Image Enhancement by Composite Diffusion Process

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Abstract

Image enhancement is considered in the context of a new kind of nonlinear diffusion-composite diffusion, which can simultaneously enhance, sharpen and denoise images corrupted by additive noise. First, a class of fourth-order PDEs are proposed to optimize the trade-off between noise removal and feature preservation. Then, by adding the diffusion direction function in diffusion coefficient composite diffusion is proposed, which can switch the diffusion process between backward and forward mode according to the type of noise. Finally, the effectiveness of the method is emphasized by various experimental results for real images.

Keywords: Image enhancement, Impulse detection, Composite diffusion, Partial differential equation.

1. Introduction

Image denoising, enhancement and sharpening have been an important and active area of research in the general fields of image processing and computer vision. The success of many applications, such as robotics, medical imaging and quality control depends in many cases on the results of these operations. Since images cannot be described as stationary processes, it is useful to consider local adaptive filters. These filters are efficiently modeled as solutions of partial differential equations (PDEs).

Nonlinear diffusion processes have been widely used over the past decade in edge preserving denoising. Perona and Malik [1] in their seminal contribution proposed a nonlinear adaptive diffusion process, where diffusion takes place with a variable diffusion coefficient in order to reduce the smoothing effect near edges. The diffusion coefficient in the P-M process was chosen to be a decreasing function of the gradient of the image. This operation selectively lowpass filters regions that do not contain large gradients. Some drawbacks and limitations of the original model have been mentioned in the literature [2] [3]. You et al. [4] investigated the blocky effects of P-M model and proposed [5] a higher-order nonlinear diffusion. Gilboa et al. [6] further extended the nonlinear PDE-based filtering methods by combining backward and forward diffusion processes, which minimizes the effect of amplification of noise. Although their methods are more effective than original P-M model, they still allowed some additive noise to interfere with the process.

In this paper, we propose a class of fourth-order nonlinear diffusion processes to avoid blocky effects. We then generalize the analysis about diffusion coefficient of [6] by introducing diffusion direction function into diffusion coefficient and propose a new kind of nonlinear diffusion-composite diffusion, which can control the diffusion process and switch between backward and forward mode according to the type of noise.

2. Fourth-order nonlinear diffusion

Let I denote the image intensity function. To quantify the image intensity surface variation around a point we use the directional curvature metric. Fix a point and a direction at this point. The change of the normal in this direction gives the directional curvature. Tumblin and Turk [7] have constructed a descriptor from a non-directional measure of curvature magnitude

$$m^2 = 0.5 \cdot \left(I_{xx}^2 + I_{yy}^2 \right) + I_{xy}^2. \tag{1}$$

Let's consider the following functional defined in the space of continuous images over a support of Ω

$$E(I) = \int_\Omega F\left(m^2\right) d\Omega \tag{2}$$

where $I \in C^4(\Omega)$. We require that the function $F(\cdot) \geq 0$ and is an increasing function $F'(\cdot) > 0$, so that the functional is an increasing function with respect to the smoothness of the image as measured by m^2. Therefore the minimization of the functional is equivalent to smoothing the image.

The equivalent Euler equation is

$$\frac{\partial^2}{\partial x^2}\left(F'\left(m^2\right) I_{xx}\right) + 2\frac{\partial^2}{\partial x \partial y}\left(F'\left(m^2\right) I_{xy}\right) + \frac{\partial^2}{\partial y^2}\left(F'\left(m^2\right) I_{yy}\right) = 0, \tag{3}$$

where $F'\left(m^2\right)$ is diffusion coefficient of P-M model

$$F'\left(m^2\right)=\frac{1}{1+m^2/K^2}. \tag{4}$$

The Euler equation may be solved through the following gradient descent procedure

$$\frac{\partial I}{\partial t}=-\left[\frac{\partial^2}{\partial x^2}\left(F'\left(m^2\right)I_{xx}\right)+ 2\frac{\partial^2}{\partial x\partial y}\left(F'\left(m^2\right)I_{xy}\right)+\frac{\partial^2}{\partial y^2}\left(F'\left(m^2\right)I_{yy}\right)\right] \tag{5}$$

with the observed image as initial condition. Equation (5) is the new nonlinear diffusion process proposed in this paper.

