Analysis of Time-frequency Conglomeration of Hilbert-Huang Transform*

Zhang Haiyong[1] Zhu Lijuan[1] Zou Yankun[2]

[1] Dalian Naval Academy, Dalian 116018 China
[2] Dalian University of Technology, Dalian 116024 China

Abstract

The mathematical expression of Hilbert-Huang time-frequency spectrum is introduced and the time-frequency conglomeration of it is discussed. The mathematical model of depicting time-frequency conglomeration with information entropy, based on information theory, is set up. Calculation and simulation results of a given example confirm Hilbert-Huang time-frequency spectrum has better time-frequency conglomeration when compared with short-time Fourier transform, wavelet analysis, and Wigner-Ville distribution. Finally, measures to improve the time-frequency conglomeration of Hilbert-Huang spectrum are presented. It is of great significance to perfecting the theory of Hilbert-Huang transform.

Keywords: Time-frequency conglomeration, Hilbert-Huang transform, information entropy.

1. Introduction

Huang et al. [1] developed a new method of time-frequency analysis. The method is composed of two parts, Empirical Mode Decomposition (EMD) and Hilbert transform. Owing to the great contribution by Huang, the new method is named "Hilbert-Huang Transform" (HHT). The key part of the method is EMD method with which the signal is decomposed into a finite and often small number of "Intrinsic Mode Function" (IMF) that admit well-behaved Hilbert transform. With the Hilbert transform, the IMFs yield instantaneous frequencies as a function of time. The final result is a 3-dimensional amplitude (energy) - frequency-time spectrum designated as Hilbert-Huang spectrum. Time-frequency conglomeration is usually used to evaluate the performance of time-frequency analysis methods. Traditionally, time-frequency conglomeration is depicted through unaided eye observation, and therefore lack quantitative references. In this paper, time-frequency conglomeration is depicted quantitatively based on information theory with information entropy as the index, and the performance of Hilbert-Huang spectrum is evaluated in comparison with traditional methods of time-frequency analysis, such as short-time Fourier transform, Wigner-Ville distribution and wavelet analysis and etc.

2. Time-frequency representation of Hilbert-Huang transform

In joint time frequency analysis, the frequency must be the function of time; therefore, we need the definition of instantaneous frequency. For an arbitrary time series, $X(t)$, we can always have its Hibert Transform, $Y(t)$ as

$$Y(t)=\frac{1}{\pi}\,p\int_{-\infty}^{\infty}\frac{X(t)}{t-\tau}\,d\tau \tag{1}$$

Where P indicate Cauchy principle value. This transformation exists for all class L^p . With this definition, $X(t)$ and $Y(t)$ form the complex conjugate pair. So we have an analytic signal, $Z(t)$ as

$$Z(t)=X(t)+iY(t)=a(t)e^{i\theta(t)} \tag{2}$$

In which,

$$\begin{aligned} a(t)&=\left[X(t)^2+Y(t)^2\right]^{\frac{1}{2}} \\ \theta(t)&=\arctan\frac{Y(t)}{X(t)} \end{aligned} \tag{3}$$

Thus, the instantaneous frequency is defined as:

$$\omega=\frac{d\theta(t)}{dt} \tag{4}$$

In principle, some limitations on the signal are necessary, for the instantaneous frequency given in (4) is a single value function of time. At any given time, there is only one frequency value; therefore, it can only represent one component. For this purpose, Huang et al. [1] proposed a class of functions designated as Intrinsic Mode Functions (IMFs).

* This study is supported by the National Natural Science Foundation of China (NSFC) under contract No.60472109

For a complicated data, we can have more than one instantaneous frequency at a time locally. In order to use the definition of instantaneous frequency, an arbitrary data set have to been reduced into IMF components from which an instantaneous frequency value can be assigned to each IMF component. Therefor, Huang et al. presents the Empirical Mode Decomposition method [1] to reduce the data into the needed IMF's. Through EMD, $X(t)$ is given by [1]

$$X(t) = \sum_{j=1}^{n} c_j + r_n \tag{5}$$

In which, $c_j (j=1,\ldots,n)$ are IMFs, and r_n is the residue which can be either the mean trend or a constant.

