Using the Hough Transform to Determine the Roundness (Smoothness) of Rock Particles

G R Drevin
North-West University
Potchefstroom, South Africa

Abstract

A number of subjective and objective methods have been proposed to determine the roundness of rock particles, roundness being one of three properties describing the shape of a particle.

The objective roundness measures are mainly based on the Fourier transform of the polar co-ordinates of particles' edge elements. Low-pass l-ters are used to smooth the pro les of rock particle, roundness is then determined from the di erences between the original and smooth pro les.

In this paper the use of the Hough transform as an alternative objective method is proposed. While a large number of variations of the Hough transform were investigated only those variations that gave the best results are discussed in this paper.

It is shown that using both convex and concave curvatures as opposed to only convex curvatures do not have an in uence on the roundness value of a particle.

Keywords: Hough transform, roundness.

1. Introduction

The shape of sedimentary particles is an important property in determining their transport history and their behaviour in hydrodynamic systems (e.g. [11]). For this reason it is important to have an objective quantitative measure of particle shape so that the changes in shape as well as the di erences between the shapes of di erent populations can readily be identi ed.

Three independent properties are used to describe the shape of a particle [2]:

1. Form describes the overall shape or sphericity of a particle; it is a measure of the extent to which a particle approaches a sphere in shape.
2. Roundness is a measure of the extent to which the edges and corners of a particle has been rounded.
3. Surface texture describes the markings on the surface of a particle.

Both subjective and objective methods have been proposed to determine the roundness of rock particles. An objective technique for measuring the roundness of sedimentary particles is presented in this paper.

Quantitatively roundness can be de ned as the ratio of the mean radius of curvature of the corners to the radius of the maximum inscribed circle [17]:

$$\frac{\frac{1}{N}\sum_{i=1}^{N} r(i)}{R} \tag{1}$$

where $r(i)$ is the curvature of the ith corner and R is the radius of the maximum inscribed circle. A number of other quantitative measures have been proposed. However, mean roundness is only measured by the procedure proposed by Wadell [17].

The use of the Fourier transform to determine roundness has been proposed by a number of authors (e.g. [3], [4], [5], [6], [8] and [16]). The basic method is to calculate the Fourier transform of the polar co-ordinates of the edge elements of the pro le of a particle. The roundness is then based either on a low-pass ltering of the Fourier transform ([3],

[6], [8] and [16]) or the energy distribution of the Fourier coefficients ([4], [5]).

A number of pebble comparison charts have been developed to aid in the determination of roundness. Examples of these charts are those proposed by Krumbein [12], Powers [14] and Russell and Taylor [15]. Of the charts available for visual estimation Krumbein's is preferred as it has the largest number of roundness classes [2]. The argument for using the chart with the largest number of classes is that it is closest to a continuous measure. It is, however, required that adjacent classes can be distinguished [2].

2. The Hough Transform

The Hough transform [9] has been generalized to not only detect lines in images, but circles (e.g. [7], [10]) and arbitrary shapes (e.g. [1]) as well. For the detection of circles the parameterization

$$(x(i) - a)^2 + (y(i) - b)^2 = c^2 \qquad (2)$$

is used resulting in each edge pixel $(x(i), y(i))$ being transformed into a surface (cone) in the three-dimensional parameter space (a, b, c). The cones of edge pixels which form a circle will then intersect in a point, (a_0, b_0, c_0), which defines the circle with centre (a_0, b_0) and radius c_0.

In practice the parameter space is represented by a three-dimensional array of accumulators, $A(a, b, c)$, which are initially set equal to zero. For each edge pixel, with co-ordinates $(x(i), y(i))$, values of c are calculated for all values of a and b using (2). The accumulator corresponding to each parameter triplet (a, b, c) obtained in this way is then incremented. The value of the accumulator at (a, b, c) is the number of edge pixels lying on the circle with centre (a, b) and radius c. After this process has been repeated for all edge pixels the accumulators are thresholded. The accumulators that are greater than the threshold represent the circles that are present in the image.