Let's refer to an image whose intensity function satisfies the equation of a plane as a planar image. The measure m^2 of such an image is zero, so it satisfies the Euler equation (3). Therefore, a planar image is a stationary point of Euler equation.

Due to the non-negativity of function $F(\cdot)$, the functional $E(I)$ is bounded below

$$E(I)\geq 0. \tag{6}$$

Since $F\left(m^2\right)$ is an increasing function of m^2, its global minimum is at $m^2 \equiv 0$.

There may exist other local and/or global minima. In the following, we show that piecewise planar images are such minima.

Let Ω_i, $i=1,\ldots n$, be a partition of Ω. We define a piecewise planar image as

$$I(x,y)=\sum_{i=1}^{n} I_i(x,y), \tag{7}$$

where

$$I_i(x,y)=\begin{cases} planar\ \ image & (x,y)\in\Omega_i \\ 0 & otherwise \end{cases} \tag{8}$$

and $I_i(x,y)\in C^4(\Omega_i)$. We require that the planar images in (8) be such that the combined image $I(x,y)$ is continuous. Note that any two adjacent I_i and I_j must be on different planes; otherwise, we can combine them as one. Let's denote $\partial\Omega_i$, as the boundary of partition Ω_i, then $\Omega_i-\partial\Omega_i$ is the interior of Ω_i. It is obvious that

$$\nabla I_i = constant\ ,\ \nabla^2 I_i = 0\ . \tag{9}$$

Therefore $m^2(x,y)$ is zero in $\Omega-\partial\Omega$, where $\partial\Omega=\bigcup\partial\Omega_i$. Since it is required that any two adjacent I_i and I_j be on different planes, we have

$$\nabla I_i \neq \nabla I_j \tag{10}$$

for any two adjacent partitions Ω_i and Ω_j. This indicates that the gradient is not continuous at the boundary $\partial\Omega$. Then we have

$$\nabla^2 I(x,y)=\infty \quad m^2(x,y)=\infty \tag{11}$$

for all $(x,y)\in\partial\Omega$. We require that $F'(\infty)=0$, so we have

$$\frac{\partial^2}{\partial x^2}\left(F'\left(m^2\right)I_{xx}\right)+2\frac{\partial^2}{\partial x\partial y}\left(F'\left(m^2\right)I_{xy}\right) +\frac{\partial^2}{\partial y^2}\left(F'\left(m^2\right)I_{yy}\right)=0 \tag{12}$$

for all $(x,y)\in\Omega$. Therefore, a piecewise planar image satisfies the Euler equation.

As discussed in [4] the diffusion process of P-M type evolves an observed image toward a piecewise level image and is, therefore, the major reason for the diffusion process to suffer from blocky effects. The proposed fourth-order PDE, however, evolves an observed image toward a piecewise planar (including planar) image which is a better approximation to natural images.

3. Composite diffusion

Gilboa et al. [6] proposed several forward-and-backward (FAB) diffusion coefficients to switch diffusion from a forward to a backward mode to enhance features. They focused on enhancing and sharpening blurry images, while still allowing additive noise to interfere with the process.

In this paper, we propose a new kind of nonlinear diffusion by introducing diffusion direction function into the FAB diffusion coefficient. Considering the following formula of the diffusion coefficient in the form of

$$c(m)=\frac{1}{1+\left(\dfrac{m}{k_f}\right)^p}-sgn\cdot\frac{\alpha}{1+\left(\dfrac{(m-k_b)}{\omega}\right)^{2q}}. \tag{13}$$

Exponent parameters p, q were chosen to be 2 and 1 respectively, and $[k_f,k_b,\omega]=[1,10,5]*MAL$ as defined in [6], where MAL is mean absolute laplaceian calculated in a window.