Having obtained the IMF components, we will have no difficulties in applying the Hilbert Transform to each component, and compute the instantaneous frequency according to Eq.(4). After performing the Hilbert transform to each IMF component, the data can be express in the following form:

$$X(t) = \sum_{j=1}^{n} a_j(t) e^{i\omega_j(t)t} \tag{6}$$

Eq.(6) also enables us to represent the amplitude and the instantaneous frequency as functions of time in a three-dimensional plot, in which the amplitude can be contoured on the frequency-time plane. This frequency-time distribution of the amplitude is designated as the Hilbert-Huang spectrum, $H(\omega,t)$ as

$$H(\omega,t) = \sum b_j a_j(t) e^{i\omega_j(t)t} \tag{7}$$

when $\omega_j(t) = \omega$, $b_j=1$, or $b_j=0$.

As is shown in Eq.(7), the time-frequency representation of HHT is a linear time-frequency one which satisfies superposition principle. Bilinear time-frequency distribution such as Wigner-Ville distribution differs from linear time-frequency representation, and it breaks linear superposition principle and cause some cross-term interference. Generally, the cross-term interference is severe. At present, some methods [2] for suppressing the cross-term interference have been proposed, but most of them are at a sacrifice of the resolution.

In HHT method, the main conceptual innovations are the introduction of `intrinsic mode functions' based on local properties of the signal, which makes the instantaneous frequency meaningful; and the introduction of the instantaneous frequencies for complicated data sets, which eliminate the need for spurious harmonics to represent nonlinear and non-stationary signals. Furthermore, the frequency definition method of HHT is in consonance with classical definition method of frequency (derivative of signal phase), and differs from the scale definition method of frequency of Wavelet transform. Thereby, it can give accurate expression of frequency change of signal.

3. Quantitative description of time-frequency conglomeration

Time-frequency conglomeration is an important reference of evaluating time-frequency distribution. The better the time-frequency conglomeration is, the more densely the energy spread in time-frequency plane. Therefore, we can describe time-frequency conglomeration through the sparsity of energy spreading time-frequency plane. Information entropy is a very effective tool of describing the sparsity [3]. Information entropy is a measure of describing the uncertainty of event, and it is given by

$$S(P) = -k \sum_{i=1}^{n} p_i \log p_i \tag{8}$$

In which, $P = (p_1, p_2, \cdots, p_n)$ is the probability distribution of a series of uncertain event, and k is a constant.

By virtue of the theory, the most uncertain probability distribution has the maximum of information entropy. The more the probability distribution approximates equiprobable distribution, the bigger the value of information entropy is. The value of information entropy reflects the uniformity of probability distribution. If some time-frequency distribution is processed into a probability distribution series, its information entropy reflects the sparsity of the time-frequency distribution, i.e. time-frequency conglomeration. As for the time-frequency representation for the same signal based on different time-frequency methods, the smaller the information entropy is, the better it is.

The information entropy of time-frequency representation is calculated in the following procedure:

∗ Normalize time-frequency representation as

$$h(\omega,t) = H(\omega,t) / \max\_H \tag{9}$$

in which, $H(\omega,t)$ is time-frequency representation, and $\max\_H$ is its maximum.

∗ Partition the time-frequency plane according to frequency or time, and statistically analyze the normalized amplitude values to calculate the probability of the amplitude values.

∗ Calculate the information entropy of time-frequency representation by Eq.(8).

3.1. Example

A frequency modulated signal is shown in Fig.1. In this signal, its frequency linearly decreases between 0 and 250*ms*, and linearly increases between 250 *ms* and 500*ms*. We analyze the signal with short-time Fourier transform, Wigner-Ville distribution, Morlet wavelet spectrum, and HHT method, and the results are shown in Fig.2 to Fig.5.

