Application of burst vibrothermography to characterize planar vertical cracks

Arantza Mendioroz*, Ricardo Celorrio, Ángel Cifuentes, Lander Zatón, Agustín Salazar

*Departamento de Física Aplicada I, Escuela de Ingeniería de Bilbao, Universidad del País Vasco UPV/EHU, Alameda Urquijo s/n, 48013 Bilbao, Spain; Departamento de Matemática Aplicada, EINA/IUMA, Universidad de Zaragoza, Campus Río Ebro, Edificio Torres Quevedo, 50018 Zaragoza, Spain; Instituto Politécnico Nacional, CICATA Legaria, Av. Legaria No. 694 Col. Irrigación, 11500 Ciudad de México, México

ABSTRACT

We present a method to characterize vertical cracks in a fast way using burst vibrothermography. In this technique the sample is excited by ultrasounds and, at the defect, rubbing of the contacting surfaces produces heat that can be detected as a temperature rise at the surface using an infrared camera. In this work, first we present the solution of the direct problem, i.e., the calculation of the surface temperature distribution produced by a vertical heat source representing a crack excited by an ultrasound burst, and we choose the information that will be used to characterize the crack, namely, one thermogram and one timing-graph. Next we address the inverse problem, consisting of finding the heat source distribution that is responsible for the observed surface temperature. This inverse problem is ill-posed, and a naïve inversion process is unstable. We propose to use three penalty terms, based on zero order Tikhonov and Total Variation functionals and the Lasso method, to stabilize the inversion. By inverting synthetic data, we analyze the performance of the algorithm as a function of the depth of the heat source and we study the effect of the burst duration and noise level in the data on the quality of the reconstructions. Finally, we invert experimental data taken in samples containing calibrated heat sources. The results show that it is possible to characterize vertical cracks down to depths of 6 mm in AISI 304 stainless steel.

Keywords: vibrothermography, nondestructive evaluation, infrared thermography, defect characterization, inverse problem.

1. INTRODUCTION

Since the spread of affordable infrared video cameras in the eighties, infrared thermography has been used as a nondestructive technique to detect defects such as cracks, delaminations, disbonds, etc. Optically excited thermography with homogeneous illumination has been widely developed to detect and characterize planar defects parallel to the sample surface such as delaminations and inserts in composite structures often used in aerospace industry. Vertical cracks, on the contrary, are elusive to this configuration because they barely scatter thermal waves diffusing perpendicular to the sample surface. For this reason, detection of vertical cracks has been implemented with optically excited thermography using focused instead of homogeneous illumination. In this configuration cracks are revealed because of the thermal resistance they produce. This thermal resistance leads to an asymmetry in surface temperature profiles perpendicular to the crack, which presents a temperature jump at the crack. The signature is less prominent as the width of the crack decreases. On the contrary, ultrasound excited thermography or vibrothermography is very well adapted for the detection of kissing cracks. In vibrothermography the sample is excited with ultrasounds and, at the defects, the vibration of the two contacting faces dissipates heat, turning the defect into an inner heat source. The heat generated at the defect diffuses inside the sample and eventually reaches the sample surface. Thus, the presence of the crack is identified, under ultrasound excitation, as a temperature rise at the surface. Several research groups have devoted efforts to understand the mechanisms for heat generation, and the optimum experimental conditions giving rise to higher temperature rises at the sample surface.
Beyond the ability shown by the technique to detect defects in many kinds of materials, from metals to composites and ceramics\textsuperscript{9,12-13}, exploring the capabilities of the technique to estimate the size and location of the defect quantitatively is an important task. Knowledge of the size and location is highly valuable to make rejection decisions in nondestructive testing. Along these lines, many efforts have been devoted to relate crack size and temperature rise, mainly using finite element modeling\textsuperscript{14}, and to establish probabilities of detection\textsuperscript{15}.

In a previous work, authors showed that it is possible to characterize the area and location of flat vertical cracks combining lock-in vibrothermography data obtained at several modulation frequencies\textsuperscript{16-17}. In this regime, the ultrasound intensity is modulated at a frequency much lower than the ultrasound's frequency and the amplitude and phase of the oscillating component (at the modulation frequency) of the temperature in image sequence are extracted. In references 16 and 17 we solved the inverse problem consisting of finding the heat source distribution that gives rise to the observed amplitude and phase thermograms, without any assumption on the shape of the crack, and with the only prior knowledge of the plane containing the crack. This is an ill-posed inverse problem, characterized by an extreme sensitivity of the solution to errors or noise in the data or the model, which makes the inversion algorithm very unstable. We stabilized the inversion by introducing a penalty term based on Total Variation functional and showed that it is possible to characterize vertical cracks using lock-in vibrothermography. The drawback of this approach is that data taking is rather time consuming since, in order to penetrate deep into the material, low modulation frequencies (long excitation periods) are needed, and only one frequency is proved at a time.

