$\mathbf{F}_{msy}$ as the Golden Rule of Fishery Harvesting*

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ABSTRACT: The FAO Fishery Glossary defines F_{msy} as "*the fishing mortality rate which, if applied constantly, would result in Maximum Sustainable Yield*". The objective of this paper is to characterize F_{msy} as the golden rule of fishery harvesting. In particular we formalize F_{msy} as the solution of an optimal problem that maximizes the stationary yield of a fishery which is modelled considering that: *i)* the fish population is structured by ages and *ii)* captures and fishing effort are related through the Baranov catch equation (1918).

Key Words: Age-structured fishery models, fishery harvesting, reference points, F_{msy}, Baranov catch equation.

JEL Classification: Q22.

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1 Introduction

The Maximum Sustainable Yield (MSY) is a concept that has been extensively applied in fishery economics mostly using biomass (or surplus production) models. These models are based on two main characteristics: *i)* the population size and the growth of the fishery are specified in terms of the total biomass, and *ii)* production is considered as a linear function of fishing effort. Gordon (1954) and Schaefer (1957) pioneered this approach and many economic issues have been successfully analyzed with this framework[1].

However, many management decisions are based on biological reference points that require more complex biological data than just biomass[2]. F_{msy} is one of the reference points used by management agencies to regulate many fisheries. For instance the International Council of Exploitation of the Seas (ICES) estimates F_{msy} for most European fisheries.

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This mathematical formalization could help to extend some contributions to fishery economics which are based on biomass approach to models that explicitly consider the age-structured behind the dynamics of fish populations. Furthermore, these theoretical results can easily be implemented empirically because fishery agencies have estimations about the age-structured model used.

The paper proceeds as follows. Section 2 shows the main features of the age-structured model that we use. In Section 3 we characterize F_{msy} as the

[1] Most of these advances appears in the seminal book by Clark (1976, 1990).

[2] Caddy and Mahon (1995) conduct an extensive survey on reference points used in fisheries management.

[3] Virtual population analysis is a general method for fish stock assessment that it was introduced by Gulland (1965) based on older works. It is now widely used. For instance the USA and Canada use the Adaptive Framework (ADAPT) which is a variety of VPA based on minimizing the sum of squares over any number of indices of abundance to find best fit parameters. However the European Commission uses Extended Survivor Analysis (XSA), a method that does not include biomass indices for fitting the VPA. See Lassen and Medley (2000) for an extensive survey of VPA.

fishing rate that maximizes the fishery stationary yield, taking into account the age-structured model. Section 3 concludes the article.

2 Model Features

We consider an age-structured model which is commonly used in Virtual Population Analysis for stock assessment.

Assume that the fish stock is broken down into A cohorts. That is in each period t, there are $A-1$ initial old cohorts and a new cohort is born. Let z_t^a be the mortality rate that affects the population of fish of a^{th} age in the t^{th} period. This mortality rate can be decomposed into natural mortality (non human component), m^a, and mortality due to fishing, $p^a F_t$,

$$z_t^a = p^a F_t + m^a,$$

where p^a is the selectivity parameter of age a.

The stock dynamics are determined by

$$N_{t+1}^{a+1} = e^{-z_t^a} N_t^a,$$

$$N_{t+1}^1 = \overline{N^1},$$

where N_t^a is the number of fish of a^{th} age at the beginning of the t^{th} period.

The size of a new cohort (recruitment), N_{t+1}^1 , depends on the spawning stock biomass of the previous year, SSB_t,

$$N_{t+1}^1 = \Psi(SSB_t),$$

where Ψ denotes the stock recruits $(S-R)$ relationship. Furthermore, the spawning stock biomass, SSB, is a function of the stock weight distribution, ω, and the maturity fraction, μ, of each age,

$$SSB_t = \sum_{a=1}^{A} \mu^a \omega^a N_t^a.$$

Finally, the fishing yield for age a is given by Baranov's equation (1918)

$$Y_t^a = \omega^a \frac{p^a F_t}{z_t^a}(1 - e^{-z_t^a}) N_t^a.$$

In this biological model a stationary path of fishing mortality, $F = F_t = F_{t-1}$, generates a **stationary age structured population** characterized by

1. The stock population for each age $a = 1, ...A$, is given by

$$N^a = N^1 \phi^a(F),$$

where

$$\phi^a(F) = \begin{cases} 1 & \text{for } a = 1, \\ \Pi_{i=1}^{a-1} e^{-p^i F - m^i} & \text{for } a = 2, ...A, \end{cases}$$

can be interpreted as the accumulated probability of a recruit reaching age a for that stationary fishing mortality rate F.

2. The stationary recruit population N^1 satisfies the stationary $S - R$ relationship, that is

$$N^1 = \Psi\left(\sum_{a=1}^{A} \mu^a \omega^a \phi^a(F) N^1\right).$$

In order to guarantee the existence of a unique stationary population associated with a stationary fishing mortality, F, it is necessary that the $S - R$ relationship be invertible. That is,

$$\frac{\Psi^{-1}[N^1]}{N^1} = \sum_{a=1}^{A} \mu^a \omega^a \phi^a(F).$$

3 F_{msy} as the solution of an optimization problem

F_{msy} is the fishing mortality target of many regional and national fishery management authorities. The FAO Fishery Glossary defines F_{msy} as the fishing mortality rate which, if applied constantly, would result in Maximum Sustainable Yield.

