Contour Extraction Based On Minimum Description Length

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Abstract:

Abstract: In this paper, we propose novelty way for contour extraction based on minimum description length. It be represented by minimization an energy function This function can describe properties of the image as a random field, the region homogeneity properties and the regularity properties of the contour curve. Based on this energy function, we propose the two-step approach to solve this minimization energy function by employing curve evolution, and level set method.

Keywords: Minimum Description Length, Curve evolution, Level Set, Active Contour.

1. Introduction

Contour estimation or extraction is among the most challenging, important, and frequently addressed fundamental problems in image processing and computer vision. The technique of *snake* or *active contour* has grown significantly in this field since the seminal work of Kass, Witkin, and Terzopoulos[1] including the development of geometric models based on curve evolutions[2], and the progression from edge-based models[3] to region-based models[4]. Comprehensive consideration of the properties of the image as a random field, the region homogeneity properties and the regularity of the contour curve; we describe this problem by using a *minimum description length* (MDL) type criterion. MDL is a criterion due to Rissanen [5][7], based on coding theoretical considerations; although it was not conceived within a Bayesian framework, MDL can be interpreted as corresponding to the adoption of a certain prior. Recently, MDL-type criteria have been successfully used for several problems in computer vision and image processing.

Firstly, we choose probability distribution mode as description language for the image with random field feature, and the corresponding description length is the negative logarithm of the probability distribution function of the image data; And then, we choose the gradient map as the description language for the region homogeneity properties, and the summation of the signed gradient inside the region as the description length; Lastly, the contour curve's smoothness is selected as the description language for the contour curve regularity, and the corresponding description length is the length of the boundary curve.

This paper is organized as follow. In section II, we present the basic formulation of the MDL principle. In section III, we descript our energy function for contour extraction problem in detail, and the two-step approach to solve this minimization this energy function be presented in section IV. In section V, the way is described how to get the initial contour curve by the edge map of the image. Finally, we give the result of experiment and conclusion in Section VI with summary of the paper and some further research directions.

2. Minimum Description Length Principle

MDL is an information-theoretical principle proposed by Rissanen[5][7] that allows the generalization of ML estimation to cases where not only the parameters but also their numbers are unknown. The ML estimate of a k-dimensional parameter vector $\theta_{(k)}$, given observation x, id defined as $\hat{\theta}_{(k)ML} = \arg\ \max\{p(x \mid \theta_{(k)})\}$ (subscript (k) indicates that a vector is k-dimensional). Underlying MDL is a coding theoretic interpretation of ML estimation. If, based on the probabilistic model $p(x \mid \theta_{(k)})$, one builds a Shannon-type code, the length of the code word for the observed data x is

$$L(x \mid \theta_{(k)}) = -\ln p(x \mid \theta_{(k)}) \quad (1)$$

with rounding effects neglected. Accordingly, given observation x, looking for $\hat{\theta}_{(k)ML}$ is the same as looking for the Shannon-type code in which x has the shortest code word; in fact, from (1)

$$\hat{\theta}_{(k)ML} = \arg\ \max_{\theta_{(k)}} \{p(x \mid \theta_{(k)})\} = \arg\ \max_{\theta_{(k)}} L(x \mid \theta_{(k)}) \quad (2)$$

Of course, it can be argued that only discrete data can have finite code lengths. However, as Rissanen recently noted [8], even the negative log of densities can be seen as code lengths; finite values may be obtained by truncating to finite precision and replacing the densities with the resulting probabilities. Abuse of the term "code length" is convenient and harmless, since the precision itself is not important.

When k is unknown, ML cannot be used; the MDL principle stipulates that one should still look for the shortest description (code length) of the data, which in fact must also include the parameters themselves. The total length of the optimal code for x, give k-dimensional $\theta_{(k)}$, is

$$L(x,\theta_{(k)},k) = L(x \mid \theta_{(k)}) + L(\theta_{(k)}) + L(k) \quad (3)$$

where $L(\theta_{(k)})$ is the code length for a k-dimensional $\theta_{(k)}$, and L(k) is the code length for k itself (usually a constant, independent of k). The MDL estimate of k and $\theta_{(k)}$ is then [after dropping L(k)]

$$(\hat{k},\hat{\theta}_{(k)})_{MDL} = \arg \min_{k,\theta_{(k)}} \left\{ -\ln p(x \mid \theta_{(k)}) + L(\theta_{(k)}) \right\} \quad (4)$$

Notice that, if $L(\theta_{(k)})$ only depends on k, then for fixed k the MDL and ML estimates coincide. As a corollary, the MDL estimate of $\theta_{(k)}$ coincides with its ML estimate given the MDL estimate of k.

