A FOCUSS based super-resolution method for ISAR imaging

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Abstract

A sparse spectrum estimation method based on FOCUSS (FOcal Underdetermined System Solver) algorithm is first presented for ISAR (Inverse Synthetic Aperture Radar) super-resolution imaging. By representing the signal with overcomplete dictionary, the sparse spectrum estimation method obtains super-resolution. And FOCUSS algorithm, as a novel class of computational strategies that combines elements of both the direct cost optimization and neural network methods, has a good convergence characteristic and a rapid convergence rate, which provides a relatively inexpensive way to accurately reconstruct sparse signals. Applying the proposed method into simulating radar data can obtain ISAR images of good quality. It shows that the proposed method is an efficient and a promising ISAR super-resolution imaging method.

Keywords: FOCUSS, super-resolution, ISAR, sparse spectrum estimation.

1. Introduction

Inverse synthetic aperture radar (ISAR) imaging is becoming increasingly important in civilian and military applications. ISAR is a well-established technique for reconstructing high-resolution radar images of targets: the resolution in range is obtained by transmitting large bandwidth signal and the resolution in cross-range is achieved by utilizing the relative motion between the target and the radar.

The conventional image formation algorithm based on Fourier transform (FFT) is known to suffer from poor resolution. What's more, it does not work properly when [1]: very high-resolution images are required or the target maneuvers. Therefore many modern spectral estimation techniques have been proposed for ISAR imaging. The adaptive FIR filtering approaches such as Capon and APES [2] can yield more accurate spectral estimates with lower sidelobes and narrower spectral estimates than the FFT. Also super-resolution imaging methods based on Hopfield network and ESPRIT [3] are proposed. However the algorithms of these methods are complex and the computation burdens are big. Therefore other new super-resolution methods should be considered.

Recently, Chen and Donoho have presented a novel super-resolution method named sparse spectrum estimation [4] for real-valued signal. It has found many successful applications.

However how is the method extended to complex-valued signals? FOCUSS [6] algorithm supplies an effective tool for sparse signal representation not only to real-valued signals but also to complex-valued signals. It starts by finding a low-resolution estimate of the sparse signal, and then, this solution is pruned to a sparse signal representation.

As ISAR super-resolution imaging problem can be modeled as a sparse spectrum estimation of complex-valued signals, the FOCUSS based sparse spectrum estimation method is first proposed for ISAR imaging in this paper.

2. ISAR imaging model

Supposing motion compensation has been done to ISAR data, the ISAR image formation problem is transformed into the turntable imaging. The geometry of radar and target is shown in Fig.1

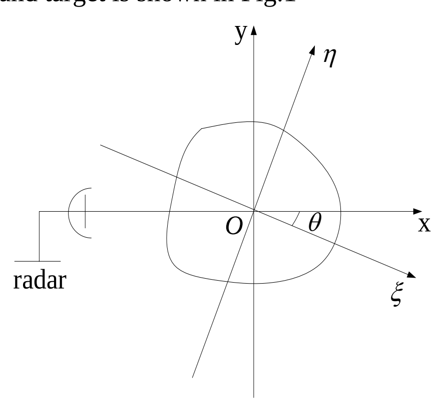

Fig. 1: The geometry of radar and target

O is the rotating center, and axis η and ξ are fixed on the target. Axis x is the radar line of sight, i.e., range direction, and axis y is cross-range direction. θ is the target's rotating angle respect to radar. Then the spatial spectrum of reflectivity function of the target $G(k_x,k_y)$ and reflectivity intensity of the target $g(\xi,\eta)$ are Fourier pairs [3]

$$\begin{cases} G(k_x,k_y)=\iint_s g(\xi,\eta)\exp(-j2\pi(\xi k_x+\eta k_y))d\xi d\eta \\ g(\xi,\eta)=\iint_\Omega G(k_x,k_y)\exp(j2\pi(k_x\xi+k_y\eta))dk_x dk_y \end{cases} \quad (1)$$

Where Ω is the area of spatial spectrum and s is the target's projection on the imaging plane; $k_x=\frac{2}{c}f\cos\theta$, $k_y=\frac{2}{c}f\sin\theta$; c —the speed of light, f — signal frequency, f_0 — the carrier frequency, B — bandwidth of signal. When the variation of θ is small and $f_0\geq 10B$, $k_x=\frac{2}{c}f$, $k_y=\frac{2}{c}f_0\theta$. Let

$$G_\xi(k_y)=\int_{\Omega k_x} G(k_x,k_y)\exp(j2\pi k_x\xi)dk_x \quad (2)$$

From formula (1) and the supposing conditions

$$g_\xi(\eta)=g(\xi,\eta)=\int_{\Omega k_y} G_\xi(k_y)\exp(j2\pi k_y\eta)dk_y$$
$$\int_{\Delta\theta} G_\xi(\theta)\exp(j\frac{4\pi}{c}f_0\theta\eta)d\theta \quad (3)$$

Where $\Delta\theta$ —the total rotating angle; $G_\xi(\theta)$ — when the target is at θ, the sum of complex reflectivity intensity of every scatter in range cell ξ.

