Triplet Lens Design using Hybrid Coded NSGA2

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Abstract

A triplet lens is minimized for two objectives, Resolution(R) and Distortion (D). NSGA2 with hybrid encoding for radii of curvatures was employed. A number of diverse solutions were obtained which functioned as an excellent starting point for the inbuilt local optimization routines of the optical design program CODE V*®. Earlier multi-objective lens optimization studies were not successful in obtaining diverse solutions. Here I discuss the results and advantages of hybrid encoding for Optical Design.

Keywords: NSGA2, triplets, lens design, optical design, hybrid encoding

1. Introduction

In lens design, the traditional approach is to select a suitable starting point by manual first order calculations and then optimize the design by minimizing the third order aberrations. The success of the local optimization program depends on the selection of starting point. Even if we choose different starting points, we may not obtain competing Pareto-Optimal designs even with single objective global optimization programs. Earlier studies on Multi-Objective Evolutionary approach to lens design by Ono [1] were not successful in obtaining a diverse Pareto-Optimal front. In Figure 1, mapping from Constructional Parameter Space X to the Objective Function Space F is shown. Three categories of solutions mapped from X to F on a particular Pareto-optimal front can be observed. When two far away points in X map to two nearby points in F, we get a choice for manufacturing the system. When two nearby points in X map to two nearby points in F, it is a trivial situation. We do not have any benefit over manufacturability. When two nearby points in X map to two far away points in F, we have an undesirable situation. Slight perturbation of parameters during manufacturing can lead to severe performance degrading.

An optical system [2] will have a number of constructional parameters which specify the system. These include the radius of curvatures of the lens elements, the distance between various surfaces, position of stops etc. Let the system be having M aberrations defined over a space of N constructional parameters. The symbols and definitions used are those used by Vasiljevic [3].

The merit function (objective function) Φ for single objective optimization is usually defined by

$$\Phi=\sum_{i=1}^{M} f_i^2(x_1, x_2, ...x_N)$$

Where f_i are the aberrations and X is a vector of constructional parameter. X= $(x_1, x_2, ...x_N)$ refers to a point in the parameter space. There can be a number of equality and inequality constraints imposed among the constructional parameters. When we have multi-objective optimization, merit function shown above will be split into component objective functions.

Here a three element lens system with air gaps in between the elements is sought to be optimized for two different image quality metrics, Distortion (D) and Resolution (R). For this, I chose to minimize the Seidel aberration coefficients for spherical aberration (SA) and distortion (DST) calculated using the optical design software CODE V®. Thereafter, the good Pareto-Optimal solutions are used as starting points for further optimization. The nature and diversity of the results are discussed. The necessity to invoke hybrid encoding for radii of curvature is discussed.

Constructional parameters for the lens were supplied by NSGA2 [4, 5] during optimization. We have 13 continuous variables. They are two air gaps, three lens thicknesses, one distance corresponding to that from the last surface to the image, six radii of curvatures and the distance between the image and the imaging screen. Object is placed on the left side of the system. The sign of radii of curvature is positive if the rays travel from the vertex to the center of curvature. Otherwise it is negative. A flat surface has an infinite radius of curvature. Curvature is inversely proportional to the radius of curvature. The radii of curvature are allowed to vary between -200 mm and

200 mm. The thicknesses were varied from 0.01 mm to 30 mm. The distance between image and the imaging screen were allowed to be varied between -0.001 mm and 0.001 mm. The six radii of curvatures were hybrid coded where as all other seven variables were real coded. In hybrid coding, each radii of curvature has a sign and magnitude. The magnitudes are real coded and the signs are binary coded. A six digit binary number is chosen which can represent values from 0 to 63. For example, if the value of the second digit is 1, I choose a negative sign for the second radius. If it is 0, I keep a positive sign. So I have 13 real coded variables and 1 binary coded variable for the problem. Now magnitudes of the radii of curvatures are allowed to vary between 20 mm and 200 mm. The right encoding choice [6] is an important issue in genetic algorithm research.

