{pmatrix}
1 & -0.0917 & 1 \\
0.0570 & -0.0142 & 1 \\
0.3371 & 0.0992 & -0.0256 & 1 \\
\end{pmatrix}
$$
Since the innovations of the interest rate equation are close to being uncorrelated to the other innovations, the order of factorization makes no difference in analyzing the impulse response functions of the Fed funds rate.

Figure 4 shows the estimated impulse response functions with upper and lower two standard error bounds of the term spread, Fed funds rate, inflation and the economic activity index for the term of office of Chairman Greenspan.\footnote{We only show the impulse response functions for the Greenspan period because this period is well characterized by a single regime. However, frequent regime switches appear in the pre-Greenspan period, which implies that the impulse response functions are less informative in the latter case.} Responses are displayed over an expanse of 48 months. Looking down a column we see the responses of a particular variable. For instance, the graph in the third row and second column corresponds to the Fed funds rate response to an innovation in the inflation equation. We observe that the largest and most persistent responses of the funds rate are due to innovations from the economic activity and term spread equations. Moreover, a contractionary monetary policy shock (a positive innovation in the Fed funds rate equation) is followed by a rise in the inflation rate. This effect resembles the \textit{price puzzle} with the inflation rate instead of the consumer price index. Usually, the inclusion of a commodity price index or other assets prices in the VAR helps to mitigate this problem. The fact that an inflation puzzle still emerges when including the term spread in the VAR together with the large body of empirical evidence on the term spread ability to predict inflation suggest that the cost channel of monetary policy proposed by Barth and Ramey (2001) works.\footnote{Barth and Ramey (2001) argue that a contactionary monetary shock may affect aggregate supply as well as aggregate demand. Thus, an increase in interest rates raises the cost of holding inventories, which is a negative supply shock that increases prices and reduces output. Under this view, the price puzzle is simply evidence of the cost channel rather than evidence that the VAR is misspecified. See Walsh (2003) and the references therein for further explanations on the issues involved in the price puzzle.}
Figure 4: Impulse responses of the VAR for the Greenspan period
3 SINGLE EQUATION ESTIMATION OF THE FRF

In this section, I estimate an FRF that allows for the presence of a Markov-switching process characterizing the parameters of the rule and considers the term spread and interest rate smoothing in addition to the variables included in the standard Taylor rule. We call the single equation specification of the FRF augmented Taylor rule (ATR). Formally, the ATR is given by

\[ r_t = \rho_0(s_t) + \rho_1(s_t) sp_{t-1} + \rho_2(s_t) \pi_{t-1} + \rho_3(s_t) y_{t-1} + \rho_4 r_{t-1} + \sigma(s_t) u_t, \]  

(5)

where \( u_t \) is a standard normal random variable and \( s_t \) is either 1, 2 or 3 and follows the same process specified above. As pointed out above, we study an ATR that only includes lagged variables since monthly data is used. Arguably, this allows for a closer match between the information set available to the researcher and real time data used by the Fed when implementing monetary policy. Moreover, considering only lagged variables in the ATR allows us a neater comparison with the estimation results of the FRF found in the previous section using the MSVAR.

Table 2. Estimation results for the ATR (5)

