

Lattice Computing in Hybrid Intelligent Systems

Manuel Graña

Computational Intelligence Group, UPV/EHU

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Summary of the talk

- Introduce Lattice Computing paradigm
- Focus on Lattice Autoassociative Memories
- Applications involving hybridization
 - Hyperspectral image unmixing
 - Face recognition
 - MRI classification
 - fMRI processing
 - Multivariate Mathematical Morphology
 - Hyperspectral image
 - brain networks on resting state fMRI

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 - Lattice Computing
 - Lattice Computing Approaches
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 - LAAM definitions and properties
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 - Diffusion MRI data classification
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 - Application resting state fMRI
 - Spatial-spectral classification
- 4 Concluding remarks

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Lattice Computing

Definition

Lattice Computing is the class of algorithms built on the basis of Lattice Theory.

- define computations in the ring of the real valued spaces endowed with some (inf, sup) lattice operators ($\mathbb{R}^n, \vee, \wedge, +$),
- or use lattice theory to produce generalizations or fusions of conventional approaches.

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Mathematical Morphology

- Classical application of lattice theory to signal and image processing
- Filtering and detection
 - Erosion and dilation operators corresponding to infimum and supremum
 - non-linear convolution-like processes with structural elements
 - Opening and closing basic filters
 - segmentation by morphological gradient and watershed
 - detection by top-hat, hit-and-miss

Formal Concept Analysis

- Application of lattice theory to semantic analysis
- Ontology induction from data
 - intensional (attributes) and extensional (instances) representation of concepts
 - building the lattice induced by the partial order of concepts

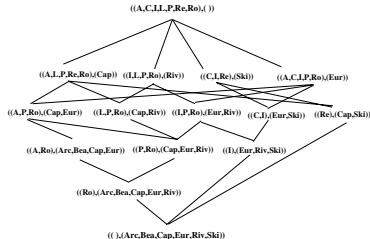


Fig. 1. Concept Lattice of the *European Cities* context.

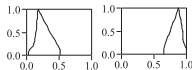
Lattice Associative Memories

- Building learning algorithms with morphological operators
- Associative Memories
 - Store and recall patterns
 - Dual memories from infimum and supremum operators
 - Nice properties:
 - infinite storage capacity of real valued patterns
 - robustness against erosive/dilative noise
 - not-nice: sensitivity to general additive noise

Kaburlasos' Lattice Interval Numbers

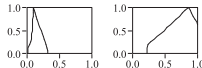
- A new general data type: Intervals Numbers
 - many conventional data types can be mapped into IN
 - the valuation function allows to define error measures
 - define the variations of conventional learning algorithms
 - generalization of Fuzzy-ART
 - lattice Self Organizing Map

(a) IF x_1 is and x_2 is THEN



$$y = 0.13 - 0.66x_1 + 0.19x_2$$

(b) IF x_1 is and x_2 is THEN



$$y = -0.10 - 1.04x_1 + 0.50x_2$$

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LAAM definitions

- LAAMs are auto-associative neural networks
 - neuron functional activations built on morphological (lattice) operations.
- LAAMs present interesting properties such as perfect recall, unlimited storage and one-step convergence.
- Proposed by Ritter et al.^{1 2}
- We found applications besides image storage and retrieval

¹G. X. Ritter, P. Sussner, and J. L. Diaz-de Leon. Morphological associative memories. Neural Networks, IEEE Transactions on, 9(2):281–293, 1998.

²G. X. Ritter, J. L. Diaz-de Leon, and P. Sussner. Morphological bidirectional associative memories. Neural Networks, 12(6):851–867, 1999.

LAAM definitions

- input/output pairs of patterns

$$(X, Y) = \left\{ \left(\mathbf{x}^{\xi}, \mathbf{y}^{\xi} \right) ; \xi = 1, \dots, k \right\}$$

- a linear heteroassociative neural network

$$W = \sum_{\xi} \mathbf{y}^{\xi} \cdot \left(\mathbf{x}^{\xi} \right)'.$$

- erosive and dilative LAMs, respectively

$$W_{XY} = \bigwedge_{\xi=1}^k \left[\mathbf{y}^{\xi} \times \left(-\mathbf{x}^{\xi} \right)' \right] \text{ and } M_{XY} = \bigvee_{\xi=1}^k \left[\mathbf{y}^{\xi} \times \left(-\mathbf{x}^{\xi} \right)' \right],$$

where \times is any of the \boxtimes or \boxdot operators,

LAAM definitions

- operator \boxplus denotes the max matrix product

$$C = A \boxplus B = [c_{ij}] \Leftrightarrow c_{ij} = \bigvee_{k=1..n} \{a_{ik} + b_{kj}\},$$

- operator \boxminus denotes the min matrix product

$$C = A \boxminus B = [c_{ij}] \Leftrightarrow c_{ij} = \bigwedge_{k=1..n} \{a_{ik} + b_{kj}\}.$$

LAAM definitions and properties

Definition

When $X = Y$ then W_{XX} and M_{XX} are called Lattice Auto-Associative Memories (LAAMs).

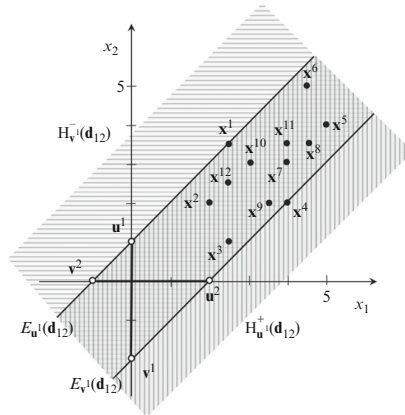
- perfect recall for an unlimited number of real-valued stored patterns

$$W_{XX} \boxtimes X = X = M_{XX} \boxtimes X$$

- convergence in one step for any input pattern
 - if $W_{XX} \boxtimes \mathbf{z} = \mathbf{v}$ then $W_{XX} \boxtimes \mathbf{v} = \mathbf{v}$
 - if $M_{XX} \boxtimes \mathbf{z} = \mathbf{u}$ then $M_{XX} \boxtimes \mathbf{u} = \mathbf{u}$.