The following are the parameters employed. Number of generations is 34, Population size is 130, Cross over probability is 1, real coded mutation probability is 0.0769, Uniform Binary Cross over is employed, binary coded mutation probability is 0.166667 and the cross over parameter in SBX operator is 100.

The NSGA2 code downloaded from Deb's [7] web site was used with necessary modifications for hybrid coding. I plan to employ other algorithms also in my future work. NSGA2 written in C interacts with CODE V® through the COM interface of Visual Studio. Since the interface is very slow, I could at most achieve two lens evaluations per minute. For practical applications, it is necessary to rewrite the optimization code in the scripting language of CODE V® to enable faster computation.

1.1. Hybrid encoding

The following factors necessitated hybrid coding for radii as opposed to real coding. There is an abrupt change in the value of the radius of curvature as it changes from positive to negative in sign. A flat surface can be considered as having a radius of either positive or negative infinity. Very small values of radii are not realizable. They correspond to infinite curvature. Whenever one of the six radii is in that range, we get undesired candidates. It is very difficult to get good solutions when any of the radii of curvature is less than 10 mm in absolute value. In the beginning, when candidates are randomly generated with radius between -200 mm and 200 mm, the probability that it lies between -10 mm and 10 mm is 0.05 for any one radius. Probability that any one of the six radii lie between -10 mm and 10 mm is approximately 0.3, so among the initial generation, 30% of the population members are bound to be bad candidates. Only the remaining 70% are the possible good candidates. In any given generation after the evolution of a Pareto-front, the cross-over takes place between selected candidates. The probability that a selected pair has at least one radius with opposite signs is approximately $(1-(0.5)^6)=0.984$. Their children will possibly belong to the prohibited region between -10 mm and +10 mm. The probability for their magnitudes to differ by at least 20 mm is 0.79.The cross-over is likely to result in children with radii in the prohibited region between -10 mm and 10 mm if the magnitudes of the radii differ by less than 20 mm. This is given by 1-0.79 = 0.21. Hence the probability for a selected pair on Pareto-Optimal front generating undesired children after cross over is given by $(0.984)(0.21)=0.207$. This essentially reduces the effectiveness of cross over by a factor of 20%.

During cross over, to reach from positive values of radius to negative values of radius, we have to cross positive infinity and negative infinity. Only a hyper cube around the origin of the system (where positive and negative infinities are placed side by side) is the allowed region for the radii. To overcome this difficulty, Hybrid encoding is introduced for the radii.

2. Results

The values of R and D (also denoted by f0 and f1 respectively) were rounded off leaving only one decimal place. The convergence was inspected from time to time and the iterations were stopped after obtaining potentially good and diverse Pareto-Optimal front. The solutions were considered as good if either any one of the objectives is less than or equal to 1 or the sum of the two objectives is less than or equal to 4. I obtained 87 good solutions forming the Pareto-Fronts. They are shown in fig. 2. The diversity is shown in fig. 3. Of these 87, I eliminated solutions that gave rise to negative thickness for stop to image distance. There were 19 solutions remaining. They are plotted in fig. 4. The diversity is shown in fig. 5.

These diverse solutions (Table 1) were used as a starting point for the local optimization routine in CODE V ®called Automatic Design (Table 2). There after, the proprietary global optimization program called Global Synthesis® (Table 3) was used to obtain still better solutions. In these optimizations, single objective optimization was employed. The objective function used is the default CODE V® objective function which includes a weighted sum of squares of all aberrations, not just spherical aberration and distortion. They resulted in a diverse set of good designs. The discussion of all the characteristics solutions relevant to Optical Engineering is beyond the scope of this paper.

In table 1, we see that solutions 2 and 12 map to regions of code 56 and 45 respectively. But they have nearly same values for objective functions. Now we have a choice for manufacturing the system. The F/# is a measure of light gathering capacity of the optical system. Systems with lower F/# s are desirable because they are faster. These lenses have F/# s 4.4 and 1.73 respectively. They are shown in Figures 6 and 7 respectively.