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Estimate</th>
<th>Stand. error</th>
<th>Parameter</th>
<th>Estimate</th>
<th>Stand. error</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \rho_0(1) )</td>
<td>-0.2335</td>
<td>0.0289</td>
<td>( \rho_4(2) )</td>
<td>0.9182</td>
<td>0.0616</td>
</tr>
<tr>
<td>( \rho_0(2) )</td>
<td>-0.5833</td>
<td>0.5785</td>
<td>( \rho_4(3) )</td>
<td>1.0222</td>
<td>0.0088</td>
</tr>
<tr>
<td>( \rho_0(3) )</td>
<td>-0.4625</td>
<td>0.0494</td>
<td>( \sigma(1) )</td>
<td>0.0916</td>
<td>0.0059</td>
</tr>
<tr>
<td>( \rho_1(1) )</td>
<td>0.1613</td>
<td>0.0177</td>
<td>( \sigma(2) )</td>
<td>1.3278</td>
<td>0.1589</td>
</tr>
<tr>
<td>( \rho_1(2) )</td>
<td>0.3787</td>
<td>0.1922</td>
<td>( \sigma(3) )</td>
<td>0.1037</td>
<td>0.0064</td>
</tr>
<tr>
<td>( \rho_1(3) )</td>
<td>0.0939</td>
<td>0.0196</td>
<td>( \rho_{11} )</td>
<td>0.9595</td>
<td>0.0080</td>
</tr>
<tr>
<td>( \rho_2(1) )</td>
<td>0.0538</td>
<td>0.0068</td>
<td>( \rho_{12} )</td>
<td>0.0013</td>
<td>0.0010</td>
</tr>
<tr>
<td>( \rho_2(2) )</td>
<td>0.2411</td>
<td>0.0796</td>
<td>( \rho_{21} )</td>
<td>0.0762</td>
<td>0.0175</td>
</tr>
<tr>
<td>( \rho_2(3) )</td>
<td>0.0302</td>
<td>0.0091</td>
<td>( \rho_{22} )</td>
<td>0.8408</td>
<td>0.0222</td>
</tr>
<tr>
<td>( \rho_3(1) )</td>
<td>0.1852</td>
<td>0.0154</td>
<td>( \rho_{32} )</td>
<td>0.0020</td>
<td>0.0019</td>
</tr>
<tr>
<td>( \rho_3(2) )</td>
<td>0.7781</td>
<td>0.2749</td>
<td>( \rho_{33} )</td>
<td>0.9546</td>
<td>0.0104</td>
</tr>
<tr>
<td>( \rho_3(3) )</td>
<td>0.1780</td>
<td>0.0152</td>
<td>( \log )</td>
<td>76.485</td>
<td></td>
</tr>
<tr>
<td>( \rho_4(1) )</td>
<td>1.0221</td>
<td>0.0072</td>
<td>likelihood</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table 2 shows the estimation results for the ATR (5). The estimation results are similar to those found above when estimating the MSVAR (1), but some noticeable differences emerge. These results can be summarized as follows. First, the volatility of innovations in (5) is significantly higher in the second regime than in the other two (i.e., \( \sigma(2) \) is significantly larger than \( \sigma(1) \)).
and \( \sigma(3) \)). Moreover, \( \sigma(1) \) and \( \sigma(3) \) are not significantly different. Second, the coefficients associated with the lagged interest rate are not significantly different from one in any regime, as was the case above when estimating the MSVAR. Third, comparing the responses of the Fed funds rate to the alternative variables displayed in Table 2 with those described in equations (2)-(4) we observe that all qualitative conclusions hold. However, there is a remarkable difference: there are significant funds rate responses to inflation in regime 1 and the term spread in regime 3, respectively, when estimating (5).

Figure 5 shows that the two models fit the actual path of the Fed funds rate reasonably well. Moreover, comparing Figures 3 and 6, we can observe that the two models provide similar allocations of time periods to the alternative regimes. Thus, funds rate dynamics are characterized by frequent switches between regimes during the pre-Greenspan period (1967:5 to 1987:8) whereas the term of office of Chairman Greenspan can be almost entirely attributed to regime 1, although for the first part of that term frequent switches between regimes 1 and 3 are present when estimating the ATR (5) that do not show up when estimating the MSVAR (1).

The estimated errors obtained from the MSVAR (1) and the ATR (5) are highly correlated (i.e., the correlation coefficient takes the value 0.924). This result is a direct consequence of the low estimated correlation coefficients between the innovations in the MSVAR (1). Interestingly, the estimated errors from the ATR (5) and the MSVAR (1) are rather much correlated than the error terms from different VAR specifications compared by Rudebusch (1998).

The correlations between the smoothed probabilities obtained from the ATR (5) and annualized inflation and the economic activity index, respectively, are almost identical with those found in the previous section with one exception: regime 3 shows a rather low correlation with the economic activity index.15

Next I estimate the model imposing the restrictions that the term spread does not determine the changes in the funds rate under any regime. The likelihood ratio test statistic associated with the null hypothesis that the changes in the funds rate are not determined by the term spread in any regime takes the value 29.42. This statistic is distributed as a \( \chi^2(3) \), which implies rejection of the null hypothesis at any standard significance level.

I further estimate a parsimonious version of the ATR (5) imposing the

---

15The correlations between annualized inflation and the smoothed probabilities of regime 1, 2 and 3 obtained from the ATR (5) are \(-0.5039, 0.5803, \) and \(-0.0155 \) respectively. The correlations between the economic activity index and the smoothed probabilities of regime 1, 2 and 3 obtained from the ATR (5) are \(0.1251, -0.2536, \) and \(0.1134 \), respectively.
Figure 5: Actual and Fitted Short-Term Rate
Figure 6: Smoothed Probabilities of the ATR (5) and Inflation
following restrictions: (i) the response of the Fed funds rate on its lagged value is invariant across regimes (i.e., $\rho_4(1) = \rho_4(2) = \rho_4(3)$), (ii) the responses of the funds rate on lagged economic activity index are identical in the two low volatility regimes (i.e., $\rho_3(1) = \rho_3(3)$) and (iii) the sizes of innovations in regimes 1 and 3 are identical (i.e., $\sigma(1) = \sigma(3)$). The four restrictions imposed by this parsimonious version of the ATR can also be tested using a likelihood ratio test. The likelihood ratio statistic, which is distributed as a $\chi^2(4)$, takes the value 3.74. This statistic implies that the restrictions imposed by the parsimonious ATR are not rejected at any standard critical value.