To overcome this drawback, in this work we explore the ability of burst vibrothermography, which is a much faster technique, to characterize planar vertical cracks. Our purpose is to determine the size and depth of the crack with only prior knowledge of the plane where it is contained. In section 2 we calculate the evolution of the surface temperature distribution produced by a vertical heat source emitting a constant flux over a short period of time and we propose the select of a subset of all available data to characterize the crack. In section 3 we describe our approach to solve the inverse problem and we introduce an extension of the regularization strategy presented in references 16 and 17 to stabilize the inverse problem. We show the performance of the algorithm and we analyze its limits by inverting noisy synthetic data. In section 4 we present the experimental set-up, the samples containing calibrated heat sources we have prepared and we show inversions of experimental data. Finally, we summarize and conclude.

2. DIRECT PROBLEM AND SIMULATIONS

In this section we calculate the evolution of the surface temperature distribution produced by a buried vertical heat source emitting a homogeneous and constant flux over a short time interval $\tau$. It represents a vertical crack excited by an ultrasound burst of duration $\tau$. The geometry is depicted in Fig. 1a.

![Figure 1. (a) Geometry of a sample containing an irregular buried vertical crack. Cross-section of a rectangular crack.](image)

First, we consider a point and instantaneous heat source located at $\vec{r}'$ and emitting an energy $H$ at time $t'$ in an infinite medium of thermal diffusivity $D$, density $\rho$ and specific heat capacity $c$. The geometry is depicted in Fig. 1a. The temperature at any position and time is given by\textsuperscript{18}:

$$T(\vec{r},t) = \frac{H}{8\rho c \left[ \pi D(t-t') \right]^{3/2}} e^{-\frac{\rho c t}{4D(t-t')^2}}$$

Then we generalize the result to the case in which the point source emits a constant power $P$ during a time interval $[0,\tau]$. The expressions for the temperature at any point of the medium, with thermal conductivity $K$, are given by\textsuperscript{9-20}:
\[ T(\vec{r},t) = \frac{P}{4\pi K} \text{Erfc} \left( \frac{\vec{r} - \vec{r}_0}{\sqrt{4Dt}} \right) \quad 0 \leq t \leq \tau \] (2a)

\[ T(\vec{r},t) = \frac{P}{4\pi K} \left\{ \text{Erfc} \left( \frac{\vec{r} - \vec{r}_0}{\sqrt{4Dt}} \right) - \text{Erfc} \left( \frac{|\vec{r} - \vec{r}_0|}{\sqrt{4D(t-\tau)}} \right) \right\} \quad \tau < t \] (2b)

We finally take into account the spatial extension of the heat source and consider that the sample is semi-infinite. We do so by integrating expressions 2a and 2b assuming a constant flux \( Q \) (in order to mimic experimental data taken on calibrated heat sources) over the area \( \Omega \) covered by the heat sources and considering adiabatic conditions at the surface. Under such assumption, we take into consideration the presence of the surface by applying the images method, in which the effect of the surface is equivalent to the effect of reflected images of the heat sources. Then, the expressions of the evolution of the surface temperature distribution are given by:

\[ T(\vec{r}_\alpha,t) = \int_{\Omega} \frac{Q}{2\pi K} \text{Erfc} \left( \frac{\vec{r}_\alpha - \vec{r}}{\sqrt{4Dt}} \right) ds \quad 0 \leq t \leq \tau \] (3a)

\[ T(\vec{r}_\alpha,t) = \int_{\Omega} \frac{Q}{2\pi K} \left\{ \text{Erfc} \left( \frac{\vec{r}_\alpha - \vec{r}}{\sqrt{4Dt}} \right) - \text{Erfc} \left( \frac{\vec{r}_\alpha - \vec{r}}{\sqrt{4D(t-\tau)}} \right) \right\} ds \quad \tau < t \] (3b)