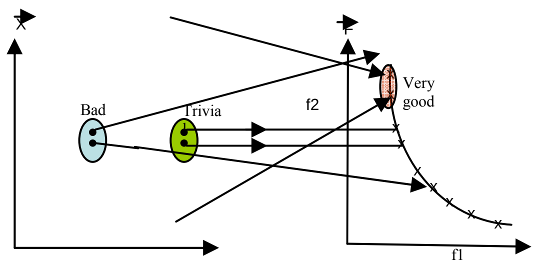

**Figure1: Mapping from Constructional Parameter Space X to the Objective Function Space F is shown. This figure shows three categories of solutions mapped from X to F on a particular Pareto-optimal front.

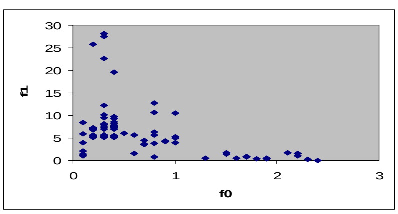

Figure2. A diverse set of good solutions belonging to different Pareto-Fronts is shown.

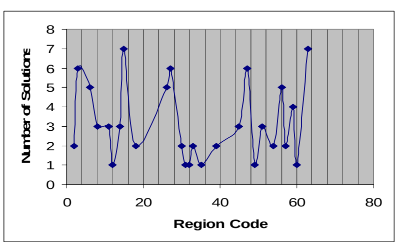

Figure3. Number of Solutions plotted against Region code. Show diversity explicitly.

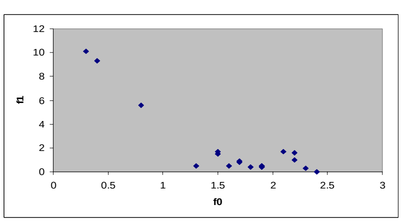

Figure4. A diverse set of good solutions belonging to different Pareto-Fronts is shown when only positive value for the stop to image distance is accepted.

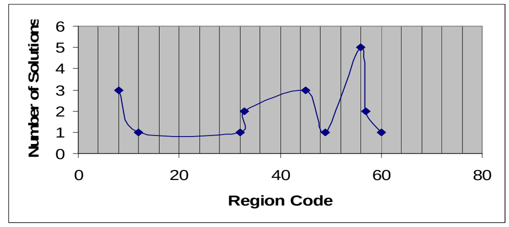

Figure5. Number of Solutions plotted against Region code when only positive value for the stop to image distance is accepted. Show diversity explicitly.

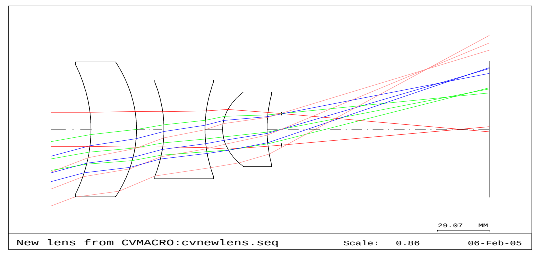

Figure6. This corresponds to the solution 2 in Table 1, belongs to region 56, has F/# 4.4, SA=-1.98 and DST= -0.44. The image is too curved.

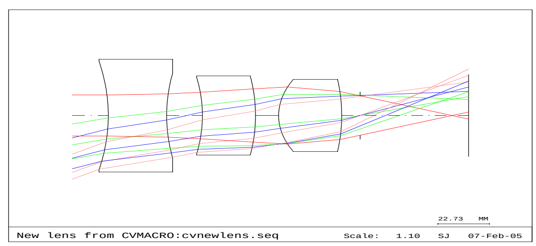

Figure7. This corresponds to the solution 12 in Table 1, belongs to region 45, has F/# 1.73, SA=-1.9 and DST= -0.49. The image suffers from astigmatism and coma, but image is flat.

