Study on Similarity Measure of Super uadrics

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Abstract

Superquadrics can describe a wide variety of complex and realistic 3D shapes effectively with compact parameters. However, there are few research focused on the similarity measure for 3D shape of superquadric models, which is one key problem in superquadric-based 3D object recognition. In this paper, the similarity measure of superquadrics is studied and two novel approaches are proposed for solving this problem efficiently, i.e. 3D shape descriptors based on spherical harmonics and moment invariant. Experiments have been carried out for evaluating and comparing the performance of two proposed approaches. The research lays a foundation for 3D object recognition with superquadric models.

Keywords: 3D recognition, superquadrics, similarity measure, spherical harmonics, 3D moment invariant

1. Introduction

3D object recognition is one of the major tasks in many industrial applications and research areas, such as robotics and computer vision. Superquadrics as a family of parametric shapes can describe a wide variety of complex and realistic 3D shapes effectively with compact parameters, which have popular use in computer vision and graphics [1,2,3]. However, there are few research focused on the similarity measure of superquadric (SQ) 3D shape. In [4], the volume difference of two SQs was proposed and implemented for the similarity measure. However, there exist two main problems in the method: one is the necessary coordinate system normalization of the target SQ and that of model database; the other is the inconsistency of superquadric volume differences and their shape similarity. Therefore, the lack of efficient similarity measure method of superquadrics has been a serious bottleneck in developing SQ-based 3D object recognition systems.

This paper studies the similarity measure problem of superquadric models and proposes two novel approaches based on spherical harmonics (SH-based) and 3D moment invariant (MI-based) respectively. The processing flowchart of the proposed approaches is illustrated in Fig.1. The approaches are processed through three stages: first, superquadric fitting is implemented to 3D data; second, the 3D spherical harmonic descriptor and 3D moment invariant are extracted from superquadric parameters obtained; thirdly, the similarity measure of two SQs is performed by computing the L_2 difference between the obtained 3D rotation invariant descriptors.

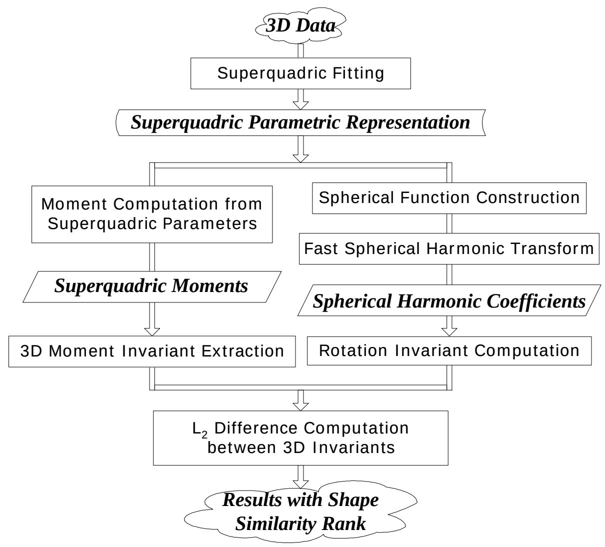

Fig.1: Processing flowchart of superquadric similarity measure based on spherical harmonics and moment invariant

2. Super uadric Models

A basic superquadric surface may be defined by an implicit equation:

$$(x,y,z)=((\tfrac{x}{a_x})^{2/\ \ 2}+(\tfrac{y}{a_y})^{2/\ \ 2})^{\ 2/\ \ 1}+(\tfrac{z}{a_z})^{2/\ \ 1}-1=0 \quad (1)$$

The modeling power of superquadrics is augmented by applying various deformations, such as bending, tapering to the basic superquadrics [1,2].

Usually, an inside-outside function is defined from (1) for superquadric fitting computation. The expanded inside-outside function with 15 parameters

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is defined for a superquadric in general position and orientation with linear tapering and bending [2].

$$F(X,Y,Z)= \quad (2)$$
$$F(X,Y,Z; a_x, a_y, a_z, \ _1, \ _2, \phi, \theta, \psi, p_x, p_y, p_z; \ _x, \ _y; \ , \alpha)$$

where a_x, a_y, a_z are defined for the size, $_1$, $_2$ for the shape, ϕ, θ, ψ for the orientation, p_x, p_y, p_z for the position in space, $_x$, $_y$ for tapering deformation, and $,\alpha$ for bending deformation control.