Luo *et al* [13] proposed the use of the Hough transform for the determination of roundness of objects. In their proposal they use the radius method which is a fast implementation of the Hough transform. The memory requirements of the radius method is also much lower than is the case with

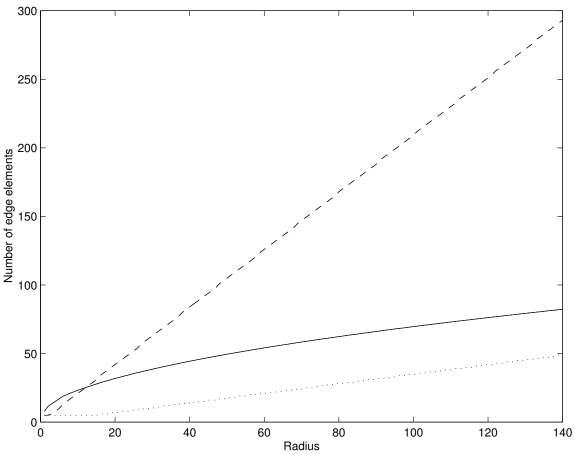

Figure 1: Thresholds used with the Hough transform: $T(c) = 2c\arccos(1 - \frac{T_n+R_n}{c})$ with $T_n + R_n = 6$ (solid line) and $T(c) = \frac{2\pi c}{\alpha}$ with $\alpha = 3$ (dashed line) and $\alpha = 18$ (dotted line).

the standard Hough transform. This method consists of the following steps:

Step 1 A temporary one-dimensional accumulator, $A(c)$, is set to zero.

Step 2 For a point, (a, b), c is calculated for every edge pixel $(x(i), y(i))$ (using 2), and the accumulator $A(c)$ incremented. The value of the accumulator at c is the number of edge pixels lying on the circle with centre (a, b) and radius c.

Step 3 The accumulator $A(c)$ is then thresholded using a thresholding function, $T(c)$.

Step 4 Of the parts of $A(c)$ that remain, the radius representing the smallest circle is stored at (a, b) in a two-dimensional array, $B(a, b)$.

Step 5 After Steps 2 to 4 have been completed for all possible centres (a, b) the array $B(a, b)$ is scanned to find the smallest circle at each of the corners of the particle.

Step 6 The inverses of the radii of these circles are then averaged and normalized by the inverse of the radius of the largest inscribed circle resulting in a single value describing the roundness of the particle.

Method	Threshold (Step 3)	Circles (Step 5)	Norm. (Step 6)
Hough1	(4), $T_n + R_n = 6$	Smallest	Inv. Radii
Hough2	(3), $\alpha = 18$	Largest	Radii
Hough3	(3), $\alpha = 7$	All	Inv. radii
Hough4	(3), $\alpha = 3$	All	Radii
Hough5	(3), $\alpha = 3$	All	Radii

Table 1: Hough transform methods used. Both convex and concave curvatures were used for Hough5 while only convex curvatures were used in the rest of the methods. Norm. is the normalization method that was used in Step 6 while Range is the minimum and maximum circle radii that were used.

Different threshold functions can be used in Step 3. The threshold

$$T(c) = \frac{2\pi c}{\alpha} \quad (3)$$

requires that a curve fit at least the proportion $\frac{1}{\alpha}$ of the circumference of the circle. Luo *et al* [13] proposed a threshold which takes into account the noise and thickness of the curve

$$T(c) = 2c\arccos(1 - \frac{T_n + R_n}{c}) \quad (4)$$

with T_n the thickness of the curve and R_n the roughness of the curve caused by noise. This threshold can discriminate between an arc of a circle and an imperfect segment of a straight line. Both threshold functions ((3) with $\alpha = 3$ and $\alpha = 18$ and (4) with $T_n + R_n = 6$) are shown in Figure 1.

The method proposed by Luo *et al* [13] was implemented and tested on the Krumbein chart resulting in a correlation of 0.88. This result does not compare very favourably with the correlation of 0.95 obtained by Diepenbroek *et al* [3] using the Fourier transform.

Variations on the method were then investigated, the best results that were achieved are are discussed in this paper. The method of Luo *et al* [13] is labelled as "Hough1", while the best variations are labelled as "Hough2" to "Hough5" (Table 1).

The following aspects of the method as described above were varied:

Step 3 Thresholding was done with the following threshold functions:

1. Equation 3. To obtain the optimal value of α, α was varied between 3 and 28. This corresponds to curves that fit between a $\frac{1}{3}$'rd and a $\frac{1}{28}$'th of the circumference of a circle. The α values that resulted in the best correlation are shown in Table 1.
2. Equation 4 with $T_n + R_n = 6$, as was proposed by Luo *et al* [13].

Step 5 Selection of circles at each corner was done as follows:

1. The smallest circle at each corner, as proposed by Luo *et al* [13].
2. The largest circle at each corner.
3. All the circles that remain after the thresholding of Step 3.