Fixed points of M_{XX} and W_{XX}^a

^aG.X.Ritter,G.Urcid,“Lattice algebra approach to endmember determination in hyperspectral imagery,” in P. Hawkes (Ed.), Advances in imaging and electron physics, Vol. 160, 113–169. Elsevier, Burlington, MA (2010)



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Linear Mixing Model

- Linear Mixing Model (LMM):

$$\mathbf{x} = \sum_{i=1}^M a_i \mathbf{e}_i + \mathbf{w} = \mathbf{E}\mathbf{a} + \mathbf{w}, \quad (1)$$

\mathbf{x} is the d -dimension measured vector,

- \mathbf{E} is the $d \times M$ matrix whose columns are the d -dimension *endmembers* $\mathbf{e}_i, i = 1, \dots, M$,
 - defining a convex region covering the measured data.
- \mathbf{a} is the M -dimension abundance vector, and
 - abundance coefficients must be non-negative
 $a_i \geq 0, i = 1, \dots, M$,
 - fully additive to 1: $\sum_{i=1}^M a_i = 1$.
- \mathbf{w} is the d -dimension additive observation noise vector.

Endmember induction Algorithm

Definition

Endmember Induction algorithms (EIA) induce the set of endmembers E from the data X

- Types of EIA
 - Geometric: searching for simplex covering
 - Algebraic (PCA, ICA, NNMF)
 - Lattice computing: lattice independence equivalence to affine independence

Ritter's EIA

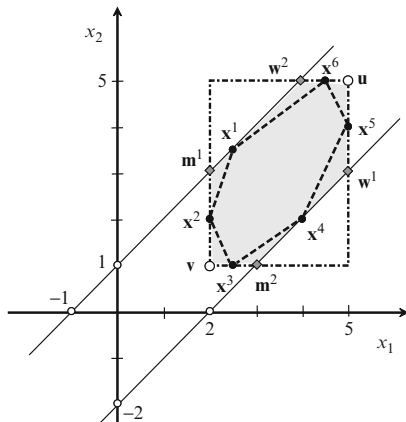
Algorithm 2 Endmember Threshold Selection Algorithm (ETSA) based on [27,28]

- (1) Given a set of vectors $X = \{\mathbf{x}^1, \dots, \mathbf{x}^k\} \subset \mathbb{R}^n$ compute the min and max auto-associative memories W_{XX} M_{XX} from the data. Their column vector sets W and M will be the candidate endmembers.
- (2) Register W and M relative to the data set adding the maximum and minimum values of the data dimensions (bands in the hyperspectral image). Obtain \overline{W} and \overline{M} as follows:
 - (a) Compute $u_i = \bigvee_{\xi=1}^n x_i^\xi$ and $v_i = \bigwedge_{\xi=1}^n x_i^\xi$.
 - (b) Compute $\overline{\mathbf{m}}^i = \mathbf{m}^i + v_i$ and $\overline{\mathbf{w}}^i = \mathbf{w}^i + u_i$
- (3) Remove lattice dependent vectors from the joint set $\overline{W} \cup \overline{M}$.
- (4) Compute the standard deviation along each dimension of the candidate end-member vectors, denoted by the vector $\vec{\sigma} = \{\sigma_1, \dots, \sigma_n\}$.
- (5) Assume the first vector in the set $\mathbf{v}_1 \in \overline{W} \cup \overline{M}$ as the first endmember, $E = \{\mathbf{v}_1\}$
- (6) Iterate for the remaining vectors $\mathbf{v} \in \overline{W} \cup \overline{M}$
 - (a) If $\|\mathbf{v} - \mathbf{e}\| < \gamma \vec{\sigma}$ for any $\mathbf{e} \in E$ then discard \mathbf{v} otherwise include \mathbf{v} in E

Figure : A realization of Ritter's EIA

Convex Polytope from Ritter's EIA^a

^aG.X.Ritter,G.Urcid,“Lattice algebra approach to endmember determination in hyperspectral imagery,” in P. Hawkes (Ed.), Advances in imaging and electron physics, Vol. 160, 113–169. Elsevier, Burlington, MA (2010)



Ritter's EIA endmembers in RGB images^a

^aG. Urcid, JC Valdiviezo-N, GX Ritter, Lattice algebra approach to color image segmentation, JMIV 2012

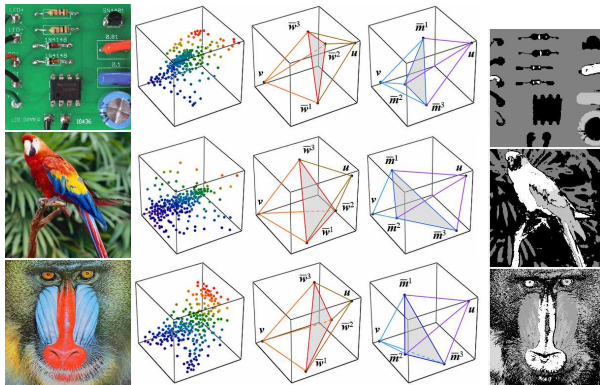


Figure 1. Color images from Ritter's EIA in RGB images

Graña's EIA^a

^aM. Granña, I. Villaverde, J.O. Maldonado, C. Hernandez, Two lattice computing approaches for the unsupervised segmentation of hyperspectral images, Neurocomputing 72 (2009) 2111–2120