3. Super uadric Similarity Measure

The similarity measure of superquadrics in the flowchart (Fig.1) is processed through three stages, which are expanded in the following sections. Since many methods have been proposed to recover the parameters of superquadrics robustly from 3D data [2], we do not describe superquadric fitting in the paper.

This paper proposes two approaches for SQ similarity measure, which implement 3D shape descriptors based on spherical harmonics and moment invariant. These descriptors are 3D rotation invariant, which is very important in recognition systems.

3.1. E tracting spherical harmonic SH based descriptor

According to the spherical harmonic theory, a rotation invariant descriptor can be defined, which is to describe a spherical function in terms of energy it contains at different frequencies. Since these values do not change when the spherical function is rotated, the resulting descriptor is rotation invariant [5].

Mathematically, a spherical function with bandwidth can be determined by ^{2} coefficients c_l^m $0 \ l \ ,|m| \ l$ of its spherical harmonic expansion:

$$c_l^m = \frac{\sqrt{2\pi}}{2} \sum_{j=0}^{2-1} \sum_{k=0}^{2-1} a_j^{(\ )} \ (\theta_j, \phi_k) e^{-im\phi_k} P_l^m(\cos\theta_j) \quad (3)$$

where $(\theta_j, \phi_k) = (\pi(2j+1)/4 \ , \pi k/ \ )$ $j,k = 0,\cdots,2 \ -1$ are chosen as the samples called *Che yshev nodes*, and $a_j^{(\ )}$ are the weights [6].

- **Sampling algorithm searching Chebyshe nodes on super uadric surface**

In this paper, a novel sampling algorithm is proposed for searching Chebyshev nodes on superquadric surface, which are the intersections of the rays from superquadric centroid regularly casting in both longitudinal and latitudinal directions and superquadric surface. First, voxelize the superquadric s bounding sphere into grid by concentric spheres; then, based on the superquadric inside-outside function (2), locate the voxel regions which contain the intersections of the ray vectors and superquadric surface; finally, for more precisely locating the intersections, re-divide all the regions located in the former search, and make a finer search same as the former processing.

After obtaining the Chebyshev nodes, we compute the distances between the superquadric centroid and Chebyshev nodes as values of the discrete spherical function (θ_j, ϕ_k), $(\theta_j, \phi_k) = (\pi(2j+1)/4 \ , \pi k/ \ )$ and $j,k = 0,\cdots,2 \ -1$. B is bandwidth of (θ_j, ϕ_k) that describes the 3D shape of a superquadric model.

- **E tracting rotation in ariant with fast Spherical Harmonic Transform SHT**

Based on the mathematical theory of spherical harmonics, the spherical function obtained may be decomposed as a sum of its harmonic components, which are determined by the corresponding spherical harmonic coefficients. The harmonic coefficients corresponding to the l th frequency of may be calculated by Eq.(3), and fast SHT [6] is implemented to compute all coefficients for $|m| \ l, l = 0,\cdots, \ -1$.

Due to the fact that rotation does not change the L_2-norm of the harmonic components, we compute the L_2-norm for each frequency component, thus a rotation invariant for can be defined as the collection of scalars:

$$g^0, g^1, \cdots; g^l, \cdots; g^{\ -1} \quad (4)$$

where $g^l = \|c_l\|, c_l = [c_l^{-l}, \cdots, c_l^0, \cdots, c_l^l]$, $l = 0,\cdots, \ -1$.

3.2. E tracting moment in ariant MI based descriptor

Moment invariants are important shape descriptors in computer vision, which have been widely used in shape recognition [7,8].