Function sgn is the diffusion direction function, which controls the diffusion mode and in this paper we choose it to be the impulse detector

$$sgn = \begin{cases} -h & if\ IM\_DETE \\ l & otherwise \end{cases}, \qquad (14)$$

where h and l are positive parameters of amplification, and IM_DETE is the impulsive noise detector of [8]

$$IM\_DETE = \left(\left\lfloor R\left(x_{ij}\right) \le s \right\rfloor \vee \left\lfloor R\left(x_{ij}\right) \ge N - s + 1 \right\rfloor \right) \wedge \left(d_{ij} \ge \Theta\right), \quad (15)$$

where $R(x)$ is the function that returns the rank of an element x in the variational series, N is the length of the variational series, s and Θ are thresholds, and d_{ij} is the difference between the pixel of interest and its closest neighbor in the variational series and has the form of

$$d_{ij} = \begin{cases} \left| x_{ij} - Var\left[R\left(x_{ij}\right) - 1\right] \right| & if\ R\left(x_{ij}\right) > \underset{i,j}{MED} \\ \left| x_{ij} - Var\left[R\left(x_{ij}\right) + 1\right] \right| & if\ R\left(x_{ij}\right) < \underset{i,j}{MED} \\ 0 & otherwise \end{cases}, \quad (16)$$

where $Var(k)$ returns the value of the pixel whose rank is k, $\underset{i,j}{MED}$ is the median in a local window around ij^{th} pixel.

A diffusion process defined by c such as in equation (13) is called composite diffusion process. Composite diffusion can switch adaptively between forward, used for suppressing oscillations and reducing noise, and backward diffusion (at medium gradients, where singularities are expected), used for enhancing and sharpening images. When images are corrupted by impulsive noise, composite diffusion can stop the backward diffusion at relevant location and switches back to forward diffusion to reduce it.

We note that the composite diffusion processes defined above are different from P-M model and FAB diffusion processes, but it doesn't damage the experimental results in the following part.

4. Examples

Examples of image restoration and enhancement using above composite diffusion are shown below.

In Fig.1[1] we compare the results of fourth-order PDE proposed in this paper and the relevant fourth-order FAB process. The fourth-order PDE process indeed avoids blocky effects and forms clear edges, but small details are lost and the result image is a little over-smoothed, while fourth-order FAB is more effective than fourth-order PDE in edge preserving and feature enhancement.

In Fig.2 we compare the different results of fourth-order FAB and composite diffusion process. Original FAB diffusion [6] and our fourth-order FAB are both interfered by additive noise, because enhancement processes happen at medium gradients, where some additive noise is expected. While using composite diffusion, the additive noise is diminished to some extent.

5. Conclusions

Sharpening and denoising are contradictory requirements in image restoration. In this paper, we extend the previous P-M diffusion processes to higher order PDE-based local filters. By introducing diffusion direction function into diffusion coefficient, we propose a new kind of nonlinear diffusion-composite diffusion, which offers practical advantages over previous studies in enhancement of image quality.

Examples illustrate that the proposed methods work, and that sharpening and denoising can be combined together in the enhancement of images corrupted by some additive noise.

6. References

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[1] Figure 1 comes from NASA.

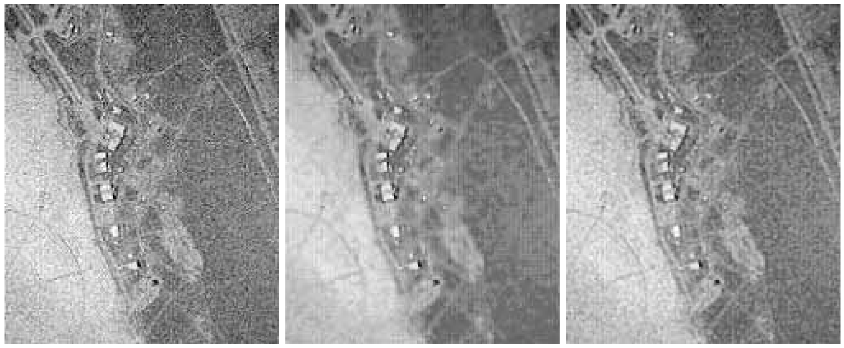

Fig. 1: Comparison between fourth-order PDE and fourth-order FAB, left – noisy, middle – fourth-order PDE, right – fourth-order FAB.

Fig. 2: Composite diffusion processes applied to canal image, with local parameter adjustment using the MAL measure, top-left – original image corrupted by salt and pepper and gaussian noise, top-right – result of fourth-order FAB applied to noisy image of top-left, bottom-left – result of composite diffusion applied to canal image corrupted by gaussian noise, bottom-right – result of composite diffusion applied to noisy image of top-left.

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