Multi-modal topological optimization of structure using ACO algorithms

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Abstract

In this study, ant colony optimization (ACO) algorithm was implemented for finding optimal solutions to multi-modal structural problems. A well-studied benchmark example in structural topology optimization problems was utilized to evaluate the proposed approach. The results indicate the effectiveness and diversity of the proposed algorithm.

Keywords: ant colony optimization, structural topology optimization

1. Introduction

The structure optimum design is a very interesting topic in the field of engineering optimization. The optimal design of structures including sizing, shape (*i.e.* configuration) and topology forms the basic issues for the structural design process. Among them, the topology optimal design may be the most important and difficult topic in structural optimization.

In the past decades a number of innovative approaches to topology optimization have been developed. The sensitivity analysis is the first approach proposed by Kibsgaard [1] for topological optimization. However, this approach has two major defects: first, it requires a good initial guess, as it demonstrated to be unstable for large variations of the domain; second, it does not allow modification of the initial domain topology (*e.g*. add or remove holes). An alternative popular method, the homogenization method first proposed by Bendsøe and Kikuchi [2] consists in dealing with a continuous density of material. In the end of this scheme, the final density is forced toward value 1 or 0 (material present or absent). Nevertheless, this approach requires the design of the homogenized operator and is insofar limited to the linear elasticity case. In addition, it cannot address loadings that apply on the actual boundary of the shape to be determined, and hardly handles optimization for multiple loadings. Recently, a simple approach to shape and topology optimization termed Evolutionary Structural Optimization (ESO) method has been developed by Xie and Steven [3]. A fundamental potential drawback of this method pointed out by Liu *et al.* [4] is the strong dependence of the solution on the mesh of finite element from which it is evolved and on the sequence of the element removal.

A possible approach to overcome those difficulties is to adopt biological-inspired computation methods imitating natural phenomena and physical processes. Among these are simulated annealing [5], immune algorithm [6] and genetic algorithms [7-8]. In this study, ant colony optimization (ACO) algorithm [9] was implemented for finding optimal solutions to multi-modal structural problems. The ACO is a relatively recent approach to solve combinatorial optimization problems mimicking the behavior of real ant colonies. It has been successfully applied to solve a number of different optimization problems such as the Traveling Salesman problem and the Vehicle Routing Problem. However, not much research has been done for structural optimization so far. Recently, Camp and Bichon [10] applied ACO algorithm optimal design cross-sectional areas (discrete variables) of space trusses. It was illustrated that their proposed ACO algorithm could design truss structure satisfying constraints while minimizing the overall weight.

2. ACO algorithm

Ant algorithms [11] are a recently developed, population-based methodology applied to numerous *NP*-hard combinatorial optimization problems. They have been inspired by the behavior of real ant colonies especially by their foraging behavior. One of the main features is the indirect communication of a colony of ants, based on pheromone trails that are a kind of distributed numeric information used to reflect their experience while solving a particular problem. Ant System, the first ACO algorithm was proposed by Droigo [11] to solve the TSP. Each ant in AS builds up a solution step-by-step employing transition rule until a solution is found. Ants that found a good solution mark their paths by laying some amount of pheromone on the edge of the path.

The Ant System used to solve the traveling salesman problem is expressed as a graph $G\langle N,E\rangle$, where N denotes the set of cities and E represents the set of edges between cities. The objective is to find the minimal length closed tour that visits each city once. Each ant is a simple agent to fulfill the task and obeys the following rules:

- It lives in a discrete-time environment.
- It chooses the next city associated with a probability which is a function of pheromone laid on the connecting edge and of the amount of the visible distance.
- It can not choose the cities which has been visited before a tour is completed.
- It lays pheromone on each edge visited when a tour is completed.

In this study, ACO algorithm was modified and adopted to solve structural topology optimization.

3. Topological optimization using ACO

Corresponding to the 2-dimensional topological optimization problems, a two-dimensional design is discretized into small, square elements ($\boldsymbol{E}(x, y)$, x=1,…,X; y=1,…,Y) where each element represents either material (with code value of 1) or void (with code value of 0). The states of the individual elements define the distribution of material and void within the domain and therefore establish the topology. This binary, material-void design domain results in a discrete search space and allows for a precise topology boundary as Fig. 1 illustrated.

