A Fuzzy Multicriteria Decision Making for Distribution Center Location Selection

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Abstract

A fuzzy multicriteria decision making (Fuzzy MCDM) approach for the selection of a distribution center (DC) is suggested. The final membership function of each DC can be developed through operations of α-cuts and interval arithmetic of fuzzy numbers. A normalization approach is applied to ensure compatibility for criteria. Finally, a ranking method is used for the ordering of all distribution centers.

Keywords: Fuzzy MCDM, Distribution Center, Normalization, Ranking

1. Introduction

A distribution center (DC) is usually supplied by the sources such as vendors, manufacturing companies, etc., and in turn it supplies the demands and/or customers. A DC can be regarded as the competency that links an enterprise with its customers and suppliers [4]. To reduce transportation cost, enhance logistic performance and operation efficiency, selecting a suitable DC location is a very important issue for both distribution and manufacturing industries.

Numerous precision-based approaches for the problems of location selection have been studied [1,9,12,14-18]. All the above methods are based on the concept of accurate measure and crisp evaluation. However, the values of the qualitative criteria are often imprecisely defined by the decision makers. Furthermore, the desired values and importance weights of criteria are usually assessed by linguistic values [19], such "medium", "fair", "high", etc. Clearly, the precision-based methods are not adequate to solve the DC location selection problem. Therefore, Chen [4] proposed a fuzzy multiple criteria decision making [2,5,10] approach to select the location of a DC. Despite many merits, Chen method [4] produces a limitation of developing a triangular shape for the multiplication of two triangular fuzzy numbers. To resolve the above limitation, this work suggests a multicriteria decision making approach for the DC location selection problem.

Through the proposed method, the membership function of the final fuzzy evaluation value of each DC can be developed via operations of interval arithmetic and of α-cuts of fuzzy numbers. Criteria are classified into benefit and cost [7]. A normalization method is applied to ensure the compatibility of all criteria. Finally, a ranking method is used to defuzzify all the final fuzzy evaluation values for the ordering of all distribution centers. The proposed method can also be applied to other fuzzy management problems.

2. Fuzzy number

Definition 1. A real fuzzy number *A*, usually assumed convex, normal and bounded, is described as any fuzzy subset of the real line *R* with membership function f_A which possesses the following properties [11]:

(a) f_A is a continuous mapping from *R* to the closed interval [0, 1];
(b) $f_A(x)$ =0, for all $x \in (-\infty, a]$;
(c) f_A is strictly increasing on $[a,b]$;
(d) $f_A(x)$ =1, for all $x \in [b,c]$;
(e) f_A is strictly decreasing on $[c,d]$;
(f) $f_A(x)$ =0, for all $x \in [d,\infty)$,

The f_A with left, $f_A^{\mathrm{L}}(x)$ and right, $f_A^{\mathrm{R}}(x)$, membership functions, of the fuzzy number *A* can also be expressed as:

$$f_A(x) = \begin{cases} f_A^{\mathrm{L}}(x), & a \le x \le b, \\ 1, & b \le x \le c, \\ f_A^{\mathrm{R}}(x), & c \le x \le d, \\ 0, & \text{otherwise}, \end{cases} \tag{1}$$

Definition 2. The α-cut of fuzzy number *A* can be defined as $A^\alpha = \{x \mid f_A(x) \ge \alpha,\}$, where $x \in R$, $\alpha \in [0,1]$. [13]

A^α can be denoted by $A^\alpha = [A_l^\alpha, A_u^\alpha]$. Given *A* and *B*, $A, B \in R^+$, some operations of *A* and

B can be expressed as follows [13]:

$$\left(A \oplus B\right)^{\alpha} = \left[A_l^{\alpha} + B_l^{\alpha}, A_u^{\alpha} + B_u^{\alpha}\right], \quad (2)$$

$$\left(A \ominus B\right)^{\alpha} = \left[A_l^{\alpha} - B_u^{\alpha}, A_u^{\alpha} - B_l^{\alpha}\right], \quad (3)$$

$$\left(A \otimes B\right)^{\alpha} = \left[A_l^{\alpha} \cdot B_l^{\alpha}, A_u^{\alpha} \cdot B_u^{\alpha}\right], \quad (4)$$

3. A Fuzzy Multicriteria Decision Making Method for DC Location Selection

Assume that a committee of k decision-makers (*i.e.*, $D_1, D_2, ..., D_k$) is responsible for evaluating m alternatives DC (*i.e.*, $A_1, A_2, ..., A_m$) under n criteria (*i.e.*, $C_1, C_2, ..., C_n$), where the ratings of alternatives under different criteria as well as the importance weights of all criteria are assessed in linguistic terms [19] represented by triangular fuzzy numbers. Both benefit and cost criteria are considered.