Equations 3a and 3b can represent the image sequence recorded by the camera at instants \( t_i \) separated by the inverse of the frame rate \( f \) of the camera, \( 1/f \). This illustrates that the number of data available in an experiment is rather large (the whole the image sequence recorded during and after the burst). Keeping in mind that the purpose of this work is to use a fast technique to characterize vertical cracks, we also seek for a fast inversion algorithm. Pursuing this goal, we select for the inversion a subset of data representative of the spatial distribution and the temporal evolution of the surface temperature: the thermogram at the end of the burst \( T(\vec{r}_\alpha,\tau) \) and the timing graph at the origin, right on top of the heat source, \( T(\vec{r} = 0,\tau) \). Finally, we normalize the data to the temperature value at the origin, obtained at the end of the burst:

\[ T_r = \frac{T(\vec{r}_\alpha,\tau)}{T(\vec{r} = 0,\tau)} \quad \text{and} \quad T_t = \frac{T(\vec{r} = 0,\tau)}{T(\vec{r} = 0,\tau)} \]  

In order to illustrate qualitatively the sensitivity of this spatial and temporal information to the geometrical parameters of the crack, in Fig. 2 we show natural logarithms of \( OX \) and \( OY \) profiles of \( T_r \) and of \( T_t \) as a function of the normalized time for two heat sources of the same width \( w = 1 \) mm and two heights \( h = 1 \) mm and \( h = 2 \) mm, both buried \( d = 1 \) mm below the surface.

As can be seen, for short bursts, the thermogram \( T_r \) and the timing graph \( T_t \) during the burst are barely sensitive to geometrical parameters of the crack. At long bursts there is not much sensitivity in \( T_r \), but \( T_t \) is rather sensitive and at intermediate bursts both \( T_r \) and \( T_t \) are sensitive. This indicates that the sensitivity of the information we will use for the inversion switches from time to space as the duration of the burst increases. Accordingly, we propose to balance space and time information in order for the quality of the inversions not to be significantly dependent on the burst duration.

Figure 2. Simulations of the surface temperature for two rectangular heat sources of the same width \( w = 1 \) mm and heights \( h = 1 \) mm (solid line) and \( h = 2 \) mm (dashed line), buried \( d = 1 \) mm, calculated for burst durations of \( \tau = 100 \) ms (blue), \( \tau = 2 \) s (green) and \( \tau = 20 \) s (red). In (a) and (b) natural logarithms of the \( T_r \) along axes \( OX \) and \( OY \), respectively. (c) Evolution of \( T_t \) as a function of the normalized time, \( t/\tau \).
3. INVERSE PROBLEM AND INVERSIONS OF SYNTHETIC DATA

3.1. Inverse problem

In the calculation of the surface temperature expressed in Equations 3a and 3b the geometrical parameters of the crack are contained in the integration limits. Hence, in order to perform the calculation, a particular geometry must be assumed. When tackling the inverse problem in a real case, we lack knowledge of the shape of the crack. For this reason, we address the inverse problem by performing the integrations in Equations 3a and 3b in the whole plane $\Pi$ containing the heat source and introducing a position dependent function $Q(r)$ to describe the heat source distribution that we want to retrieve:

$$T(\tilde{r}, t) = \int_{\Pi} \frac{Q(\tilde{r})}{2\pi K} \text{Erfc} \left[ \frac{\mid \tilde{r} \mid}{\sqrt{4Dt}} \right] \, ds, \quad 0 \leq t \leq \tau \quad (4a)$$

$$T(\tilde{r}, t) = \int_{\Pi} \frac{Q(\tilde{r})}{2\pi K} \left( \text{Erfc} \left[ \frac{\mid \tilde{r} \mid}{\sqrt{4Dt}} \right] - \text{Erfc} \left[ \frac{\mid \tilde{r} \mid}{\sqrt{4D(t-t')}} \right] \right) \, ds, \quad \tau < t \quad (4b)$$

Now, $T_r$ and $T_t$ will be calculated according to Equations 4a and 4b, and can be expressed in an operator form as:

$$\begin{bmatrix} A_r \\ \eta A_t \end{bmatrix} Q = \begin{bmatrix} T_r \\ \eta T_t \end{bmatrix}, \quad \text{with} \quad \eta = \sqrt{\frac{\text{#samples } T_r}{\text{#samples } T_t}} \quad (5)$$

where $A_r$ and $A_t$ are the operators that map the heat source distribution into $T_r$ and $T_t$, respectively and $\eta$ is a weighting factor that is introduced in order to balance the spatial and temporal information.