Recent evidence reported by Rudebusch (2002) suggests that only term spreads at the short end of the term structure (up to three-month Treasury bill spread) are able to predict Fed funds rate changes. In order to disentangle the predictive power of the term spread for funds rate changes from its independent role in the monetary policy reaction function, we estimate an ATR that includes two term spreads: First, the term spread between the 3-month Treasury bill rate and Fed funds rate; second, the term spread between the 1-year Treasury constant maturity rate and the 3-month Treasury bill rate. Formally, this ATR is given by

$$r_t = \rho_0(s_t) + \rho_1(s_t)sp_{3m,ff}_{t-1} + \rho_2(s_t)\pi_{t-1} + \rho_3(s_t)\bar{y}_{t-1} + \rho_4r_{t-1} + \rho_5(s_t)sp_{1y,3m}_{t-1} + \sigma(s_t)u_t,$$

where $sp_{3m,ff}_{t-1}$ and $sp_{1y,3m}_{t-1}$ denote the spread between the 3-month rate and Fed funds rate and the spread between the 1-year rate and the 3-month rate, respectively.

The estimation results for the ATR (6) are displayed in Table 3. Before focusing on the parameter estimates, notice that the ATR (6) becomes (5) imposing the three restrictions $\rho_3(s_t) = \rho_5(s_t)$ for $s_t = 1, 2, 3$. Therefore, we can test ATR (5) versus ATR (6) by carrying out a likelihood ratio test. The likelihood ratio test statistic, which is distributed as a $\chi^2(3)$ in this case, takes the value 7.47. This statistic implies that the ATR (5) is not rejected in favor of the ATR (6) at the 5% significance level but is rejected at the 10% significance level. Although there is only a marginal improvement in splitting the spread between the 1-year rate and Fed funds rate into the two spreads considered in ATR (6), we find that the spread between the 3-month rate and Fed funds rate is only significant in the high volatility regime (state 2) whereas the spread between the 1-year rate and 3-month rate is significant in the two low volatility regimes (states 1 and 3). These findings suggest that the term spread has an independent role in the FRF in the two low volatility regimes whereas the significance of the term spread in the high volatility

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regime is more related to the predictive ability of the term spread to detect Fed funds rate changes.\textsuperscript{16}

Table 3. Estimation results for the ATR (6)

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Estimate</th>
<th>Stand. error</th>
<th>Parameter</th>
<th>Estimate</th>
<th>Stand. error</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\rho_0(1)$</td>
<td>$-0.1583$</td>
<td>$0.0844$</td>
<td>$\rho_3(3)$</td>
<td>$0.1880$</td>
<td>$0.0179$</td>
</tr>
<tr>
<td>$\rho_0(2)$</td>
<td>$-0.6825$</td>
<td>$0.5365$</td>
<td>$\rho_5(1)$</td>
<td>$0.1852$</td>
<td>$0.0291$</td>
</tr>
<tr>
<td>$\rho_0(3)$</td>
<td>$-0.4043$</td>
<td>$0.0617$</td>
<td>$\rho_5(2)$</td>
<td>$0.3133$</td>
<td>$0.3385$</td>
</tr>
<tr>
<td>$\rho_1(1)$</td>
<td>$0.0117$</td>
<td>$0.1361$</td>
<td>$\rho_5(3)$</td>
<td>$0.1053$</td>
<td>$0.0310$</td>
</tr>
<tr>
<td>$\rho_1(2)$</td>
<td>$0.4798$</td>
<td>$0.2376$</td>
<td>$\sigma(1)$</td>
<td>$0.0831$</td>
<td>$0.0058$</td>
</tr>
<tr>
<td>$\rho_1(3)$</td>
<td>$0.0291$</td>
<td>$0.0840$</td>
<td>$\sigma(2)$</td>
<td>$1.3328$</td>
<td>$0.1862$</td>
</tr>
<tr>
<td>$\rho_2(1)$</td>
<td>$0.0600$</td>
<td>$0.0185$</td>
<td>$\sigma(3)$</td>
<td>$0.1073$</td>
<td>$0.0109$</td>
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<td>$\rho_2(2)$</td>
<td>$0.2388$</td>
<td>$0.0866$</td>
<td>$p_{11}$</td>
<td>$0.9589$</td>
<td>$0.0172$</td>
</tr>
<tr>
<td>$\rho_2(3)$</td>
<td>$0.0300$</td>
<td>$0.0164$</td>
<td>$p_{12}$</td>
<td>$0.0024$</td>
<td>$0.0050$</td>
</tr>
<tr>
<td>$\rho_3(1)$</td>
<td>$0.2047$</td>
<td>$0.0207$</td>
<td>$p_{21}$</td>
<td>$0.0655$</td>
<td>$0.0279$</td>
</tr>
<tr>
<td>$\rho_3(2)$</td>
<td>$0.7316$</td>
<td>$0.2806$</td>
<td>$p_{22}$</td>
<td>$0.8369$</td>
<td>$0.0327$</td>
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<td>$\rho_3(3)$</td>
<td>$0.1880$</td>
<td>$0.0179$</td>
<td>$p_{32}$</td>
<td>$0.0027$</td>
<td>$0.0037$</td>
</tr>
<tr>
<td>$\rho_4(1)$</td>
<td>$0.9943$</td>
<td>$0.0371$</td>
<td>$p_{33}$</td>
<td>$0.9484$</td>
<td>$0.0108$</td>
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<tr>
<td>$\rho_4(2)$</td>
<td>$0.9438$</td>
<td>$0.0835$</td>
<td>log likelihood</td>
<td>$80.222$</td>
<td></td>
</tr>
</tbody>
</table>