It can be seen that formula (2) can achieve the resolution in range direction, and in fact $G_\xi(\theta)$ is the range profile. To tell apart different scatters in cross-range direction, Fourier transform can be used according to formula (3), and the resolution is $\delta_c=\frac{c}{2f_0\Delta\theta}$, which is the conventional range-Doppler method based on FFT [3]. It can be seen that resolution in cross-range is related with the rotating angle. To obtain high resolution, the variation of θ should be increased. However when the imaging angle is increased, the scatters far from the scatter center will not be focused. Therefore the imaging method based on FFT can't obtain high-resolution image. Then super-resolution technique should be used to obtain high quality ISAR image from limited radar data.

To have a good understand how to apply super-resolution technique, it is useful to describe the problem in a concise form. From formula (3)

$$G_\xi(\theta)=\int_{s\eta} g_\xi(\eta)\exp(-j\frac{4\pi}{c}f_0\theta\eta)d\eta \quad (4)$$

In discrete form

$$b(p)=G_\xi(p)=\sum_{i=0}^{N-1} g_\xi(\eta)\exp(jp\omega_i) \quad (5)$$

Where $\omega_i=\frac{4\pi}{c}f_0\eta_i$ and there are N scatters in a range cell. During the coherent processing, θ is sampled into P points, i.e., $p=0,1,\cdots,P-1$. To obtain super-resolution in cross-range, $P<N$ is supposed. Let

$$\begin{cases} b=[b(1),b(2),\cdots,b(P-1)]^T \\ x=[g_\xi(0),g_\xi(1),\cdots,g_\xi(N-1)]^T \\ e(\omega_i)=[\exp(j0\omega_i),\exp(j1\omega_i),\cdots,\exp(j(P-1)\omega_i)]^T \end{cases} \quad (6)$$

Formula (5) can be expressed in the matrix equation form

$$Ax=b \quad (7)$$

Where $A=[e(\omega_0),e(\omega_1),\cdots,e(\omega_{N-1})]$ is the $P\times N(P<N)$ matrix operator form an unknown spectral estimation $x\in c^N$ to a limited radar data set $b\in c^P$. Here, each column of A represents a sampled exponential sinusoid of some frequency. And then the ISAR imaging problem now is just to estimate x from the observed radar data b.

3. FOCUSS algorithm

Obviously, the solutions to (7) are not unique for $P<N$, i.e., imaging from limited radar data is an underdetermined problem. Thus additional criteria should be used to select the required estimate. The sparsity of the solution is the only a priori selection criterion available here.

FOCUSS algorithm provides a relatively inexpensive way to accurately reconstruct sparse signals. In FOCUSS algorithm, the low-resolution initial estimate provides the necessary extra constraint to resolve the non-uniqueness of the problem. The basis of the basic FOCUSS algorithm lies the generalized Affine Scalling Transformation (AST)

$$q=X_{K-1}^{+}x \quad (8)$$

Where $X_{K-1}=diag(x_{k-1})$ and x_{k-1} is the solution from the previous iteration.

The iterative part of FOCUSS has appeared in earlier literature for neuroimaging and spectral estimation problems. The basic form of the FOCUSS algorithm is [6]

step1: $w_{pk}=(diag(x_{k-1}))$ (9)

step2: $q_k=(Aw_{pk})^{+}b$ (10)

step3: $x_k=w_{pk}q_k$ (11)

Where k is used in subscript to denote the current iteration step and w_{pk} denotes the a posteriori weight in each iterative step.

In fact, FOCUSS is based on the solution minimizing a weighted norm $\|w^+ x\|$, where w is a matrix. It is the solution as follows [6]

$$\begin{aligned} &find\ x = wq, \\ &where\ q : \min\|q\|, subject\ to\ \mathrm{A}wq = b \end{aligned} \quad (12)$$

Where $\|q\| = \|w^+ x\|$.