3. Energy Function for Contour Extraction

Assume that only a noisy version image **y** is observed, the image model **x** is random field model. Base on MDL, contour extraction is equivalent to minimum the energy function $|\mathrm{L}(\mathbf{y})| + |\mathrm{L}(\mathbf{y}|\mathbf{x})|$, where $L(\bullet)$ is the description language, $|\bullet|$ is the description length (or the length code word). Then $|\mathrm{L}(\mathbf{y})|$ can describe the length of the code word for the observed image **y** after given the image model, and $|\mathrm{L}(\mathbf{y}|\mathbf{x})|$ present the code length of the image model **x**.

3.1. The length of the code word for the observed image y

In view of the region homogeneity properties of the image, we can get the length of the code word for the observed image **y** from the pdf.

$$|L(\mathbf{y} \mid \mathbf{x})| = -\sum \ln p(\mathbf{y} \mid \mathbf{x}) = -\sum_{t=1}^{N} \sum_{(i,j)\in R_t} \ln P(\mathbf{y}(i,j) \mid \phi_t) \quad (5)$$

where, on the assumption that the observed image be consisted of N homogeneous regions $R_t (t = 1,\cdots,N)$, ϕ_t is the pdf's parameters of the region R_t.

3.2. The length of the code word for the image model x

Because the image is consisted of the homogeneous region, the code length for image model is equivalent to the description length for every homogenous region. So, the code length is the sum of the code length of every region's regularity and the code length of the region homogeneity properties. The code length of the region's regularity can be presented by [6]

$$\frac{b}{2}\left|\Gamma_{R_t}\right| \quad (6)$$

where $\left|\Gamma_{R_t}\right|$ is the contour curve length of the region R_t, *b* is the sum of (a) the number of bits required to encode each element in the chain code and (b) the number of bits required to encode the constant intensity and starting element, divided by the average region-boundary length.

The code length of the region homogeneity properties can be described by the sum of the signed gradient map inside the region. It can be presented by

$$-\sum_{(i,j)\in R_t} G_{.} * sign(U_p) \quad (7)$$

where, $G_{.}$ is the gradient map of the image, U_p is the degree of uncertainty of the pixel. U_p , and $sign(U_p)$ can be presented by

$$U_p = \begin{Bmatrix} -P(\mathbf{y}(i,j) \mid \phi_t) \\ -P(\mathbf{y}(i,j) \mid \hat{\phi}_t) \end{Bmatrix}$$

$$sign(U_p(i,j)) = \begin{cases} +1 \text{ if } (i,j) \in R_t \cap P(\mathbf{y}(i,j) \mid \phi_t) \geq P(\mathbf{y}(i,j) \mid \hat{\phi}_t) \\ -1 \text{ if } (i,j) \in R_t \cap P(\mathbf{y}(i,j) \mid \phi_t) < P(\mathbf{y}(i,j) \mid \hat{\phi}_t) \\ 0 \quad \text{if } (i,j) \notin R_t \end{cases}$$

where ϕ_t and $\hat{\phi}_t$ is the pdf's parameters of the region R_t and the corresponding background region.

From Eq. (6) and (7), we can get the code length for the image model **x**.

$$|L(\mathbf{x})| = \sum_{t=1}^{N} \left\{ \frac{b}{2}\left|\Gamma_{R_t}\right| - \sum_{(i,j)\in R_t} G_{.} * sign(U_p) \right\} \quad (8)$$

Sum Eq. (5) and (8), the total energy function can be represented by

$$|L(\mathbf{x})|+|L(\mathbf{y}\mid\mathbf{x})|=\sum_{t=1}^{N}\left\{\frac{b}{2}\left|\Gamma_{R_t}\right|-\sum_{(i,j)\in R_t} G_{\cdot} * sign(U_p)\right\}$$
$$-\sum_{t=1}^{N}\sum_{(i,j)\in R_t}\ln P(\mathbf{y}(i,j)\mid\phi_t)$$
$$\triangleq E(N,\phi_t,C_t) \quad (9)$$

It is obviously that the energy function can describe properties of the image as a random field, the region homogeneity properties and the regularity properties of the contour curves from about deduction.

4. Algorithm Implementation

It is too difficulty to minimize (9) in indirectly, we address this minimization problem with two-step approach according to the idea of the EM algorithm. So, this problem can be separated into two steps: (1) solve ϕ_t with N, C_t fixed; (2) solve N, C_t with ϕ_t fixed.