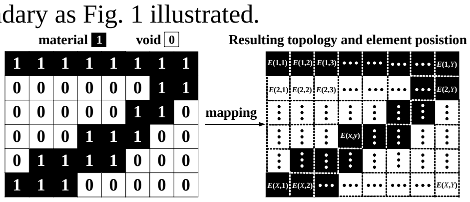

Fig. 1 Mapping from design domain to topology

The ACO algorithm for structure topology optimization follows the following steps:

(1) *Initialization*.

A population of n_a ants is placed randomly in elements of the design domain. Then the ant located at $\boldsymbol{E}(x, y)$ will forward to one of the 8 neighboring elements shown in Fig. 2 according to “element transition rule” described below. The elements where the ant has visited will be marked 1 (material).

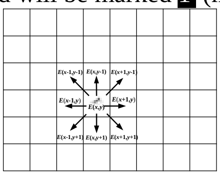

Fig. 2 Eight possible forward directions of ant

(2) *Element transition rule*.

The transition probability from element $\boldsymbol{E}(x, y)$ to element $\boldsymbol{E}(i, j)$ (one of the eight neighboring elements $\boldsymbol{E}(x\text{-}1, y\text{-}1)$, $\boldsymbol{E}(x\text{-}1, y)$, $\boldsymbol{E}(x\text{-}1, y+1)$, $\boldsymbol{E}(x, y\text{-}1)$, $\boldsymbol{E}(x, y+1)$, $\boldsymbol{E}(x+1, y\text{-}1)$, $\boldsymbol{E}(x+1, y)$, $\boldsymbol{E}(x+1, y+1)$) for the k^{th} ant is defined as:

$$p_{ij}^k(t)=\begin{cases}\dfrac{[\tau_{ij}(t)]^{\alpha}}{\sum_{h\notin \boldsymbol{tabu}}[\tau_{ih}(t)]^{\alpha}} & \text{if } j\notin \boldsymbol{tabu}\\ 0 & \text{otherwise}\end{cases}$$

where ***tabu*** is used to define the set of elements have been visited for all ants in last period (t-1) and τ_{ij} represent the concentration of pheromone laid on elements $\boldsymbol{E}(i, j)$.

(3) *Pheromone updating rule*.

The trail intensity at each element $\boldsymbol{E}(i,j)$ is updated according to the following formula

$$\tau_{ij}(t+n)=(1-\rho)\tau_{ij}(t)+\Delta\tau_{ij}+e\cdot Q/Obj^*$$

where e indicates the number of elitist ants, Q is a positive constant value, and Obj^* is the best objective value of solution/topology found from the beginning of the trail. In an effort to improve performance, “elitist ants” (similar to the elitist strategy used in genetic algorithm) introduced by Dorigo *et al*. [9] is included in the pheromone updating rule.

$$\Delta\tau_{ij}=\sum_{k=1}^{n_a}\Delta\tau_{ij}^k$$

$$\Delta\tau_{ij}^k=\sigma_{ij}Q/Obj_{Niche}^k$$

where σ_{ij} is the normalized stress value of each element $\boldsymbol{E}(i,j)$. Niche strategy is utilized to find the multiple solutions since topology optimization of structure is a kind of multi-modal problem. In this study, sharing scheme is employed to calculate the similarity for each ant/topology and described below.

$$Obj_{Niche}^k=\frac{Obj^k}{(1+penalty_k)\cdot SC_k}$$

and $$Obj^k=\frac{1}{d_k^{\max}\cdot Area_k}$$

where Obj^k and Obj_{Niche}^k denote the original objective and the shared objective value of the k^{th} ant, respectively. In addition, Obj^k indicates the k^{th} topology’s stiffness-to-weight ratio and stiffness is presented by inverse of topology’s maximum displacement ($1/d_i^{\max}$) at the point of loading application. It should be noted that the number of connected material element of topology is used as a qualitative measure of topology’s weight ($Area_k$). SC_k represents the similarities count of the k^{th} ant/topology with all other antibodies and is expressed as

$$SC_k=\sum_{j=1}^{n_a}count_{kj}\,,\ k=1,2,\dots,n_a;\ \ j=1,2,\dots,n_a$$

with $$count_{kj} = \begin{cases} 1, & if \;\; ant_{kj} < Th \\ 0, & else \end{cases}$$

$$ant_{kj} = \sqrt{(avg_k^{stress} - avg_j^{stress})^2 - (std_k^{stress} - std_j^{stress})}$$

where Th is an user-defined threshold value illustrating the allowable difference between antibodies, ant_{kj} representing distance in average-standard deviation stress space indicates a relationship between the k^{th} and the j^{th} ant/topology. Note that avg_k^{stress} and std_k^{stress} are normalized average and normalized standard deviation of stress of the k^{th} ant/topology. In this study, the $penalty_k$ of the k^{th} topology is treated as the penalty term for constraint violation and can be expressed as follows,