To understand the convergence of the algorithm, consider the objective minimized at each step

$$\|W^+ x\|^2 = \|q\|^2 = \sum_{i=1, w_i \neq 0}^{n} (\frac{x_i}{w_i})^2 \quad (13)$$

Minimization of (13) gradually reinforces some of the already prominent entries in x while suppressing the rest until they reach machine precision and become zeros. In fact, while the entries of x_k converge to zero and nonzero values, the corresponding entries in q_k converge to zeros or ones, i.e., $q_k(i) \to 0$ as $x_k(i) \to 0$, and $q_k(j) \to 1$ as $x_k(j)$ approach nonzero values [6].

By introducing two parameters, basic FOCUSS is extended into general FOCUSS. The first extension is that the entries of x_{k-1} to be raised to some power l, as shown in (14). The second extension is the use of an additional weight matrix w_{ak}, which is independent of the a posteriori constraints. This extension makes the algorithm flexible enough to be used in many different applications. It also provides a way to input a priori information. The general form of FOCUSS then is [6]

$$w_{pk} = diag(x_{k-1}^l), \qquad l \in I_+ \quad (14)$$

$$q_k = (Aw_{ak} w_{pk})^+ b \quad (15)$$

$$x_k = w_{ak} w_{pk} q_k \quad (16)$$

Where I_+ denotes the set of all positive integers.

Like classical direct cost optimization methods, FOCUSS descends a well-defined cost function, but the function is generated in the process of computation rather than being explicitly supplied [6]. Like an associative network, FOCUSS retrieves a stored fixed state in response to an input, but no learning is involved. Learning can be added, however, if desired, to fine tune the cost function.

The convergence characteristic of FOCUSS is shown by the following theorem [6].

Theorem 1:Let x^* denote a sparse solution to (7). For any x^*, there exits a neighborhood Ω around it such that for any $x_0 \in \Omega$, the FOCUSS generated sequence $\{x_k\}_{k=0}^{\infty}$ converges to x^*. The local rate of convergence is at least quadratic for the basic algorithm and at least $2l$ for the general forms.

Hence the FOCUSS algorithm has a good convergence characteristic and a rapid convergence rate. And an initialization as close to the true solution as possible should be used. Recent research shows that the algorithms minimizing the $l_{(p\leq 1)}$ diversity measures are equivalent to FOCUSS [7]. A crucial advantage of this algorithm is that a process of regularization can be built into the algorithm to deal with noise; therefore, a model of noise is not required.

4. Experiment and discussion

Now we present examples to illustrate the performance of the FOCUSS algorithm for ISAR imaging and then give some discussions on the comparison of the FOCUSS based super-resolution method and other existed approaches.

Example 1:For simulated B-727, the stepped frequency radar operates at 9GHz and has a bandwidth of 150MHz. For each pulse, 64 complex range samples are saved. Altogether 256 successive pulses are available. The pulse repetition frequency is 20KHz. The velocity fluctuation is induced by assuming that it is a since-type function of time, which can cause the reconstructed image to be blurred. Because of maneuvering of the target, only 32 successive pulses are used for imaging. Fig2 (a) is the imaging result based on FFT. As the resolution obtained by Fourier transform is limited by the data length, the quality of the image is not satisfied. Fig2 (b) is the reconstructed image by using the FOCUSS based super-resolution method. It is obvious that the latter has a much better imaging result.

Example 2:For simulated MIG-25, the stepped frequency radar operates at 9GHz and has a bandwidth of 512MHz. For each pulse, 64 complex range samples are saved. 512 successive pulses are available. The pulse repetition frequency is 15KHz. Aircraft has a fast rotational motion with a rotation rate of $10^0 / s$ Due to the fast rotation of the target, only 1/16th of the data, i.e., 32 successive pulses are used to create an instantaneous imaging. The reconstructed images by using the Fourier transform and the method proposed in the paper are shown in Fig3 (a) and Fig3 (b), respectively. Similarly, the latter method can obtain a much better image.

The computational complexity of adaptive FIR filtering approaches and other super-resolution methods tends to limit their usage in ISAR imaging. As the adaptive FIR filtering approaches such as Capon and APES require expensive computation for each frequency of interest, the implementation becomes computationally increasingly more intensive as the number of frequency samples increased. This is especially so in 2-D applications, such as when forming ISAR images. Comparing with other exited super-resolution methods, ESPRIT algorithm can reduce computation greatly as it needn't search the spectral peaks. However as ESPRIT algorithm needs eigenvalue decomposition and generalized eigenvalue decomposition, it still requires very big computation. Comparing with the existed methods, the FOCUSS based super-resolution method not only performs well in ISAR imaging but also provides a relatively inexpensive super-resolution ISAR imaging method.