Equation 5 can be used to calculate exact temperatures $T = (T_r, \eta T_t)^T$ for a given heat source distribution. However, in real situations, available information is noisy temperatures $T^\delta = (r T^\delta_r, \eta T^\delta_t)^T$, affected by a noise level $\delta$, defined as $\delta^2 = \| T^\delta - T \|^2$. This implies that in the inverse problem, the calculated temperatures (left-hand side of Equation 5) are only an approximation of the data $T^\delta$. Accordingly, finding the heat source distribution $Q^\delta$ from noisy data consists of finding an approximation of $Q$ that is the function that minimizes of the residual $R$ (discrepancy term) defined as:

$$R^2 = \left\| \begin{bmatrix} A_r \\ \eta A_t \end{bmatrix} Q^\delta - \begin{bmatrix} T_r^\delta \\ \eta T_t^\delta \end{bmatrix} \right\|^2 = \left\| T_{\text{calc}}(Q^\delta) - T^\delta \right\|^2 \quad (6)$$

As mentioned in the introduction, the ill-posed character of the inverse problem makes the naïve inversion unstable. We faced a similar problem when dealing with lock-in vibrothermography, but in that case we were using more information for the inversion: amplitude and phase thermograms obtained at nine modulation frequencies. We stabilized the inversion adding a penalty term to the residual, based on TV functional\textsuperscript{21}. Following this idea, and taking into account that in the burst regime we are using only one thermogram and one timing-graph, we have improved the stabilization by introducing three regularization terms: one based on zero order Tikhonov functional (TK\textsubscript{0})\textsuperscript{22}, one based on Total Variation (TV) and the third one based on the Lasso method\textsuperscript{23} (L1 functional\textsuperscript{24}), defined as:

$$TK_0(Q) = \int_{\Pi} |Q| \, ds \quad TV(Q) = \int_{\Pi} |\nabla Q| \, ds \quad L_1(Q) = \int_{\Pi} |Q| \, ds$$

TK\textsubscript{0} is known to search for smooth functions, TV searches for blocky functions that minimize the blocks’ contours and $L_1$ tends to minimize the area where the function has nonzero values. Each terms is multiplied by a regularization parameter, $\alpha_{TK}, \alpha_{TV}, \text{ and } \alpha_1$, respectively, all with the same initial value $\alpha$, that we reduce in each iteration $i$. The final function to be minimized is the combination of the three regularization terms plus the discrepancy term expressed in Equation (6):

$$R^2 = \alpha_{TK} \cdot TK_0(Q^\delta, \omega') + \alpha_{TV} \cdot TV(Q^\delta, \omega') + \alpha_1 \cdot L_1(Q^\delta, \omega') + \left\| T_{\text{calc}}(Q^\delta, \omega') - T^\delta \right\|^2$$

\[ \text{Proc. of SPIE Vol. 9861 98610E-4} \]
Of course, the introduction of the penalty terms affects the resulting heat source distribution $Q^{\delta\alpha r}$, with $\bar{\alpha} = (a_{\alpha r}, a_{\alpha v}, \alpha_i)$. Concerning the evolution of the regularization parameters, there is a lack of theoretical results establishing the optimum reduction rates. We have performed many inversions with synthetic data seeking for the optimum decay rates and we have selected the following values which provide robustness to the inversion method:

$$
\alpha_{\alpha r} = 0.01^{-1} \alpha \quad \alpha_{\alpha v} = 0.6^{-1} \alpha \quad \alpha_i = 0.7^{-1} \alpha
$$

Finally, we stop the iteration process when the discrepancy term defined in Equation 6 reaches the noise level in the data, which is known as the stopping Morozov’s criterion.$^{25}$

### 3.2. Inversion of synthetic data

In the following, we analyze the performance of the inversion algorithm by inverting synthetic $T_r$ and $T_v$ data calculated for a standard 1 mm$^2$ square heat source ($w = h = 1$ mm) buried at different depths in a material with the thermal properties of AISI 304 ($K = 16$ W/mK, $D = 4$ mm$^2$/s). In particular, we analyze the effect of the burst duration and the noise in the data on the quality of the reconstruction.