Time-frequency plots shown in Fig.2 to Fig.5 all reflect that frequency varies with time. Wigner-Ville distribution and Hilbert-Huang spectrum can clearly identify frequency hopping time at 250*ms*, and both have excellent time-frequency conglomeration. But Wigner-Ville distribution causes severe cross-term interferences, and the false information directly affects the observation. The end swings of HHT method are caused by the Gibbs phenomenon.

Short-time Fourier analysis cannot clearly reflect the time of frequency hopping and its time-frequency conglomeration is very bad. Because time-frequency plane of short-time Fourier analysis is partitioned with grid, it is very difficult to find a suited short-time window function so that the signals can get the best analysis results within the time width owing to the influence of Heisenberg inequation.

The result of wavelet analysis is not perfect: frequency and energy are not well localized, and the conglomeration is bad, especially at low frequency stage. Once the basic wavelet is selected, one will have to use it to analyze all the data, therefore we cannot get the optimal analysis result. The frequency resolution of wavelet analysis is high at low frequency stage, and time resolution is high at high frequency stage, which lead to the fact that the time-frequency of wavelet analysis cannot clearly reflect the hopping at 250*ms*.

Information entropy of time-frequency distribution (TFD) based on different methods is shown in Table 1. The information entropy of Hilbert-Huang spectrum is clearly less than that of the three other time-frequency representations. Because of the effect of cross-term interferences, information entropy of Wigner-Ville distribution is maximal. The same results have been obtained through the analysis of many signals (data and figures omitted).