4 CONCLUSIONS

This paper implements Markov regime switching procedures à la Hamilton to analyze the stability of the Fed’s reaction function over the last thirty-five years. The estimation results show the existence of three different regimes. Two of them (say regime 1 and 3) are characterized by low volatility and the other (regime 2) by high volatility. Moreover, the scales of the responses of the funds rate to movements in the rate of inflation and the economic activity index are much smaller in the low volatility regimes than in the high volatility regime. On the one hand, regime 1 is also associated with low inflationary periods and rather large funds rate responses to term spread movements whereas regime 3 is more associated with expansionary periods than with low inflationary periods, although the evidence is much weaker in the latter case. On the other hand, the high volatility regime is strongly related to high inflation and recessions.

The estimation results also show robust empirical evidence that a Fed reaction function that includes the term spread has remained relatively stable.\textsuperscript{16}The estimation results for the rest of the parameters are similar to those obtained in estimating the ATR (5), so we do not discuss them further.
in explaining the dynamics of the funds rate during the term of office of Chairman Greenspan. However, for the pre-Greenspan period the estimation results of the Fed’s reaction function is unstable, showing frequent switches between three rather different regimes.

APPENDIX

This appendix briefly summarizes the recursive algorithm implemented in the maximum likelihood estimation procedure. I pay attention to the three-state case considered in this paper. Let $\theta$ denote the vector of parameters. Let $\hat{\xi}_{t/t}$ denote the $3 \times 1$ vector containing the researcher inference about the values of $s_t (=1, 2, 3)$ based on data obtained through date $t$ and conditional on a given value for $\theta$. Finally, let $\hat{\xi}_{t+1/t}$ denote the $3 \times 1$ vector containing the one-period forecast about the values of $s_{t+1} (=1, 2, 3)$ based on data obtained through date $t$. Hamilton (1994, chap. 22) shows that the optimal inference and the one-period forecast can be solved recursively from the two following equations:

$$
\hat{\xi}_{t/t} = \frac{\hat{\xi}_{t-1/t} \odot \eta_t}{1' (\hat{\xi}_{t-1/t} \odot \eta_t)}, \\
\hat{\xi}_{t+1/t} = P \hat{\xi}_{t/t}
$$

where the symbol $\odot$ denotes element-by-element multiplication, $1$ denotes a $3 \times 1$ vector of 1s, $P$ is the $3 \times 3$ transition probability matrix and $\eta_t$ is a $3 \times 1$ vector containing the three conditional densities, one for each state. For the MSVAR (1),

$$
\eta_t = \begin{bmatrix} \eta_t(1) \\ \eta_t(2) \\ \eta_t(3) \end{bmatrix}
$$

where each element of $\eta_t$ is given by the probability density function of the multivariate normal distribution

$$
\eta_t(s_t) = \frac{1}{(2\pi)^{3/2} |\Omega(s_t)|^{1/2}} \exp\left\{ -\frac{1}{2} [Z_t - \Upsilon(s_t) - B(s_t)Z_{t-1}]' \\
[\Omega(s_t)]^{-1} [Z_t - \Upsilon(s_t) - B(s_t)Z_{t-1}] \right\},
$$

for $s_t = 1, 2, 3$.

The log likelihood function $L(\theta)$ for the data set evaluated at a value of $\theta$ can be computed as a by-product of the former recursive algorithm from the following expression

$$
L(\theta) = \sum_{t=1}^{T} \log \left[ 1' (\hat{\xi}_{t-1/t} \odot \eta_t) \right].
$$

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The value of $\theta$ that maximizes $\mathcal{L}(\theta)$ is found using the maximum likelihood routine programmed in GAUSS. The Broyden-Fletcher-Goldfarb-Shanno numerical method is used to update the Hessian at each iteration of the maximization routine.
References