Algorithm 3 Endmember Induction Heuristic Algorithm (EIHA)

- (1) Shift the data sample to zero mean
 $\{\mathbf{f}^c(i) = \mathbf{f}(i) - \vec{\mu}; i = 1, \dots, n\}.$
 - (2) Initialize the set of vertices $E = \{\mathbf{e}_1\}$ with a randomly picked sample. Initialize the set of lattice independent binary signatures $X = \{\mathbf{x}_1\} = \{(e_k^1 > 0; k = 1, \dots, d)\}$
 - (3) Construct the AMM's based on the lattice independent binary signatures: M_{XX} and W_{XX} .
 - (4) For each pixel $\mathbf{f}^c(i)$
 - (a) compute the noise corrections sign vectors $\mathbf{f}^+(i) = (\mathbf{f}^c(i) + \alpha \vec{\sigma} > 0)$ and $\mathbf{f}^-(i) = (\mathbf{f}^c(i) - \alpha \vec{\sigma} > 0)$
 - (b) compute $y^+ = M_{XX} \boxtimes \mathbf{f}^+(i)$
 - (c) compute $y^- = W_{XX} \boxtimes \mathbf{f}^-(i)$
 - (d) if $y^+ \notin X$ or $y^- \notin X$ then $\mathbf{f}^c(i)$ is a new vertex to be added to E , execute once 3 with the new E and resume the exploration of the data sample.
 - (e) if $y^+ \in X$ and $\mathbf{f}^c(i) > \mathbf{e}_{y^+}$ the pixel spectral signature is more extreme than the stored vertex, then substitute \mathbf{e}_{y^+} with $\mathbf{f}^c(i)$.
 - (f) if $y^- \in X$ and $\mathbf{f}^c(i) < \mathbf{e}_{y^-}$ the new data point is more extreme than the stored vertex, then substitute \mathbf{e}_{y^-} with $\mathbf{f}^c(i)$.
 - (5) The final set of endmembers is the set of original data vectors $\mathbf{f}(i)$ corresponding to the sign vectors selected as members of E .
-

Hyperspectral images

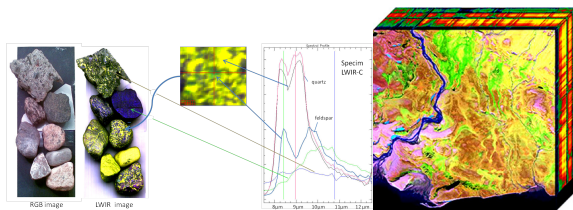
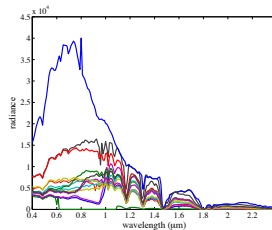


Figure : Hyperspectral imaging, source: wikipedia

Hyperspectral image unmixing



(a)



(b)

Figure : (a) patch of washington dc image, (c) EIHA endmembers

Hyperspectral image unmixing

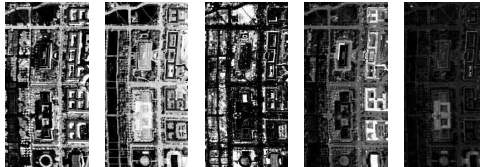


Figure : LSU abundances from Whashington DC patch

Lattice Independent Component Analysis (LICA)

- A non-linear version of Independent Component Analysis
 - Statistical Independence – $>$ Lattice independence
 - Endmembers == Lattice Independent sources
 - Abundance computation == feature extraction

LICA

Algorithm 1 LICA feature extraction .

1. Given training data matrix

$$X_{TR} = \{\mathbf{x}_j; j = 1, \dots, m\} \in \mathbb{R}^{N \times m}$$

and testing data matrix

$$X_{TE} = \{\mathbf{x}_j; j = 1, \dots, m/3\} \in \mathbb{R}^{N \times m/3}$$

2. Apply on X_{TR} an EIA to induce the set of k endmembers

$$E = \{\mathbf{e}_j; j = 1, \dots, k\}$$

3. Unmix train and test data: $A_{TR} = E^\# X_{TR}^T$ and $A_{TE} = E^\# X_{TE}^T$.

Application examples

- Focus on recent works in our reasearch group
- LICA applications
 - Face recognition: feature extraction
 - DWI data classification Alzheimer's Disease
- Multivariate Mathematical Morphology
 - resting state fMRI processing
 - hyperspectral image spectral-spatial classification

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Face recognition

- 1st Experiment comparing LICA with PCA, ICA, LDA³
- Classification by Extreme Learning Machines, Random Forest and SVM
- Four unbalanced face databases from the FERET database

³Ion Marques, Manuel Graña, Face recognition with Lattice Independent Component Analysis and Extreme Learning Machines Soft Computing, 2012, Volume 16, Issue 9, pp 1525-1537