- **Moment computation of super uadrics**

The 3D moment of order $n = p + \ + r$, $n \in N$ of a 3D density function (x,y,z) is defined as Riemann integral where $p, \ , r = 0,1,2,\cdots$

$$M_{p \ r} = \int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} x^p y \ z^r \ (x,y,z) dxdydz \quad (5)$$

For the superquadric, since we are interested in its solid moments, we set $(x,y,z) = 1$ inside the SQ and $(x,y,z) = 0$ outside. Due to the symmetry of a basic SQ with respect to xy plane and the origin of the coordinate system, it is easy to note that

$$p \text{ is odd} \quad \text{is odd} \quad r \text{ is odd} \quad M_{p \ r} = 0 \quad (6)$$

For the case when all of p, and r are even, a closed form expression for 3D Cartesian moment of order $p + \; + r$ of a superquadric in its canonical coordinate system [9] can be derived as:

$$M_{p\;r} = \frac{2}{p + \; + 2} a_x^{p+1} a_y^{\;+1} a_z^{r+1} \;_1 \;_2 \cdot \;((r+1)\frac{\;_1}{2}, (p + \; + 2)\frac{\;_1}{2} + 1) \cdot \;((\; + 1)\frac{\;_2}{2}, (p+1)\frac{\;_2}{2}) \quad (7)$$

where (x, y) is Beta function. Furthermore, the 3D Cartesian moment of a globally deformed superquadric in general position and orientation can be computed as a combination of 3D Cartesian moments of the corresponding nondeformed superquadric in canonical coordinate system [9].

- **E tracting 3D moment in ariant descriptor**

In this paper, two kinds of moment-based features are implemented for shape description [7,8]. We select five features as the first 3D moment invariant [7] $MI1 = [\;_1, \;_2, \;_3, \;_4, \;_5]$.

$$\;_1 = M_{200} + M_{020} + M_{002}$$

$$\;_2 = M_{200}M_{020} + M_{200}M_{002} + M_{020}M_{002} - M_{101}^2 - M_{110}^2 - M_{011}^2$$

$$\;_3 = M_{200}M_{020}M_{002} - M_{002}M_{110}^2 + 2M_{110}M_{101}M_{011} - M_{020}M_{101}^2 - M_{200}M_{011}^2$$

$$\;_4 = \;_1^2 / \;_2$$

$$\;_5 = \;_3 / \;_2\;_1$$

Another longer version of 3D moment invariant [8] $MI2 = [I_1, I_2, I_3, I_4, I_5, I_6, I_7, I_8, I_9, I_{10}, I_{11}]$ can be derived from group theory, where $I_i, i = 1, \cdots, 11$ are the combination of $M_{p\;r}, p, \;, r = 1,2,3$.

3.3. Similarity measure computation

For the similarity measure computation of two superquadrics S_1 and S_2, we calculate the L_2 difference between the two corresponding 3D rotation invariant descriptors obtained as the similarity measure metrics.

4. E perimental Results

In our experiments, we evaluate and compare the effectiveness of the two proposed approaches. The experiments include two groups, the superquadric similarity matching and the superquadric clustering based on the Geon theory [10].

4.1. Super uadric similarity matching

A model database with both basic and deformed SQs is constructed for matching simulations. Some models are selected from database for matching experiments. Fig.2 shows three matching results for basic and deformed SQs calculated by MI1, MI2 and SH based descriptors respectively. In each result, the top-left model is the query model for matching, and the set of superquadric models are arranged in a left to right, top to bottom order based on their similarity ranks to the query model with the similarity metric labeled below.

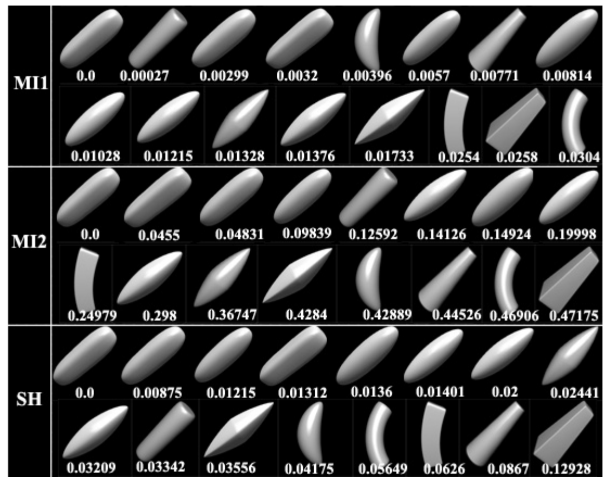


Fig.2: Marching results on the superquadrics

4.2. Super uadric clustering

Based on RBC theory [10] and the superquadric symmetry property, we choose ten geon categories for our experiment (Fig.3). The geon models selected are classified on the basis of three qualitative geometrical attributes: axis shape (**s**-straight or **b**-bent), cross-section edge shape (**s**-straight or **c**-curved), cross-section size sweeping function (**c**-constant, **t**-tapering or **id**-increasing and decreasing).