3.1 Aggregate ratings and perform normalization

Let $x_{ijt} = \left(a_{ijt}, b_{ijt}, c_{ijt}\right)$, $x_{ijt} \in R^{+}$, $i = 1,2,...,m$, $j = 1,2,...,n$, $t = 1,2,...,k$, be the rating assigned to alternative DC A_i by decision-maker D_t for criterion C_j. The aggregated rating, $x_{ij} = \left(a_{ij}, b_{ij}, c_{ij}\right)$, of alternative DC A_i under criterion C_j assessed by the committee of k decision-makers can be evaluated as [4,5,7]:

$$x_{ij} = \left(1/k\right) \otimes \left(x_{ij1} \oplus x_{ij2} \oplus \cdots \oplus x_{ijk}\right) \quad (5)$$

where $a_{ij} = \sum_{t=1}^{k} a_{ijt} / k$, $b_{ij} = \sum_{t=1}^{k} b_{ijt} / k$, $c_{ij} = \sum_{t=1}^{k} c_{ijt} / k$.

To ensure compatibility between benefit and cost criteria, the following normalization formulas [3]are applied. This method makes the ranges of normalized triangular fuzzy numbers in [0,1].

$$r_{ij} = \left(\frac{a_{ij} - a_j^*}{c_j^* - a_j^*}, \frac{b_{ij} - a_j^*}{c_j^* - a_j^*}, \frac{c_{ij} - a_j^*}{c_j^* - a_j^*}\right), \quad j \in B, \quad (6)$$

$$r_{ij} = \left(\frac{c_j^* - c_{ij}}{c_j^* - a_j^*}, \frac{c_j^* - b_{ij}}{c_j^* - a_j^*}, \frac{c_j^* - a_{ij}}{c_j^* - a_j^*}\right), \quad j \in C, \quad (7)$$

where r_{ij} is the normalized value of x_{ij}, and $a_j^* = \min_i a_{ij}$, $c_j^* = \max_i c_{ij}$, for each j.

3.2 Aggregate importance weights

Let $w_{jt} = \left(o_{jt}, p_{jt}, q_{jt}\right)$, $w_{jt} \in R^{+}$, $j = 1,2,...,n$, $t = 1,2,...,k$, be the importance weight given by decision-maker D_t for criterion C_j. The aggregated importance weight, $w_j = \left(o_j, p_j, q_j\right)$, of criterion C_j assessed by the committee of k decision-makers can be evaluated as [4,5,7]:

$$w_j = \left(1/k\right) \otimes \left(w_{j1} \oplus w_{j2} \oplus \cdots \oplus w_{jk}\right) \quad (8)$$

where $o_j = \sum_{t=1}^{k} o_{jt} / k$, $p_j = \sum_{t=1}^{k} p_{jt} / k$, $q_j = \sum_{t=1}^{k} q_{jt} / k$.

3.3 Produce final membership functions

The final fuzzy evaluation value of each alternative DC A_i can be obtained by using the simple additive weighting (SAW) concept [5] as follows:

$$P_i = \sum_{j=1}^{n} r_{ij} \otimes w_j, \quad i = 1 \sim m, \quad (9)$$

where P_i is the final fuzzy evaluation value of each alternative DC A_i.