4.1.Solving for ϕ_t, given N and C_t

Our first step is estimate the parameters ϕ_t of the pdf for fixed N and C_t. The minimization w.r.t ϕ_t can be rewritten as

$$E(\phi_t)=-\sum_{t=1}^{N}\sum_{(i,j)\in R_t}\ln P(\mathbf{y}(i,j)\mid\phi_t) \quad (10)$$

In fact, minimization in (10) is the ML estimates for maximization joint density function. For Gauss-Markov random field image model with parameter $\phi_t=[\mu_t,\sigma_t^2]$, this step consists simply of computing the every region sample mean and variance. For other random field image model, we can use the correlative way to estimation the parameter of the image model.

4.2.Solving N and C_t, given ϕ_t

At this case, (9) can be formulated as

$$E(N,C_t)=\sum_{t=1}^{N}\left\{\frac{b}{2}\left|\Gamma_{R_t}\right|-\sum_{(i,j)\in R_t} G_{\cdot} * sign(U_p)\right\}$$
$$-\sum_{t=1}^{N}\sum_{(i,j)\in R_t}\ln P(\mathbf{y}(i,j)\mid\phi_t) \quad (11)$$

According to the variational principle, we can get equations of motions for any point V on the contour curve.

$$\frac{dV}{dt}=-b\kappa_{V(R_t)}\vec{\mathbf{N}}_{V(R_t)}$$
$$+G_V sign\left(\ln p(\mathbf{y}_V\mid\phi_t)-\ln p(\mathbf{y}_V\mid\widehat{\phi}_t)\right)\vec{\mathbf{N}}_{V(R_t)}$$
$$+\left(\ln p(\mathbf{y}_V\mid\phi_t)-\ln p(\mathbf{y}_V\mid\widehat{\phi}_t)\right)\vec{\mathbf{N}}_{V(R_t)} \quad (12)$$

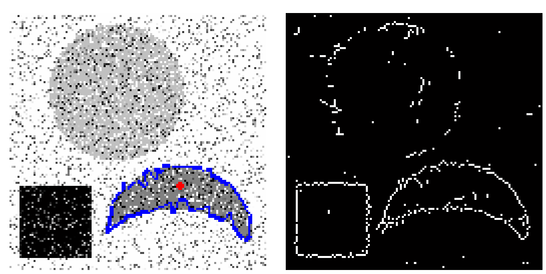
where $\kappa_{V(R_t)}$ is the curvature of the contour curve of the region R_t at the point V, $\vec{\mathbf{N}}_{V(R_t)}$ is the outside normal of the contour curve of the region R_t at the point V, G_V is the gradient at pixel V, and the $\mathbf{y}_v$ is the observed image at the pixel V. Eq. (12) can be solved by curve evolution principle. It can be presented by level set equation.

$$\frac{\partial\varphi}{\partial t}=\left(-b\kappa+G_V sign\left(\ln p(\mathbf{y}_V\mid\phi_t)-\ln p(\mathbf{y}_V\mid\widehat{\phi}_t)\right)\right)|\nabla\varphi|$$
$$+\left(\left(\ln p(\mathbf{y}_V\mid\phi_t)-\ln p(\mathbf{y}_V\mid\widehat{\phi}_t)\right)\right)|\nabla\varphi| \quad (13)$$

where the contour curve C, which be consisted of the points V in the index (12), is the zero level set of the level set function φ. We carry out above two steps alternately until the algorithm convergence.

5. Initial Contour Curve

To solve the PDE of the index (13), we must give the initial contour curve. The traditional way is to obtain the initial contour by manual. The edge detection is the technology of the image segmentation based on the image's local characteristic, e.g., the differential operator and canny operator. The edge detection can give the edge map of the image, but the edge points which be detected by these ways is complete independence for each other and they can't to form the significant curve for our contour extraction. In the fact, the edge map can give great information for the contour extraction.



(a) (b)

Fig.1 (a)The original image with seed point (the red point) and the initial contour obtained by our way (the blue line); (b) the edge map of (a)

In this manuscript, we only need one given point (seed point) to get the initial contour curve. At firstly, the edge map can be easily obtained by the edge detection (e.g., Canny detection); then the initial curve can be form in deasil or anticlockwise from the edge points which is the nearest point away from the seed point along the some direction. Although the initial curve obtained by this way is not very accurate, we only need the approximately contour curve in this step. Fig. 1 illustrate this way, Fig. 1(a) is the original image and the red point is the seed point, Fig. 1(b) is the edge map of the original image, the blue line in Fig. 1(a) is the initial contour curve obtained by this way.