Face data processing pipeline

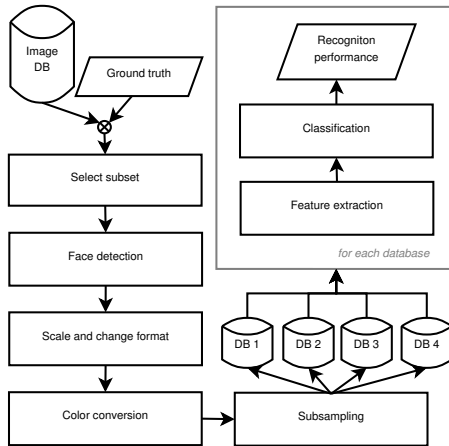


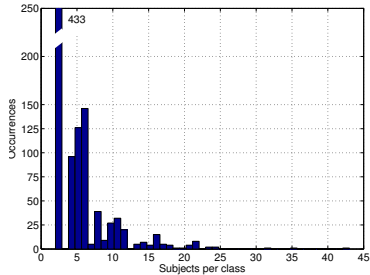
Figure 1

Face sample



Figure : subject sample

Databases



	DB 1	DB 2	DB 3	DB 4
Number of samples	5169	3249	832	347
Number of classes	994	635	265	79
Mean (samples per class)	4.3924	3.1396	5.2835	5.2002
Standard deviation (samples per class)	5.8560	3.4498	4.9904	4.5012
Median (samples per class)	2	2	4	4
Mode (samples per class)	2	2	2	2

Figure : features of the face databases in the experiment

Face detection

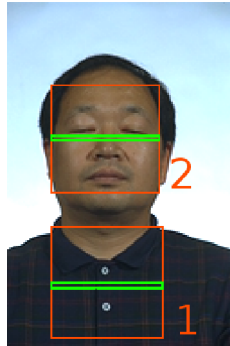


Figure : Face detection candidates by Viola's algorithm, source: SciLab, SIVP toolbox

Face bases



Figure : Rows: Instances of 5 basis from ICA Infomax, ICA Molguez & Schuster, LICA, PCA

Face recognition results

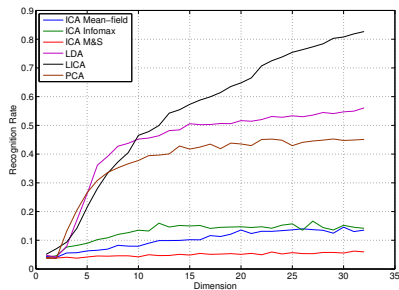
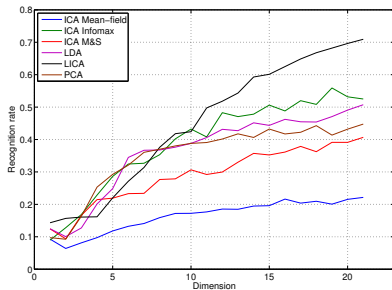


Figure : face recognition results on databases of increasing size

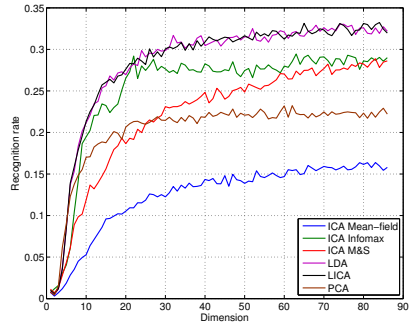
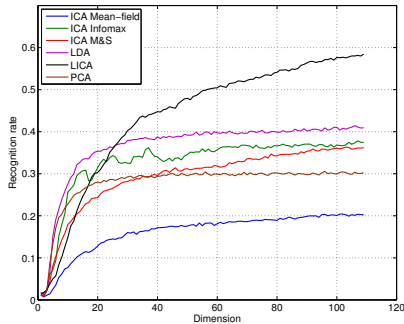


Figure : face recognition results cont.

Fusion of features

- The 2nd experiment performs the fusion of features obtained by LICA and linear algorithms⁴
- Classification by ELM
- Four different databases tested
- conclusion: LICA-fusion enhances the linear features

⁴Ion Marques, Manuel Graña Fusion of lattice independent and linear features improving face identification Neurocomputing (in press)

Fusion pipeline

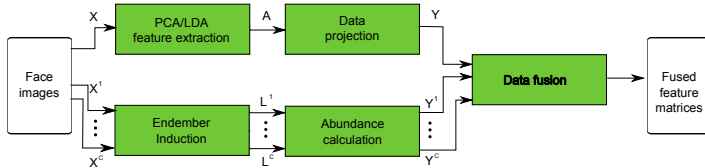


Figure : Pipeline of LICA and linear feature fusion

Feature fusion

- dataset matrix X :
$$X = \{\mathbf{x}_i^c; i = 1, \dots, n; c \in \{1, 2, \dots, C\}\} \in \mathbb{R}^{n \times N},$$
- dataset restricted to class c :
$$X^c = \{\mathbf{x}_j^c \in X; j = 1, \dots, M\} \in \mathbb{R}^{M \times N},$$
 - class restricted abundance matrix: $A^c = (E^c)^\# X^{cT},$
- data features obtained by linear features $Y = \Phi X^T$

Feature fusion (cont.)

- class restricted abundance coefficients

$$A^c = \{\mathbf{a}_i^c; i = 1, \dots, M\} \in \mathbb{R}^{M_c \times M}$$

- linear feature matrix

$$Y = \{\mathbf{y}_i^c; i = 1, \dots, n; c \in \{1, 2, \dots, C\}\} \in \mathbb{R}^{d \times n}$$

- the fused i -th feature vector $\mathbf{z}_i^c \in \mathbb{R}^d$ of a face of class c is

$$\mathbf{z}_i^c = \mathbf{a}_{j(i)}^c \parallel [y_{i, M_c+1}^c, \dots, y_{i, d}^c], \quad (2)$$

Face databases

Table : Summary characteristics of the experimental databases.