In order to illustrate the method, in Figure 3a and 3c (symbols) we show $\ln T_r$ and $\ln T_v$, respectively, corresponding to the standard $w = h = 1$ mm square, buried $d = 1$ mm below the surface, excited by a $\tau = 0.5$ s burst. Data are affected by a 5% noise level. The fittings corresponding to the recovered crack are shown in Figures 3b and 3c (solid line).

![Graphs showing natural logarithms of normalized noisy synthetic thermograms](image)

**Figure 3.** (a) Natural logarithm of the normalized noisy synthetic thermogram calculated at the end of a $\tau = 0.5$ s burst and (b) fitted thermogram of the recovered crack, corresponding to a square heat source ($w = h = 1$ mm) buried $d = 1$ mm below the surface. (c) Natural logarithms of the normalized noisy (symbols) and fitted (solid line) timing-graph. The noise in the data is 5%.

In Figure 4 (upper left) we show a black and white representation of the recovered heat source distribution in the search plane $x = 0$, white being maximum heat flux and black absence of heat sources. The real contour is depicted with a red line.

First, we analyze the effect of the burst duration on the quality of the reconstructions. To this purpose we have inverted synthetic data generated for the standard $w = h = 1$ mm, buried at different depths between 0.1 and 8 mm, excited with bursts ranging from $\tau = 0.5$ to $8$ s. The noise in the data is 5% in all cases. As an example, in Figure 4 we show reconstructions for $d = 1$ and 8 mm, and $\tau = 0.5, 4,$ and $8$ s.
Figure 4. Black and white representation of the reconstructions obtained from synthetic data with 5% noise corresponding to squares \((w = h = 1 \text{ mm})\) buried at depths of \(d = 1\) and 8 mm. Burst durations of \(\tau = 0.5, 4\) and 8 s are analyzed. The real contours are depicted in red.

As can be observed, the duration of the burst does not significantly affect the quality of the reconstructions, although the retrieved heat source distribution of the deep heat sources excited with long bursts look slightly wider than the real heat source. On the other hand, for deep heat sources the (normalized) instant at which the timing-graph starts being sensitive enough to reveal geometrical differences, is delayed as the burst is reduced (see Figure 2c). This limits the shortest usable burst, which, in real experimental situations, can be even further limited by the signal level. Anyway, the reconstructions in Figure 4 are very accurate and prove the robustness of the method for burst durations typically used in experimental situations. It must be noted, however, that having experimental data corresponding to 1 mm\(^2\) heat sources buried beyond 5 mm with a noise level of 5%, as the one represented in the lower part of Figure 4, is rather idealistic. For this reason, we have analyzed the quality of the reconstructions as a function of the depth of the standard heat source, considering that data are affected by three different noise levels, namely, 5%, 10%, and 15%. In Figure 5 we show the results we have obtained for \(d = 0.5, 2,\) and 8 mm with a \(\tau = 1\) s burst. As can be observed, the quality of the reconstructions is reduced as the noise level in the data increases, the degradation being more remarkable for increasing depths. If we define an accurate reconstruction as one whose area does not exceed 1.5 times the real area, we can say that, with a noise level of 10% (15%), the maximum depth is limited to about 2 mm (0.5 mm).

Figure 5. Black and white representation of the reconstructions obtained from synthetic data calculated for a \(\tau = 1\) s burst, corresponding to squares \((w = h = 1 \text{ mm})\) buried at different depths \((d = 0.5, 2,\) and 8 mm). Three noise levels are analyzed: (a) 5%, (b) 10%, and (c) 15%. The real contours are depicted in red.
4. EXPERIMENTS AND INVERSIONS OF EXPERIMENTAL DATA

We have also checked the performance of the algorithm by inverting synthetic data on samples containing calibrated heat sources. The samples consist of two AISI 304 stainless steel parts with a flat common surface. We put in between a 38 $\mu$m thick copper slab of known dimensions and we press the two parts with screws. In Fig. 6 we show a sketch of the samples. When we launch the ultrasounds, the rubbing between the copper slab and the steel surfaces provides the calibrated heat source. In order to avoid any contact between the two steel parts we put two more copper slabs at the back side of the sample, so that they do not disturb the temperature distribution at the measuring surface.