$$penalty_k = \sum_{j=1}^{N_C} amount_j^k \cdot \sum_{j=1}^{N_C} count_j^k$$

with $$amount_j^k = \begin{cases} \dfrac{|g_j^k|}{g_a} - 1 & if \quad |g_j^k| > g_a \\ 0 & else \end{cases}$$

$$count_j^k = \begin{cases} 1 & if \quad |g_j^k| > g_a \\ 0 & else \end{cases}$$

where $penalty_k$ represents the penalty value for the k^{th} ant/topology; N_c is the total number of equality and inequality constraints; $amount_j^k$ and $count_j^k$ correspond to the normalized values of the summation and total number of the *jth* constraint violation, respectively; g_j^k denotes the equality and/or inequality constraint violations whereas g_a indicates the allowable constraint.

(4) *Connectivity analysis*.

For any two elements in a topology to be considered as connected they must share at least one edge while element sharing only one corner are considered as disconnected. A topology contained disconnected elements will undergo a structure modification procedure. In this procedure, the removal of disconnected elements or the adding of elements to neighboring disconnected element will be done randomly until the discontinuous structure is compensated. The continuous topology will be further analyzed via the finite element computation to obtain the required displacements and stresses. To reduce computation time, elements with a stress value lower than the user-defined level of average stress (which do not break continuity requirements) will be removed from the structure, and the corresponding element set to a binary value of 0 (void).

(5) *Memories of multiple colonies*.

In this paper, multiple-colony memories were employed to save the multi-modal topologies found. The number of memories was predefined and the solutions saved in memories will be replaced by the best solution found currently if it satisfies the following conditions:

$$Obj^k > Obj^{memory} \;\; \bigcap \;\; SC_{memory}^k \geq \delta$$

where Obj^k and Obj^{memory} denote the objective value of the k^{th} ant/topology and that of the ants/topologies saved in memories, respectively. The variable SC_{memory}^k represents their associated similarities and δ is a predefined similarity threshold.

(6) *Stopping criteria*.

Whenever the k^{th} ant finishes visiting all the basic elements (supporting and loading elements), it will complete a tour and stop. The trail of the k^{th} ant constitutes a structural topology. A new iteration will start after all the n_a ants complete their tours. Moreover, a new run time will begin and the pheromone values of ants reset to initial value after N iteration.

4. Simulation and Discussions

In this study, a topological optimization example was employed to evaluate the proposed algorithm. This example describes the optimization of a cantilevered plate (0.001*m* thickness with aspect ratio 1.6) subjected to a downward concentrated loading applied at an FE node on its right hand bottom side with its stress being constrained to 200*MPa*. The design domain is shown in Fig. 3.

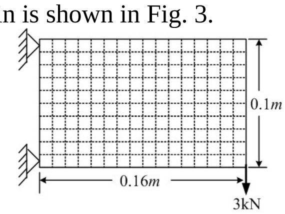

Fig. 3 The design domain and boundary conditions

The design domain is discretized according to a 20×32 element mesh FE model. The support nodes on both end of left hand side are defined to have zero displacement in the FE analysis. The unit cell material is assumed isotropic with Young's modulus equal to 200*Gpa* and Poisson coefficient equal to 0.33.

Table 1 lists the parameters employed in the proposed ant algorithm.

Table 1 Parameters employed in truss simulation

Number of ants n_a	60
Number of iterations N	60
Run time	5
Number of elitist ants e	5
Initial pheromone value τ_0	0.001
Pheromone decay factor ρ	0.05
Similarity threshold δ	0.75
Threshold value Th	0.1
Q	4.0
α	1

One execution of the computer model requires around 18,000 functional evaluations (60 ants by 60 iterations and 5 run times), taking approximately 120 minutes with a Pentium 4 processor running at 1.5GHz. Numerous significant multi-modal topologies (from lightest weight to heaviest weight) are derived from the ant memory pool. Fig. 4 demonstrates part of the structure topologies derived using the proposed ant algorithm.