Name	Number of images	Number of subjects	Variations
AT&T Database of Faces	400	40	Pose, expression, light*
MUCT Face Database	3755	276	Pose, expression, light
PICS (Stirling)	312	36	Pose, expression
Yale Face Database	165	15	Expression, light, glasses

Face feature fusion results

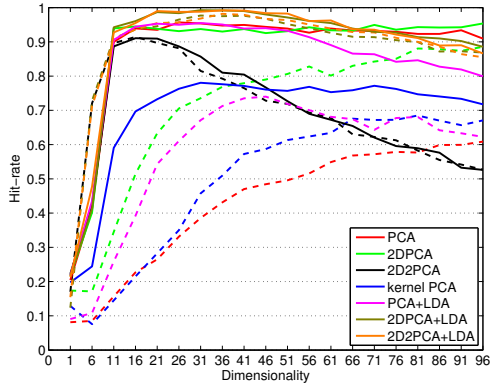


Figure : Recognition rate using ELM classifier for the AT&T database. Dotted lines correspond to standard feature extraction methods. Solid lines correspond to feature fusion approach.

Face feature fusion results (cont.)

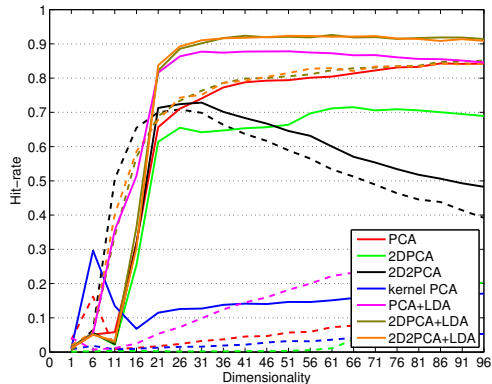


Figure : Recognition rate using ELM classifier for the MUCT database. Dotted lines correspond to standard feature extraction methods. Solid lines correspond to feature fusion approach.

Face feature fusion results (cont.)

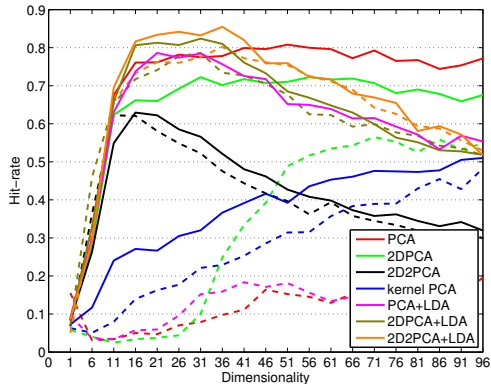


Figure : Recognition rate using ELM classifier for the PICS database. Dotted lines correspond to standard feature extraction methods. Solid lines correspond to feature fusion approach.

Face feature fusion results (cont.)

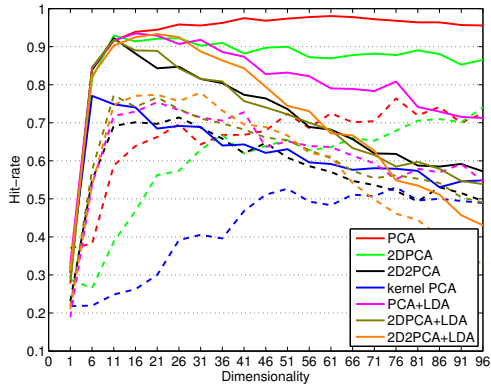


Figure : Recognition rate using ELM classifier for the Yalefaces database. Dotted lines correspond to standard feature extraction methods. Solid lines correspond feature fusion.

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Diffusion MRI data classification

- the experiment's goal is the discrimination of Alzheimer's disease (AD) patients from diffusion MRI data ⁵
- database collected by collaborating clinicians at Hospital Santiago, Vitoria
- Classification by SVM, RVM, 1-NN
- LICA residuals are used for feature selection
 - localization of voxel sites for classification with clinical significance
 - classification performance

⁵M. Termenon, M. Graña, A. Besga, J. Echeveste, A. Gonzalez-Pinto, Lattice Independent Component Analysis feature selection on Diffusion Weighted Imaging for Alzheimer's Disease Classification, Neurocomputing (online)

DWI, DTI and FA, MD

- Diffusion Weighted Imaging (DWI) measures the diffusion of water molecules inside the brain along several directions
 - *in vivo* information about the integrity of the White Matter (WM) fibers.
- Diffusion Tensor Imaging (DTI) is the diffusion covariance tensor at each voxel.
- Scalar diffusion measures computed from DTI are
 - Fractional Anisotropy (FA) privileged diffusion direction
 - Mean Diffusivity (MD), magnitude of the diffusion process
- DTI studies about WM abnormalities in AD have found differences between AD patients and controls

Preprocessing pipeline

Algorithm 1 T1 and DWI data processing pipeline to obtain spatially normalized FA data.

1. Convert DICOM to nifti
 2. Skull stripping T1-weighted volumes
 3. Affine registration of T1-weighted skull stripped volumes to template MNI152.
 4. Correct DWI scans.
 5. Obtain skull stripped brain masks for each DWI corrected scans.
 6. Apply diffusion tensor analysis computing DTI and FA.
 7. Rigid registration 6DoF of FA data to T1-weighted normalized volumes, from Step3.
-

LICA for feature detection in FA

- Recall the Linear Mixing Model $X = AS + \epsilon$,
- estimation of the abundance matrix, i.e. by LSU $\hat{A} = XS^\#$, or FCLSU
- The residual error is $R = (X - \hat{A}S)^2$.
- vowelwise across subjects: compute $P(i, j, k)$ as the Pearson's correlation of $R(i, j, k)$ with the categorical variable (AD=1, HC=0)
 - Feature sites $|P(i, j, k)| > P_\alpha$
 - where P_α is the α -percentile of the e.p.d. of $P(i, j, k)$