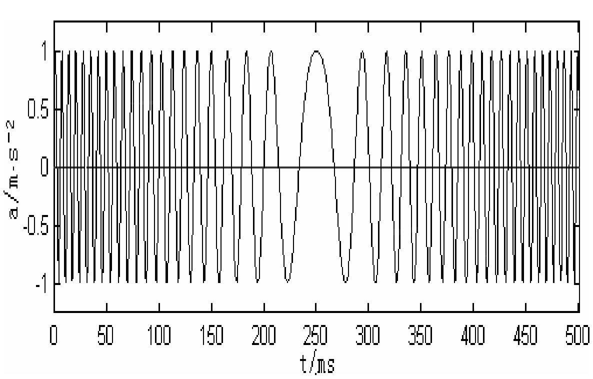

Fig.1 Frequency modulation signal

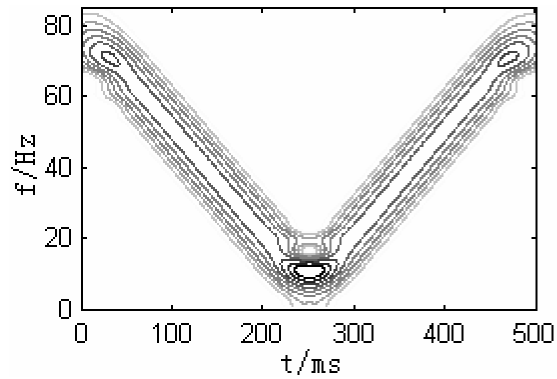

Fig.2 TFD by short-time Fourier transform

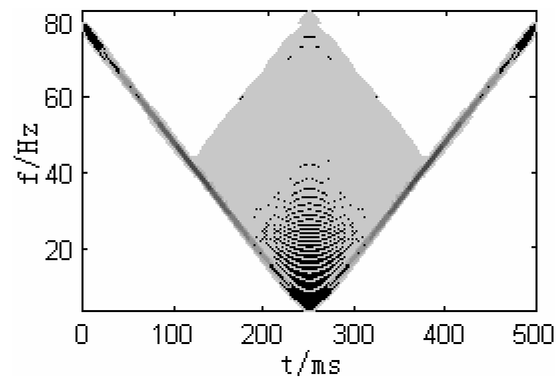

Fig.3 TFD by Wigner_Ville distribution

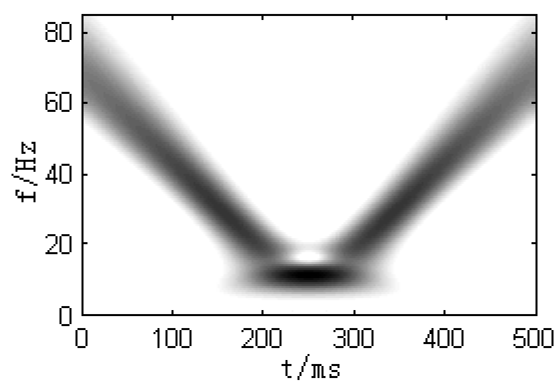

Fig.4 TFD by morlet wavelet method

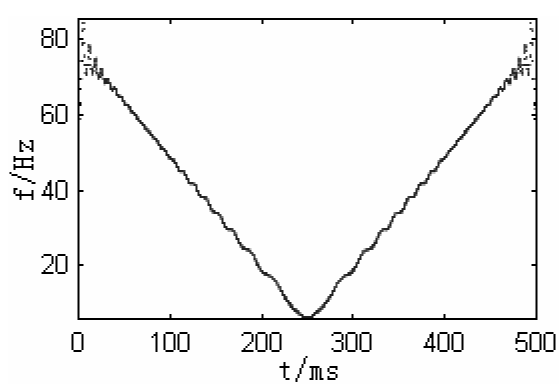

Fig.5 TFD by HHT method

Table 1 Information entropy of time-frequency distribution based on different methods

	Information entropy
Short-time Fourier analysis	1.6693×10^5
Wigner-Ville distribution	8.3597×10^5
Wavelet analysis	1.6690×10^5
Hilbert-Huang transform	2.08×10^4

4. Improvement of time-frequency conglomeration of HHT

The key to HHT is the algorithm of EMD which influences the accuracy of the signal decomposition and in turn influence the conglomeration of time-frequency. The algorithm of EMD is effective, but spline fitting, and end effects need improvement [1].

To improve spline fitting of EMD algorithm, one good way is to lessen the times of spline fitting and improve the method of getting local-mean. For instance, local-mean can be obtained through time-varying filter, that is, local-mean can be obtained by way of designing a filter to some successive extrema of all the local-mean extrema, the coefficient of filter varies with the time intervals and distance of local-mean extrema, thus to increase the accuracy of the decomposition; local-mean can also be obtained by means of the local-mean of all the two neighbouring extrma which is based on all the data of two neighbouring extrema without using envelop or successive extrema. All the original data used enhances the precision of the local-mean. In the two methods mentioned above, spline fitting is done once after the local-mean of all the extrema is deprived. Another way is to improve the algorithm of spline fitting.

The best way to restrain end effects is to find a feasible method to extend the ends of a given signal and then get two additional maximum extrema and minimum extrema in the two ends of the data. Time series predicting method, vague predicting method and neural network predicting method and so on can be used to improve the current methods which work by adding characteristic waves.

5. Conclusion

In this paper, the mathematical expression of Hilbert-Huang spectrum is introduced and the mathematical model of time-frequency conglomeration was set up based on information theory. Calculation and simulation results confirm Hilbert-Huang spectrum has better time-frequency conglomeration when compared with short-time Fourier transform, wavelet analysis, and Wigner-Ville distribution. HHT method is adaptive, and it is based on the local characteristic time scale of the signal, therefore it can effectively overcome the shortcomings of the other time-frequency analysis methods available and depict precisely the non-stationary signals in time-frequency plane.

6. References

[1] N. E. Huang, Z. Shen, S. R. long, M. C. Wu, H. H. Shih, et al., The Empirical Mode Decomposition and The Hilbert Spectrum for Nonlinear and Non-stationary Time Series Analysis, *Proc. Roy. Soc.,* London. A, Vol. 454, pp. 903-995, 1998.

[2] S.E.Qian, D.P.Chen Joint Time-Frequency Analysis, Methods and Applications Prentice Hall PTR, 1996

[3] L. C. Xiong, T. L. Shi, S. Z. Yang. Parameter Optimizing of Choi-Williams Distribution and Its Application. J. Huazhong Univ. of Sci. & Tech., 2003, Vol.31(1), pp103-104.

[4] http://www.jcis.org.