Step 6 Normalization was done as follows:

1. To approximate the method of Wadell [17] the means of the radii of the circles were normalized using the radius of the largest inscribed circle.
2. The means of the inverses of the radii of the circles were normalized using the inverse of the radius of the largest inscribed circle. Using inverse radii emphasizes the contributions of the sharper corners [13].

Range The lower and upper bounds on the size of the circles that should be used were found empirically. The lower and upper bounds that gave the best results are given in Table 1 under the heading "Range".

Curvatures Most of the existing methods only make use of convex curvatures (curvatures of which the centres of curvature are within the particle) [3]. Diepenbroek *et al* [3] stated that concave as well as plane elements also indicate a deficiency in the roundness of a particle and should also be measured. To test this suggestion method Hough4 was repeated (as method Hough5) using both convex and concave corners.

Method	Range	Corr.	CoMR	e_{RMS}
Hough1	7, 81	0.88	0.968	0.138
Hough2	9, 110	0.95	0.993	0.083
Hough3	73, 302	0.91	0.973	0.114
Hough4	15, 21	0.93	0.973	0.099
Hough5	15, 24	0.93	0.969	0.105

Table 2: Results of Hough transform methods used. Corr. is the correlation between the calculated and actual roundness values, CoMR is the correlation between the mean roundness obtained for each class and the actual roundness and e_{RMS} is the root-mean-square error between the calculated and actual roundness values.

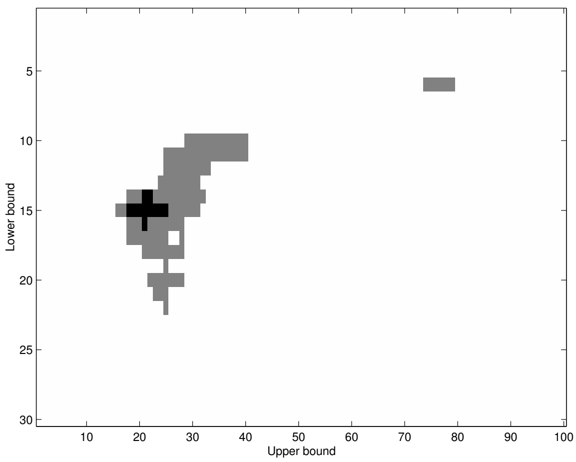

Figure 2: The (lower, upper) bound pairs which result in a correlation of 0.92 and higher (grey) and of 0.93 and higher (black) with the method Hough4. The maximum correlation obtained with Hough4 was 0.9339, with 11 (lower, upper) bound pairs resulting in a correlation of 0.93 and higher while 122 pairs resulted in a correlation of 0.92 and higher.

It should be noted that the combinations of variations of these aspects that are not discussed in this paper gave results that were worse than that obtained by the method of Luo *et al* [13].

3. Results

The results obtained are given in Table 2. Normalization of radii as opposed to inverse radii in Step 6 gave better results where (3) was used as thresholding function in Step 3 (Hough4 vs. Hough3). The combination of (3) with = 18, the use of the largest circles in Step 5 and normalization of radii in Step 6 (that is method Hough2) resulted in a correlation of 0.95 and e_{RMS} of 0.083 between the calculated and actual roundness values which was also the best performance of the Hough methods. This method also resulted in a correlation of 0.993 between the mean roundness of each class and the actual roundness.

As an alternative to using only the smallest or the largest circles in Step 5 the use of all the circles remaining after being thresholded with (3) in Step 3 was also tested. Combined with the normalization of radii, this method (Hough4) resulted in the next best performance with correlations of 0.93 and 0.973 between the calculated and actual roundness values and the mean roundness of each class and actual roundness values respectively while e_{RMS} was 0.099. The method Hough4 was further adapted to make use of both convex and concave curvatures (method Hough5) to test the recommendation made by Diepenbroek *et al* [3]. The results of these two methods were comparable with correlations of 0.933 and 0.926 for Hough4 and Hough5 respectively. Furthermore, the correlation between the roundness values obtained with these two methods was 0.98 with the correlation between their mean roundness values being 0.999. It would therefore seem su cient to determine the roundness of a particle based only on the concave corners of the particle.

Although speci c ranges of radius lengths, which give the best results for each method, are given in Table 2 it should be noted that a region of lower and upper bound combinations exist which all give acceptable results as is shown in Figure 2 for method Hough4.

4. Conclusion

A number of methods based on the Hough transform were investigated, but only one of these methods gave acceptable results. It was shown that the use of both convex and concave curvatures as opposed to only convex curvatures do not have an in uence on the roundness value of a particle.

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