LICA for feature detection in FA

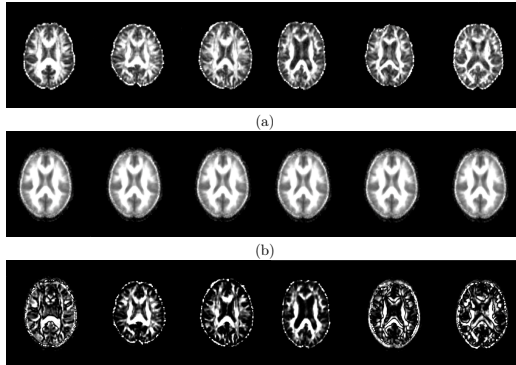


Figure : (a) original FA data, (b) reconstruction from FCLSU estimated abundances, (c) residual R

Feature localization

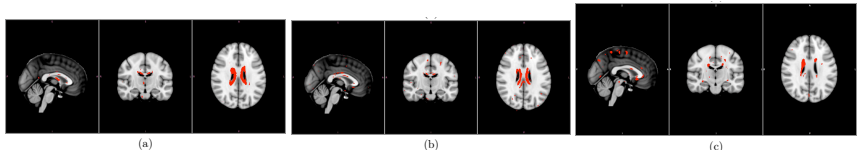


Figure : Feature localization in the brain (a) LICA residual, (b) bare FA data, (c) VBM

Feature localization

- LICA residuals produce feature localization that correspond to biomarkers in the limbic system in agreement with the medical literature,
 - hippocampus,
 - amygdala , and
 - the brainstem.

DWI Classification results

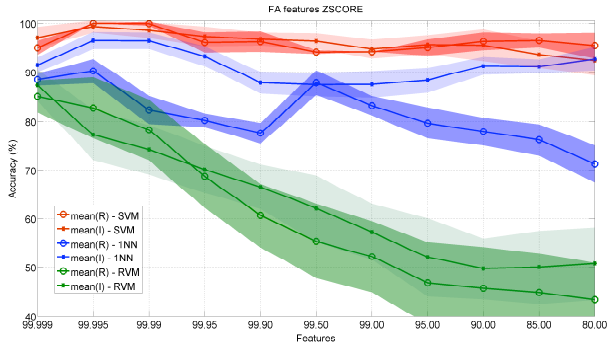


Figure : LICA vs. bare FA, accuracy results for decreasing P_{α} increasing number of features

DWI Classification results

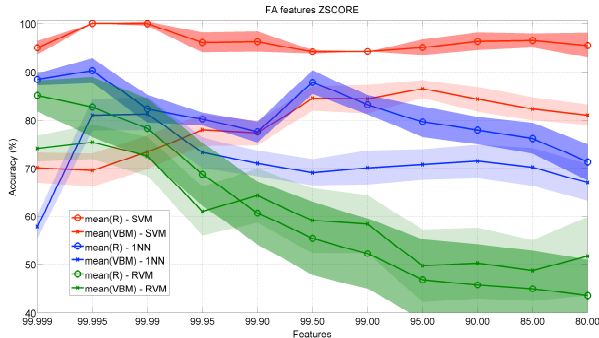


Figure : LICA vs. VBM, accuracy results for decreasing P_{α} increasing number of features

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Multivariate Mathematical Morphology

Morphological operations are mappings between complete lattices, denoted \mathbb{L} or \mathbb{M} ,

erosion is a mapping $\varepsilon : \mathbb{L} \rightarrow \mathbb{M}$ commuting with the infimum operation, $\varepsilon(\bigwedge Y) = \bigwedge_{y \in Y} \varepsilon(y)$; $\forall Y \subseteq \mathbb{L}$

dilation is a mapping $\delta : \mathbb{L} \rightarrow \mathbb{M}$ commuting with the supremum operation, $\delta(\bigvee Y) = \bigvee_{y \in Y} \delta(y)$.

gradient $g(Y) = \delta(Y) - \varepsilon(Y)$,

top-hat $t(Y) = Y - \delta(\varepsilon(Y))$.

Multivariate ordering

Definition

A h -ordering is defined by a surjective map of the original partially ordered set onto a complete lattice $h : X \rightarrow \mathbb{L}$,

- the order defined in \mathbb{L} induces a total order in X ,

$$r \leq_h r' \Leftrightarrow h(r) \leq h(r') \quad (3)$$

Definition

A h -supervised ordering is a h -ordering satisfying $h(b) = \perp$, $\forall b \in B$, and $h(f) = \top$, $\forall f \in F$,

- for background and foreground $B, F \subset X$, $B \cap F = \emptyset$,
- \perp and \top are the bottom and top elements of \mathbb{L}

Supervised erosion and dilation

Definition

The supervised erosion by structural object S is

$$\varepsilon_{h,S}(I)(p) = I(q) \text{ s.t. } I(q) = \bigwedge_h \{I(s); s \in S_p\}$$

Definition

The supervised dilation by structural object S is

$$\delta_{h,S}(I)(p) = I(q) \text{ s.t. } I(q) = \bigvee_h \{I(s); s \in S_p\}$$

where \bigwedge_h and \bigvee_h are the infimum and supremum defined by the reduced ordering \leq_h

LAAM h-function

Definition

Given $\mathbf{c} \in \mathbb{R}^n$ and $X = \{\mathbf{x}_i\}_{i=1}^K$, $\mathbf{x}_i \in \mathbb{R}^n$; the LAAM based h_X -function is

$$h_X(\mathbf{c}) = \zeta(\mathbf{x}^\#, \mathbf{c}), \quad (4)$$

- $\mathbf{x}^\# \in \mathbb{R}^n$ is a LAAM recall result

$$\mathbf{x}^\# = M_{xx} \boxtimes \mathbf{c}$$

or

$$\mathbf{x}^\# = W_{xx} \boxtimes \mathbf{c}$$

- $\zeta(\mathbf{a}, \mathbf{b})$ is the Chebyshev distance $\zeta(\mathbf{a}, \mathbf{b}) = \bigvee_i |a_i - b_i|$.