6. Experiments and Conclusion

The first example (Fig. 2) use synthetic images obeying the Gaussian-Markov random field model. The left row of the Fig. 2 is the original image with the different seed point, and the right row of the Fig. 2 is the corresponding extraction result. Our way can extract the different contour curve according the different seed point and can be used to capture the contour of the target. The second example use (Fig. 3) the real image "heart". The dot line is the initial contour provided by the user, and the line is the estimation contour.

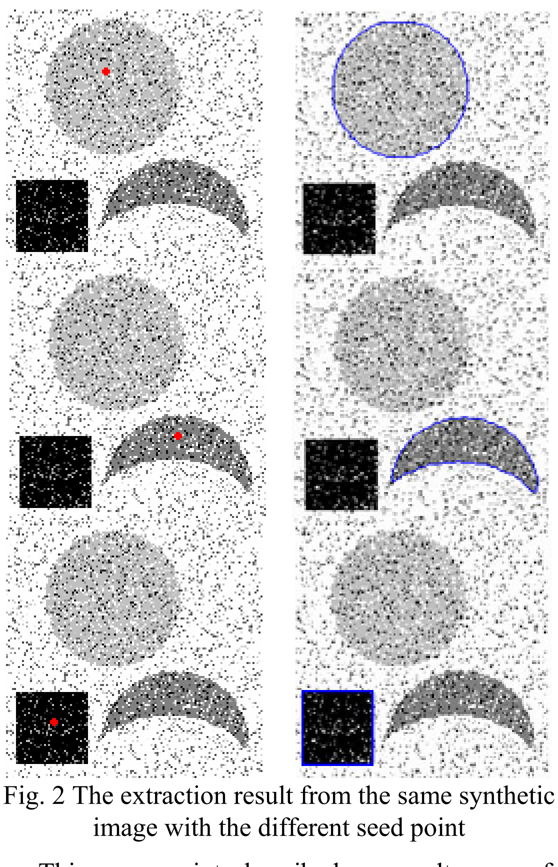


Fig. 2 The extraction result from the same synthetic image with the different seed point

This manuscript described a novelty way for contour extraction based on minimum description length. It is represented by the minimization the energy function. Based on comprehensive view of image model, region homogeneity, and the regularity of the contour curve, we proposed the new energy function for this problem. This function can describe properties of the image as a random field, the region homogeneity properties and the regularity properties of the contour curve. Based on this energy function, we propose the two-step approach to solve this minimization energy function by employing curve evolution and level set method. It can be easily used as image segmentation, image restoration, and contour representation and estimation by some little modification.

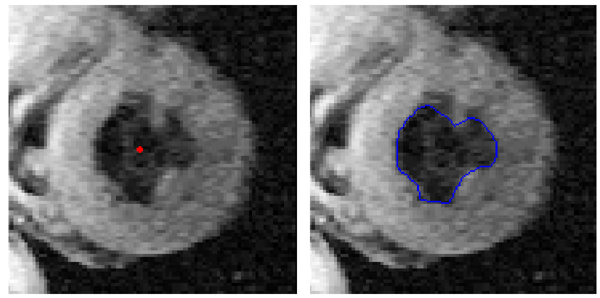


Fig. 3 Example with "heart", the left image is the original image with the seed point(red) and the right is the result

Acknowledgments: This work was supported by the National Natural Science Foundation of China (No. 60441002) And the University Key Research Project (No. 2003SZ002).

References

[1] M. Kass, A. Within and D. Terzopoulos, Snake: Active contour models, Int. J. Comput. Vis., Vol. 1, pp.321-331, 1987

[2] R. Malladi, J. Sethain, and B. Vemuri, Shape modeling with front propagation: A level set approach, IEEE Trans. PAMI, Vol. 17, pp. 158-175, 1995

[3] A.Yezzi, S. Kichenassamy, A. Kumar, P. Olver, and A. Tannenbaum, A geometric snake model for segmentation of medical imagery, IEEE Trans. Med. Image., Vol. 16, pp.199-209, 1997

[4] S. Zhu and S. Yuille, Region competition: Unifying snakes, region growing, and Bayes/MDL for multiband image segmentation, IEEE Trans. PAMI, Vol. 18, pp.884-900, 1996

[5] J. Rissanen, Universal coding, information, prediction, and estimation, IEEE Trans. Information Theory, Vol. IT-30, pp.629-636, 1984

[6] Y. G. Leclerc, Constructing simple stable descriptions for image portioning, International Journal of Computer Vision, Vol. 13, pp.73~102, 1989

[7] J. Rissanen, A universal prior for integers and estimation by minimum description length, Ann. Stat., Vol. 11, pp. 416-431, 1983

[8] J. Rissanen, Fisher information and stochastic complexity, IEEE Trans. Information Theory, Vol. IT-42, pp. 40-47, 1996