PARETO- OPTIMA FROM NSGA2

NO	*CODE*	*F/#*	*SA*	*DST*
1	49	1.7	-0.84	1.9
2	56	4.4	-1.98	-0.44
3	60	2	-0.39	3.5
4	56	4.4	-2.3	-0.57
5	56	4.5	-1.97	-0.56
6	33	1.8	-1.72	0.94
7	56	4.5	-2.2	-1.7
8	8	5.4	-2.2	-1.1
9	33	1.69	-1.8	0.48
10	32	3.1	-1.57	-1.76
11	12	2.4	-0.43	9.4
12	45	1.73	-1.9	-0.49
13	57	2.1	-1.6	1.6
14	41	1.42	-2.5	-0.08
15	45	1.59	-2.3	-0.32
16	56	4.48	-1.97	-0.558
17	8	5.5	-1.6	-0.58
18	8	5.09	-1.4	0.55
19	57	2.09	-1.7	0.88

Table1. Pareto-Optima obtained from NSGA2.

CODE V® AUTOMATIC DESIGN OPTIMIZATION

NO	*CODE*	*F/#*	*SA*	*DST*
1	49	3.4	-0.27	-2.8
2	57	17.8	-0.13	-1.36
3	61	12	-0.16	-3.1
4	57	16.5	-0.18	4
5	57	11.7	-0.23	-2.4
6	49	5.6	-0.22	-9.5
7	48	112.8	-0.1	2.4
8	9	19.1	-0.09	0.94
9	33	6.86	-0.42	-11.8
10	33	5.7	-0.47	-9.9
11	57	13.5	-0.23	-4.3
12	61	9.2	-0.28	-9
13	57	10.5	-0.27	-6.6
14	49	3.36	-0.38	-9.8
15	49	5.05	-0.24	-10.3
16	57	11.66	-0.23	-2.4
17	9	13.16	-0.09	0.28
18	9	15.9	-0.086	0.4
19	9	13	-0.08	0.62

Table2. Results of CODE V® AUTOMATIC DESIGN optimization with Pareto-Optima from NSGA2 used as starting points.

CODE V® GLOBAL SYNTHESIS

NO	*CODE*	*F/#*	*SA*	*DST*
1	49	4.2	-0.21	-2.9
2	57	26.6	-0.09	2.3
3	57	21.9	-0.11	-1.43
4	57	19.5	-0.15	3.4
5	57	15.4	-0.2	-1.4
6	49	13.5	0.29	-15.4
7	48	193.1	-0.08	-5.9
8	9	15.5	-0.08	0.29
9	0	48.4	-0.16	6.1
10	0	44.6	-0.18	8.5
11	57	20.4	-0.18	-3.2
12	57	18.9	-0.18	-2.7
13	57	17.6	-0.19	-2.5
14	49	4.6	-0.36	-14.9
15	49	5.6	-0.19	-11.7
16	57	15.4	-0.2	-1.38
17	9	12.7	-0.08	0.32
18	9	14.3	-0.08	0.07
19	9	12.8	-0.08	0.33

Table3. Results of CODE V® Global Synthesis optimization with results from AUTOMATIC DESIGN used as starting points.

3. Conclusions

I have successfully demonstrated the usefulness of NSGA2 for Optical System optimization. We can obtain alternate manufacturing choices for a given set of objective functions. This was demonstrated with Siedel aberration coefficients corresponding to spherical aberration and distortion chosen as objectives. Hybrid encoding is introduced for the radii of curvatures and they were found to be better suited for the present problem. Diversity is clearly evident in the solutions, which is a significant improvement over earlier studies [1]. The diverse Pareto-Optimal front was used as a starting point for single objective local and global optimization methods. They resulted in a range of excellent and diverse solutions. Some of their features are shown in Tables 2 and 3.

4. References

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[7] http://www.iitk.ac.in/kangal/soft.htm
This is the web site of K. Deb where NSGA2 was obtained.

[8] http://delta.cs.cinvestav.mx/~ccoello/EMOO/
This site is maintained by Dr. Carlos A.Coello Coello. It is a repository of resources on Multi-objective optimization.

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