One sided ordering

Definition

one-side LAAM-supervised ordering:

$$\forall \mathbf{x}, \mathbf{y} \in \mathbb{R}^n, \mathbf{x} \leq_X \mathbf{y} \iff h_X(\mathbf{x}) \leq h_X(\mathbf{y}). \quad (5)$$

- $h_X : \mathbb{R}^n \rightarrow \mathbb{L}_X$, where $\mathbb{L}_X = (\mathbb{R}_0^+, <)$, $\perp_X = 0$
- the Background set B s.t. $h_X(\mathbf{b}) = \perp_X = 0$
 - is the set of fixed points of the LAAM $B = \mathcal{F}(X)$

B/F ordering

Definition

The relative background/foreground supervised h -function:

$$h_r(\mathbf{c}) = h_F(\mathbf{c}) - h_B(\mathbf{c}), \quad (6)$$

Given training sets B and F

Definition

relative LAAM-supervised ordering denoted \leq_r :

$$\forall \mathbf{x}, \mathbf{y} \in \mathbb{R}^n, \mathbf{x} \leq_r \mathbf{y} \iff h_r(\mathbf{x}) \leq h_r(\mathbf{y}) \quad (7)$$

B/F ordering

- $h_r(\mathbf{c}) : \mathbb{R}^n \rightarrow \mathbb{L}_{B/F}$ where $\mathbb{L}_{B/F} = (\mathbb{R}, <)$,
 - $h_r(\mathbf{b}) > 0; \mathbf{b} \in \mathcal{F}(B)$
 - $h_r(\mathbf{f}) < 0; \mathbf{f} \in \mathcal{F}(F)$
 - no proper bottom and top elements
 - $h_r(\mathbf{c}) = 0$; decision boundary $\mathbf{c} \in C_r$

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Resting state fMRI

- Resting state fMRI data has been used to study brain functional connectivity
 - correlation of low frequency oscillations in diverse areas of the brain reveal their functional relations.
 - connections discovered are a brain fingerprint, the so-called default-mode network.
- do not impose constraints on the cognitive abilities of the subjects.
 - in the study of brain maturation there is no single cognitive task which is appropriate across the aging population.

Schizophrenia

Schizophrenia is a severe psychiatric disease that is characterized by delusions and hallucinations, loss of emotion and disrupted thinking.

Functional disconnection between brain regions is suspected to cause these symptoms, because of known aberrant effects on gray and white matter in brain regions that overlap with the default mode network.

Resting state fMRI studies have indicated aberrant default mode functional connectivity in schizophrenic patients.

Goal of our work is to find differences in connectivity between patients with and without auditory hallucinations

Experiment's goal

aim of the experiments is a proof of concept of the LAAM multivariate morphology approach

- discrimination of **healthy** control subjects, **schizophrenia** patients **with** and **without** auditory hallucinations.

results find different brain networks depending on the subject using the **same** h -function built from selected voxel seeds.

Results

expected_result network correlated with an the auditory cortex voxel:

- effect related to the auditory hallucinations.

seed voxel time series X extracted from HC.

same LAAM M_{XX} applied to HC and patients

computing both h_X and $h_{B/F}$ maps

network: peaks of the top-hat transformation

Supervised top-hat

Definition

The h -supervised top-hat is defined as follows:

$$t_{h,S}(I) = h(I) - h(\delta_{h,S}(\varepsilon_{h,S}(I))),$$

where $\varepsilon_{h,S}(I)$ and $\delta_{h,S}(I)$ are the h -supervised erosion and dilation

Materials

- resting state fMRI data obtained from 1 HC, 1SCNH, 1SCWH
- F 240 BOLD volumes and one T1-weighted
 - skull extraction
 - manually AC-PC transformed.
 - The functional images coregistered to the T1-weighted anatomical image.
 - slice timing,
 - head motion correction
 - smoothing (FWHM=4mm)
 - spatial normalization to (MNI) template
 - temporal filtering (0.01-0.08 Hz)
 - linear trend removing
 - All the subjects have less than 1mm maximum displacement and less than 1° of angular motion.

network from the one-side h -function

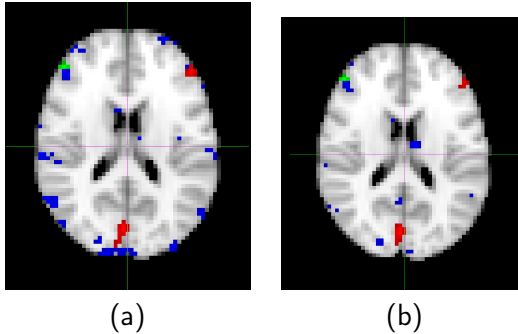


Figure : **network** computed from one-sided LAAM supervised h -function on front lobe (a) and auditory cortex (b). Red, green, blue voxel colors correspond to HC, SCNH, and SCWH, respectively.

network from the B/F h -function

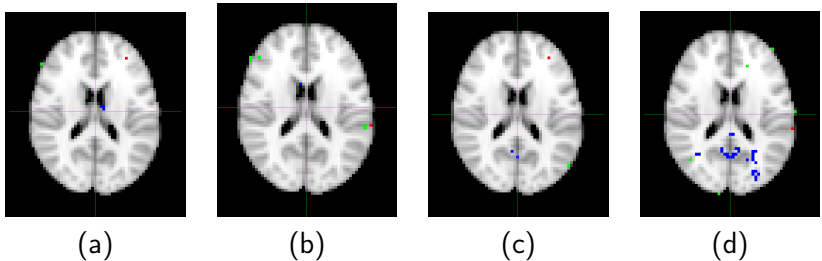


Figure : **network** computed with B/F LAAM supervised h -function from different voxel seed pairs. Red, green, blue voxel colors correspond to HC, SCNH, and SCWH, respectively.