Among all the fishing mortality stationary paths, F_{msy} can be formally defined as the fishing mortality rate where the stationary yield is at its maximum. That is, F_{msy} is the mortality rate that maximizes

$$\max_{\{F,N^1\}} \sum_{a=1}^{A} Y^a = \sum_{a=1}^{A} y^a(F)\phi^a(F)N^1, \tag{1}$$

$$s.t. \quad N^1 = \Psi\left(N^1 \sum_{a=1}^{A} \mu^a \omega^a \phi^a(F)\right),$$

where $y^a(F) = \omega^a \frac{p^a F}{p^a F + m} \left(1 - e^{-p^a F - m^a}\right)$.

The solution of problem (1) is characterized by the following first order conditions,

$$\sum_{a=1}^{A} \frac{\partial y^a(F)}{\partial F} \phi^a(F) N^1 + \sum_{a=1}^{A} y^a(F) \frac{\partial \phi^a(F)}{\partial F} N^1 + \lambda \Psi'(.) N^1 \sum_{a=1}^{A} \mu^a \omega^a \frac{\partial \phi^a(F)}{\partial F} = 0, \tag{2}$$

$$\sum_{a=1}^{A} y^a(F) \phi^a(F) - \lambda [1 - \Psi'(.) \sum_{a=1}^{A} \mu^a \omega^a \phi^a(F)] = 0, \tag{3}$$

$$N^1 = \Psi \left(N^1 \sum_{a=1}^{A} \mu^a \omega^a \phi^a(F) \right), \tag{4}$$

where $\Psi'(.)$ is valued at $N^1 \sum_{a=1}^{A} \mu^a \omega^a \phi^a(F)$ and λ is the associated Lagrange multiplier that measures the impact of new recruits on the total yield.

Substituting equation (3) into (2) and taking into account total differentiation of (4), the following expression is obtained,

$$\sum_{a=1}^{A} \frac{\partial y^a(F)}{\partial F} \phi^a(F) + \sum_{a=1}^{A} y^a(F) \frac{\partial \phi^a(F)}{\partial F} + \frac{1}{N^1(F)} \frac{\partial N^1(F)}{\partial F} \sum_{a=1}^{A} y^a(F) \phi^a(F) = 0, \tag{5}$$

where

$$\begin{aligned}
\frac{\partial y^a(F)}{\partial F} &= \omega^a \left\{ \frac{p^a m^a}{(p^a F + m^a)^2} \left(1 - e^{-p^a F - m^a}\right) + \frac{{p^a}^2 F}{p^a F + m^a} e^{-p^a F - m^a} \right\} > 0, \\
\frac{\partial \phi^a(F)}{\partial F} &= \begin{cases} 0 & \text{for } a = 1, \\ \sum_{i=1}^{a-1} (-p^i) \phi^a(F) < 0 & \text{for } a = 2, ...A, \end{cases} \\
\frac{\partial N^1(F)}{\partial F} &= \frac{N^1 \Psi' \sum_{a=1}^{A} \mu^a \omega^a \frac{\partial \phi^a(F)}{\partial F}}{1 - \Psi' \sum_{a=1}^{A} \mu^a \omega^a \phi^a(F)} \lesseqgtr 0.
\end{aligned}$$

Equation (5) determines F_{msy} and its interpretation is clear. Variations in the fishing mortality rate affect the stationary yield in three ways: through weighted catches (first sum), through the accumulated probability of a recruit reaching age a (second sum) and through the number of recruits (third sum). F_{msy} is chosen so that the three sources of variations cancel out.

Notice that an increase of the mortality rate has an ambiguous effect on the stationary recruits. In particular, this effect is negative (positive) whenever an increase in the number of recruits leads to a lower (larger) increase in the SSB.

The following statement characterizes the existence and uniqueness of F_{msy}.

Proposition 1 *If i)* $\sum_{1=2}^{A} y^a(F)\phi^a(F)N^1$ *is concave in* F *and ii) For each* F *there exists a unique* N^1*, that satisfies* $N^1 = \Psi(\sum_{a=1}^{A} \mu^a \omega^a \phi^a(F)N^1)$*, then there exists a unique* F_{msy} *that solves the maximization problem (1)*

Proof. Condition *ii)* guarantees that the set of F over which the objective function is maximized is compact. Since the objective function is continuous, the maximization problem has at least one solution. Furthermore, since $\sum_{1=2}^{A} y^a(F)\phi^a(F)N^1$ is concave, the maximum is unique.■

4 Conclusions

The Maximum Sustainable Yield (MSY) is a concept that has been extensively applied in fishery economics mostly using biomass (or surplus production) models. These models are based on two main characteristics: *i)* the population size and the growth are specified in terms of the total biomass, and *ii)* production is considered a linear function of fishing effort. Many economic issues have been successfully analyzed with this approach.

However, many management decisions are based on biological reference points that require more complex biological data. F_{msy} is one of these reference points used by management agencies to regulate the fisheries. In particular, F_{msy} is defined as the fishing mortality rate which, if applied constantly, would result in MSY.

The objective of this paper is to characterize F_{msy} as the solution of an optimal problem that maximizes the yield of a fishery taking into account that the fish population is structured by age rather than considered as a total biomass. This mathematical formalization could help to extend some contributions to fishery economics which are based on biomass models that abstract from the age-structure behind the dynamics of fish populations.

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