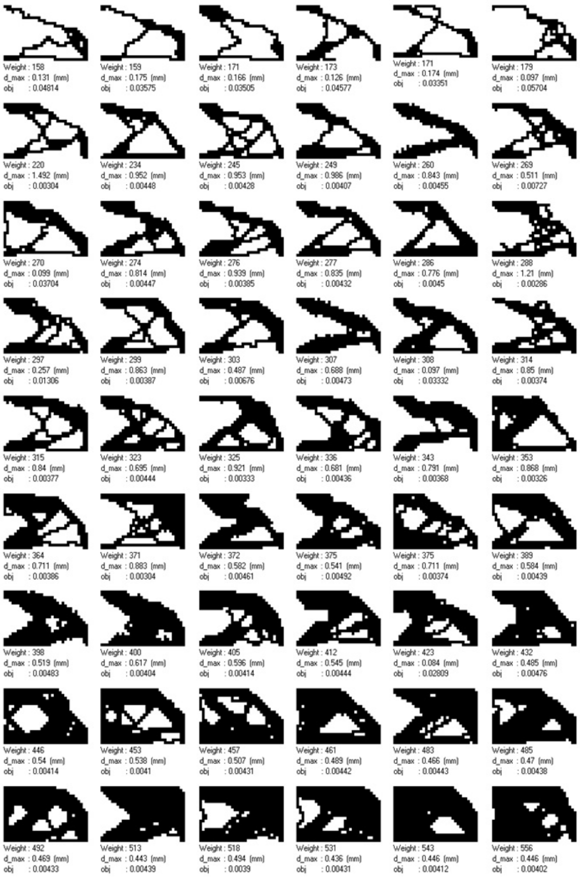

Figure 4 Family of topologies derived

As can be seen from the diverse range of resultant 54 topologies shown in Figs. 4, the structures therefore provide the designer with a set of near-optimal solutions that can be easily developed into discrete truss systems. Part of the structures shows well-defined truss-like members of constant cross sectional area with large voids between members. A proportion of these structural members have straight alignment between joints and exhibit low levels of porosity. If manufacturability is the prime consideration (*i.e.* large voids between members), the designer may choose these truss-like topologies.

5. Conclusions

In this study, a novel concept for handling multi-modal topological optimization has been presented using an ACO algorithm mimicking the behavior of real ant colonies. A well-studied benchmark example in structural topology optimization problems wase used to evaluate the proposed approach. The results show that diverse topology structures could be derived using ACO algorithm and can provide the designer with a set of possible choice.

6. References

[1] S. Kibsgaard, "Sensitivity analysis - the basis for optimization," *International Journal of Numerical Methods in Engineering*, Vol. 34, pp. 901-932, 1992.

[2] M.P. Bendsøe, and N. Kikuchi, "Generating optimal topologies in structural design using a homogenization method," *Computer Methods in Applied Mechanics Engineering*, Vol. 71, pp. 197-224, 1988.

[3] Y.M. Xie, and G.P. Steven, "A simple evolutionary procedure for structural optimization," *Computers & structures*, Vol. 49, pp. 885-896, 1993.

[4] J.S. Liu, G.T. Parks, P.J. Clarkson, Metamorphic development: a new topology optimization method for continuum structures, Struct. Multidisc. Optim. 20 (2000) 288-300.

[5] G. Anagnostou, E. Rønquist, A. Patera, A computational procedure for part design, Computer Methods in Applied Mechanics Engineering 97(1992) 33-48.

[6] G.-C. Luh, and C.-H. Cheuh, "Multi-modal topological optimization of structure using immune algorithm," *Computer Methods in Applied Mechanics and Engineering*, Vol. 193, pp. 4035-4055, 2004.

[7] M.J. Jakiela, C. Chapman, J. Duda, A. Adewuya, and K. Saitou, "Continuum structural topology design with genetic algorithms," *Computer Methods in Applied Mechanics Engineering*, Vol.186, pp. 339-356, 2000.

[8] C. Chapman, K. Saitou, and M. Jakiela, "Genetic algorithms as an approach to configuration and topology design", ASME Journal of Mechanical design, Vol. 116, pp. 1005-1012, 1994.

[9] M. Dorigo, V. Maniezzo, and A. Colorni, "The ant system: optimization by a colony of cooperating agents," *IEEE Transactions on Systems, Man, and Cybernetics – Part B*, 1996; Vol. 26, pp. 29-41, 1996.

[10] CV. Camp, and BJ. Bichon, "Design of space trusses using ant colony optimization," *Journal of Structural Engineering*, Vol. 130, pp. 741-751, 2004.

[11] M. Dorigo, and G. di Caro, "The ant colony optimization meta-heuristic," New Ideas in Optimization, pp. 11-32, McGraw-Hill, London, UK, 1999.