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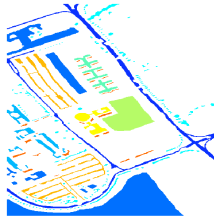
Hyperspectral image spatial-spectral classification

- the aim is building hyperspectral imagee thematic maps from spatial and spectral information
- Pixel independent SVM classification
- Multivariate mathematical morphology provide the spatial information
 - watershed regions from morphological gradient
 - assume homogeneous class
 - spatially guided regularization of SVM results

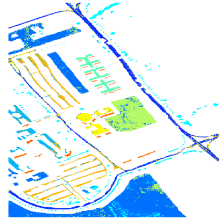
Hyperspectral image and baseline SVM classification



(a)



(b)



(c)

Figure : (a) Pavia image, (b) ground truth, (c) pixelwise SVM classification

Supervised morphological gradient

Definition

The h -supervised morphological gradient is defined as follows:

$$g_{h,S}(I) = h(\delta_{h,S}(I)) - h(\varepsilon_{h,S}(I)),$$

where $\varepsilon_{h,S}(I)$ and $\delta_{h,S}(I)$ are the h -supervised erosion and dilation

Unsupervised selection of training data

- An EIA induces a set of endmembers $E = \{\mathbf{e}_i\}_{i=1}^p$ from the image data. Compute $D = [d_{i,j}]_{i,j=1}^p$, where $d_{ij} = |\mathbf{e}_i, \mathbf{e}_j|$
- One-side h -supervised ordering
 - $X = \{\mathbf{e}_{k^*} \in E\}$ such that $k^* = \arg \min_k \left\{ \frac{1}{p-1} \sum_{i \neq k} d_{ik} \right\}_{i=1}^p$.
- Background/Foreground h -supervised orderings
 - $F = \{\mathbf{e}_{i^*} \in E\}$ and $B = \{\mathbf{e}_{j^*} \in E\}$ such that $(i^*, j^*) = \arg \max_{i,j} \{(d_{ij})\}$

Endmembers

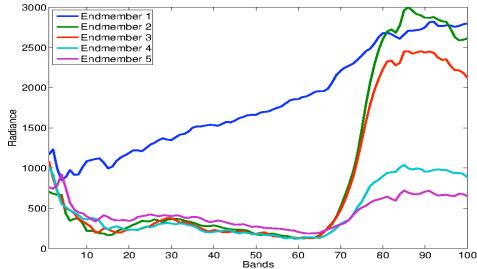


Figure : Endmembers found in the hyperspectral image

Morphological gradient results

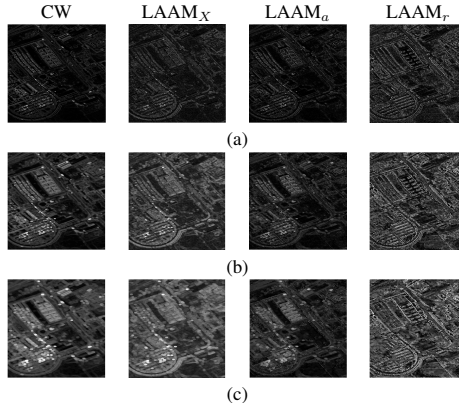
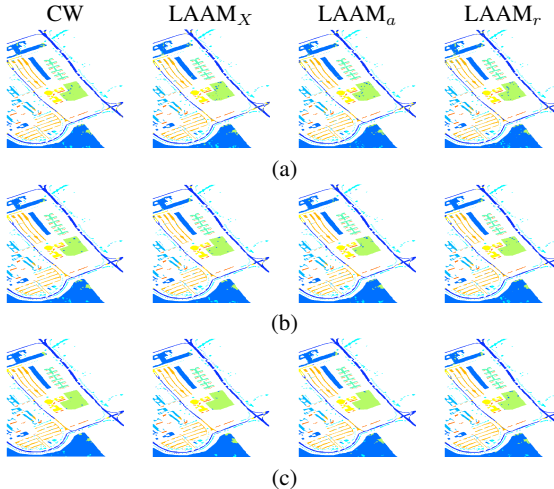


Figure : Morphological gradients with increasing structural element size

Classification results



Class specific sensitivities

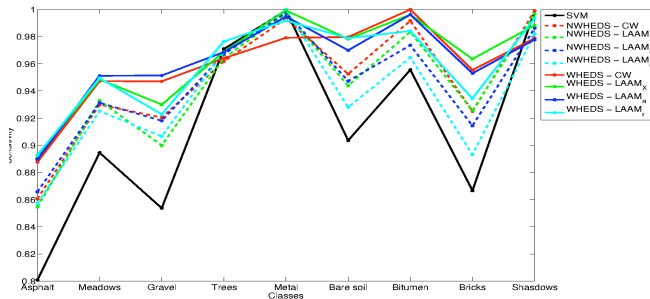


Figure : Class sensitivities, structural element of radius 3

Concluding remarks

- Lattice Computing proposes a new paradigm for the definition of Hybrid Intelligent Systems
 - does not involve statistical techniques, is model-free
 - relies only in lattice operators and addition
- I have concentrated on the LAAMs stream of research
- increasing range of practical applications with competitive results

Future work avenues

- Sparse bayesian unmixing based on Ritter's EIA
- Multi-class Supervised Multivariate Mathematica Morphology
- LICA fMRI group analysis for detection