The membership function of the P_i can be developed as follows:

Let

$$(e_{ij}, f_{ij}, g_{ij}) = \begin{cases} \left(\frac{a_{ij} - a_j^*}{c_j^* - a_j^*}, \frac{b_{ij} - a_j^*}{c_j^* - a_j^*}, \frac{c_{ij} - a_j^*}{c_j^* - a_j^*}\right), j \in B \\ \left(\frac{c_j^* - c_{ij}}{c_j^* - a_j^*}, \frac{c_j^* - b_{ij}}{c_j^* - a_j^*}, \frac{c_j^* - a_{ij}}{c_j^* - a_j^*}\right), j \in C \end{cases} \quad (10)$$

By applying Eqs. (2)~(4) to obtain the α-cuts of P_i as follows:

$$P_i^{\alpha} = \sum_{j=1}^{n} r_{ij}^{\alpha} \otimes w_j^{\alpha}$$

$$= \left[\sum_{j=1}^{n} \left(f_{ij} - e_{ij}\right)\left(p_j - o_j\right)\alpha^2 + \right.$$

$$\sum_{j=1}^{n}\left[e_{ij}\left(p_j-o_j\right)+o_j\left(f_{ij}-e_{ij}\right)\right]\alpha+\sum_{j=1}^{n}e_{ij}o_j,$$
$$\sum_{j=1}^{n}\left(f_{ij}-g_{ij}\right)\left(p_j-q_j\right)\alpha^2+$$
$$\left.\sum_{j=1}^{n}\left[g_{ij}\left(p_j-q_j\right)+q_j\left(f_{ij}-g_{ij}\right)\right]\alpha+\sum_{j=1}^{n}g_{ij}q_j\right] \tag{11}$$

We now have two equations to solve, namely:

$$\sum_{j=1}^{n}\left(f_{ij}-e_{ij}\right)\left(p_j-o_j\right)\alpha^2+ \sum_{j=1}^{n}\left[e_{ij}\left(p_j-o_j\right)+o_j\left(f_{ij}-e_{ij}\right)\right]\alpha+\sum_{j=1}^{n}e_{ij}o_j-x=0 \tag{12}$$

$$\sum_{j=1}^{n}\left(f_{ij}-g_{ij}\right)\left(p_j-q_j\right)\alpha^2+ \sum_{j=1}^{n}\left[g_{ij}\left(p_j-q_j\right)+q_j\left(f_{ij}-g_{ij}\right)\right]\alpha+\sum_{j=1}^{n}g_{ij}q_j-x=0 \tag{13}$$

Let $G_{i1}=\sum_{j=1}^{n}\left(f_{ij}-e_{ij}\right)\left(p_j-o_j\right)$,

$$H_{i1}=\sum_{j=1}^{n}\left[e_{ij}\left(p_j-o_j\right)+o_j\left(f_{ij}-e_{ij}\right)\right],$$

$$G_{i2}=\sum_{j=1}^{n}\left(f_{ij}-g_{ij}\right)\left(p_j-q_j\right),$$

$$H_{i2}=\sum_{j=1}^{n}\left[g_{ij}\left(p_j-q_j\right)+q_j\left(f_{ij}-g_{ij}\right)\right],$$

$$V_i=\sum_{j=1}^{n}e_{ij}o_j,\quad Y_i=\sum_{j=1}^{n}f_{ij}p_j,\quad Z_i=\sum_{j=1}^{n}g_{ij}q_j.$$

Equations (12) and (13) can be expressed as:

$$G_{i1}\alpha^2+H_{i1}\alpha+V_i-x=0 \tag{14}$$

$$G_{i2}\alpha^2+H_{i2}\alpha+Z_i-x=0 \tag{15}$$

Only roots in [0,1] are retained in Eqs. (14) and (15). The left membership function, *i.e.* $f_{P_i}^{\mathrm{L}}(x)$, and the right membership function, *i.e.* $f_{P_i}^{\mathrm{R}}(x)$, of the final fuzzy evaluation value P_i can be produced as follows:

$$f_{P_i}^{\mathrm{L}}(x)=\left\{-H_{i1}+\left[H_{i1}^2+4G_{i1}\left(x-V_i\right)\right]^{1/2}\right\}\Big/2G_{i1},\quad V_i\le x\le Y_i, \tag{16}$$

$$f_{P_i}^{\mathrm{R}}(x)=\left\{-H_{i2}-\left[H_{i2}^2+4G_{i2}\left(x-Z_i\right)\right]^{1/2}\right\}\Big/2G_{i2},\quad Y_i\le x\le Z_i. \tag{17}$$