5. Conclusion

We have presented a FOCUSS based sparse spectrum estimation method for ISAR super-resolution imaging. Imaging results of experimental ISAR data have shown that the proposed method can obtain much better images than the conventional imaging method. Therefore the proposed method is an effective and a promising super-resolution imaging method for ISAR.

6. References

[1] F. Berizzi, E. Dalle Mese, M. Diani, and M. Martorella, "High-resolution ISAR imaging of maneuvering targets by means of the range instantaneous Doppler technique: modeling and performance analysis," IEEE Trans on Image Processing, pp.1880-1890, 2001.

[2] J. Li, and P. Stoica, "An adaptive filtering approach to spectral estimation and SAR imaging," IEEE Trans on Signal Processing, pp. 1469-1484, 1996.

[3] Qiang Wu, Guolin Wang, and Rongqin Xu, "EPSRIT super-resolution ISAR imaging," Systems Engineering and Electronics, 21(7), pp.33-36, 1999.

[4] S. S. Chen, and D. L. Donoho, "Application of basis pursuit in spectrum estimation," In IEEE Int. Conf. Acoust., Speech, Signal Proc. IEEE Serviece Center, Piscataway, 1998.

[5] G. Zweig, "Super-resolution Fourier transform by optimization, and ISAR imaging," IEE Proc.-Radar Sonar Navig-, pp. 247-252, 2003.

[6] I. F. Gorodnitsky, and B. D. Rao, "Sparse signal reconstruction from limited data using FOCUSS:A re-weighted minimum norm algorithm," IEEE Trans. Signal Processing, pp. 600-616, Mar. 1997.

[7] B. D. Rao, and K. Kreutz-Delgado, "An affine scaling methodology for best basis selection," IEEE Trans. Sig. Proc, pp. 187-200, January 1999.

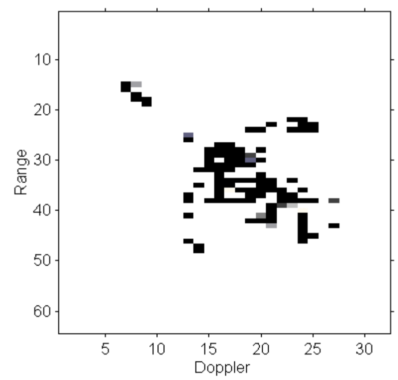

Fig.2 (a): The image of B-727 by FFT

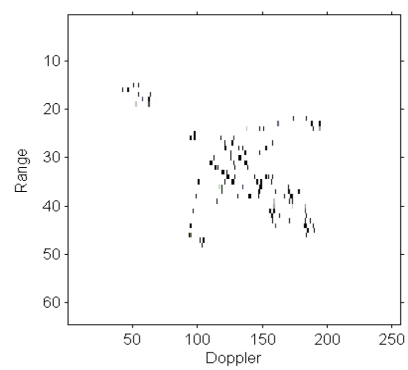

Fig.2 (b): The image of B-727 by the proposed method

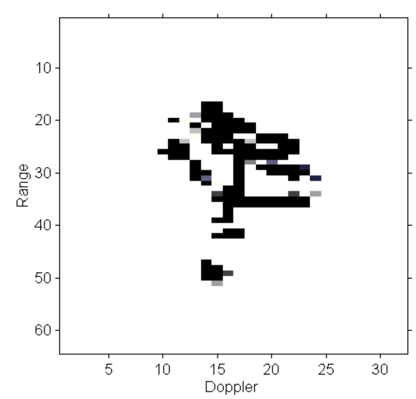

Fig.3 (a): The image of MIG-25 by FFT

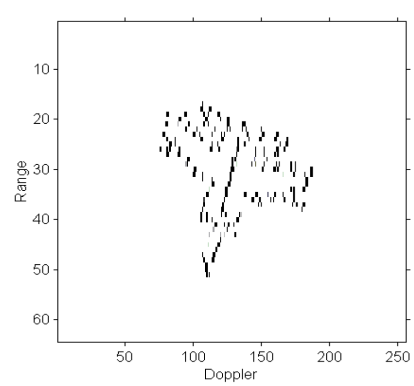

Fig.3 (b): The image of MIG-25 by the proposed method