We excite the samples using an UTvis ultrasound equipment from EDEVIS that works in the 15 to 25 kHz range. We select the optimum ultrasound frequency (about 23 kHz) by performing frequency sweeps at constant ultrasound intensity. A thin Al tape is used as coupling material between the sonotrode and the sample, whose surface is covered with black paint to improve IR emissivity. We use ultrasound powers ranging between 200 and 600 W depending on the burst duration and depth of the heat source. The typical maximum temperature rises range between 1 and 5 K. The infrared radiation coming from the sample is captured with an IR camera (JADE J550M from Cedip) working in the 3.5 - 5 $\mu$m range, provided with a 50 mm focal length lens. The camera is placed at the minimum working distance from the sample (about 35 cm). At this distance each pixel in the camera averages the radiation coming from a 135 $\mu$m side square in the sample.

We have taken data on samples containing a square copper slab ($w = h = 1.5$ mm) buried at three depths $d = 0.1, 0.57, 2$ and 3.1 mm and excited with burst $\tau = 0.5, 1, 2, 4$ and 6 s. As an example, in Figure 7 we show $\text{Ln}T_r$ and $\text{Ln}T_t$ corresponding to $d = 0.57$ mm and $\tau = 1$ s.

In Figure 8 we show the reconstructions we have obtained for $d = 0.57$ and 3.1 mm. As predicted in section 3, the results show that the duration of the burst do not significantly affect the quality of the reconstructions. The area and depth of the true copper squares are well retrieved down to depths of 3 mm.
Finally, in Figure 9 we show reconstructions of experimental data obtained with copper slabs of a different geometry: on semicircles \((R = 1.4 \text{ mm})\), buried \(d = 2.9\) and \(6 \text{ mm}\), using \(\tau = 2 \text{ s}\). As can be seen, the reconstructions from experimental data retrieve the area and depth of the heat sources quite accurately down to \(6 \text{ mm}\).

It must be mentioned that, in order to apply the Morozov criterion, a very accurate estimation of the noise in experimental data is needed. The iteration processes corresponding to the experimental reconstructions shown in Figs. 8 and 9 were not stopped using the Morozov criterion. They correspond to the iteration in which we find the best reconstruction, so they represent the best reconstruction that can be found from each data set. We are currently working using “control data” that are not introduced in the inversion algorithm, to stop the iterations. Anyway, the experimental reconstructions we have presented show that the method is able to characterize the area and location of vertical cracks accurately in a fast way: data taking and inversion procedure take about 2 minutes.

5. SUMMARY AND CONCLUSIONS

Burst vibrothermography has been used to characterize vertical cracks. We have calculated the evolution of the surface temperature distribution produced by a vertical planar heat source emitting a homogeneous and constant flux during a time interval \(\tau\). The inversion algorithm has been stabilized with a combination of \(TK_0\), \(TV\) and \(L_1\)
functionals to retrieve the size and depth of the crack from the thermogram obtained at the end of the burst and the timing-graph corresponding to the central pixel with knowledge of the plane containing the crack as the only prior information used.

The performance of the algorithm was analyzed by inverting noisy synthetic corresponding to $1 \times 1 \text{mm}^2$ heat sources, buried at different depths below the surface (down to 8 mm). The analysis of the dependence of the quality of the reconstruction on the burst duration shows that for typical experimental situations (bursts between $\tau = 0.5$ and 8 s), the reconstructions barely depend on the duration of the bursts down to depths of 8 mm, which is a demonstration of the robustness of the method. The maximum depth for which accurate reconstructions can be obtained depends on the noise in the data: 8 mm for data affected by 5%, 2 mm for 10%, and 0.5 mm for noise levels of 15%. Iterations were stopped automatically using the Morozov criterion.

Inversions of experimental data taken on samples containing calibrated heat sources show that it is possible to characterize calibrated heat sources down to 6 mm below the surface. Further work is needed to implement an automatic stopping criterion to experimental data such as the Morozov criterion. It is important to stress that the whole process (data taking and inversion) take at most 2 minute.

We think that these results are very promising regarding the possibility of characterizing real vertical cracks in a very fast manner.

ACKNOWLEDGMENTS

This work has been supported by the Ministerio de Ciencia e Innovación (MTM2013-40842-P), by Gobierno Vasco (IT619-13), by Diputación General de Aragón (Grupo Consolidado PDIE), by University of the Basque Country (UPV/EHU UFI11/55), by Instituto Politécnico Nacional CICATA-Legaria and by CONACYT Becas Mixtas Nacionales.

REFERENCES