Clearly, P_i may not yield a triangular shape. Only when $G_{i1}=0$ and $G_{i2}=0$, P_i is a triangular fuzzy number, that is $f_{P_i}^{\mathrm{L}}(x)=(x-V_i)/H_{i1}$, $V_i\le x\le Y_i$, and $f_{P_i}^{\mathrm{R}}(x)=(x-Z_i)/H_{i2}$, $Y_i\le x\le Z_i$. P_i can also be denoted as

$$P_i=(V_i,Y_i,Z_i;H_{i1},G_{i1};H_{i2},G_{i2}),\ i=1\sim m. \tag{18}$$

3.4 Defuzzification

Numerous defuzzification/ranking methods have been studied. Each method has its strength and weakness [6]. Herein, for convenience, the method of an area between the centroid and original points of a fuzzy number from [8] is applied to defuzzify all the final fuzzy evaluation values for the ordering of the alternative distribution centers.

The centroid point of a fuzzy number corresponds to an $\bar{x}$ value on the horizontal axis and an $\bar{y}$ value on the vertical axis. The centroid point ($\bar{x}$, $\bar{y}$) for a fuzzy number P in Definition 1is defined as [6,8]:

$$\bar{x}(P)=\frac{\int_a^b (xf_P^{\mathrm{L}})\,\mathrm{d}x+\int_b^c x\,\mathrm{d}x+\int_c^d (xf_P^{\mathrm{R}})\,\mathrm{d}x}{\int_a^b (f_P^{\mathrm{L}})\,\mathrm{d}x+\int_b^c \mathrm{d}x+\int_c^d (f_P^{\mathrm{R}})\,\mathrm{d}x}, \tag{19}$$

$$\bar{y}(P)=\frac{\int_0^1 (yg_P^{\mathrm{L}})\,\mathrm{d}y+\int_0^1 (yg_P^{\mathrm{R}})\,\mathrm{d}y}{\int_0^1 (g_P^{\mathrm{L}})\,\mathrm{d}y+\int_0^1 (g_P^{\mathrm{R}})\,\mathrm{d}y}, \tag{20}$$

where f_P^{L} and f_P^{R} are the left and right membership functions of fuzzy number A, respectively. g_P^{L} and g_P^{R} are the inverse functions (inverse membership functions) of f_P^{L} and f_P^{R}, respectively.

The area between the centroid point ($\bar{x},\bar{y}$) and original point (0,0) of the fuzzy number P is then defined as:

$$S(P) = \overline{xy} \quad (21)$$

where $\bar{x}$ and $\bar{y}$ is the centroid point of fuzzy number P. If $S(P_i) > S(P_j)$, then $P_i > P_j$. If $S(P_i) = S(P_j)$, then $P_i = P_j$. Finally, if $S(P_i) < S(P_j)$, then $P_i < P_j$.

4. Numerical Example

The example in Chen [4] is applied to demonstrate the feasibility of the proposed method. Detailed information is clearly listed in [4]. By Eq. [18], the final fuzzy evaluation of the three candidate locations can be produced as

P_1=(0.1111,1.5645,2.9167;0.3515,1.1019;0.1516,-1.5037),
P_2=(1.7834,3.5231,4.6905;0.3373,1.4023;0.1242,-1.2916),
P_3=(1.4794,3.1416,4.3373;0.3139,1.3483;0.1022,-1.2979).

Eq. (21) obtains the values $S(P_1)$=0.7486, $S(P_2)$=1.6708 and $S(P_3)$=1.4953. The ranking of the three alternative locations is $A_2 > A_3 > A_1$. Thus city A_2 is the best location to establish a new DC, a result which is the same as that of Chen [4].

5. Conclusion

An application of a fuzzy multicriteria decision making approach to the selection of a distribution center location is proposed. In the proposed model, the membership function of the final fuzzy evaluation value of each alternative distribution center can be developed through operations of α-cuts and interval arithmetic of fuzzy numbers. A normalization approach for the aggregated ratings of alternatives versus benefit and cost criteria is used to ensure compatibility. This method makes the ranges of normalized triangular fuzzy numbers in [0,1]. A ranking approach of an area between the centroid and original points of a fuzzy number is applied to defuzzify all the final fuzzy evaluation values for the ordering of the alternatives in order to select the best distribution center. An example has verified the feasibility of the proposed method.

The suggested method provides a new approach for the available fuzzy MCDM models. The proposed method can also be applied to other fuzzy management problems.

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