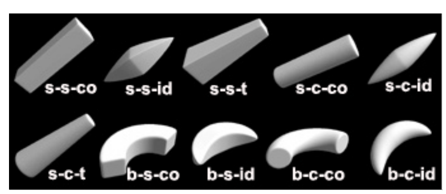
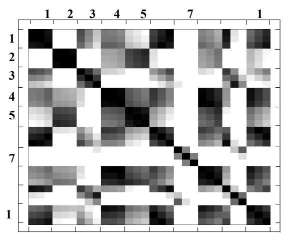


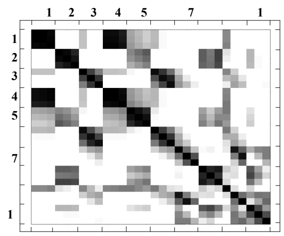
Fig.3: The set of ten geons modeled by superquadrics

We select thirty SQ models from the database belonging to the ten geon categories respectively for testing the clustering results with proposed approaches. Similarity matrix is implemented for showing the experimental results, in which the lightness of each element (i, j) is proportional to the magnitude of the computed dissimilarity between the SQ i and j. Each row and each column represent the similarity between a certain SQ and all the SQs, so the matrix is symmetric. The darker the element is, the higher similarity two relative SQs have. Some experimental results are shown by the similarity matrices in Fig.4.

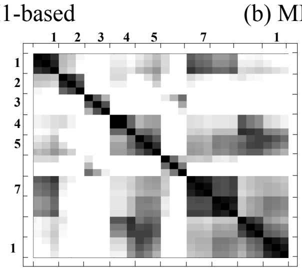
In each similarity matrix, big blocks correspond to geon categories (10 10) and little blocks to superquadric models (30 30). Each similarity matrix in Fig.4 shows a sequence of darker blocks along the main diagonal, with lighter colors in the off-diagonal matrix elements. Generally speaking, the SQs in same geon class have similar shapes, therefore, the obtained rotation invariant is similar and the corresponding L_2 difference is smaller, which corresponds to the 10 big darker blocks along the main diagonal. Experiments show that the SH-based approach can obtain the best performance and the MI1-based approach worst due to its simplicity and less features.



(a) MI1-based



(b) MI2-based



(c) SH-based

Fig.4: Similarity matrices on the basic and deformed superquadrics with geon classes.

Another experiment is implemented for further testing the performance of SH-based approach. Three geon categories, Geon5, Geon6, Geon9, are selected, and each geon category contains six SQs of the class, which are illustrated in Fig.5. Then, the SH-based approach for 3D shape similarity measure is performed on the superquadrics of the three geon categories. The experimental result is shown by the similarity matrix in Fig.6.

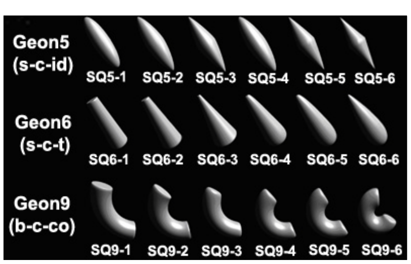


Fig.5: Three geon categories selected

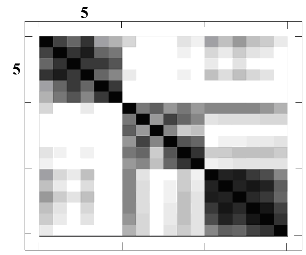


Fig.6: Similarity matrix with SH-based approach

5. Conclusions

In this paper, two novel approaches are proposed for superquadric similarity measure, which are based on spherical harmonics and 3D moment invariant respectively. Comparison experiments have been performed, which show that the SH-based approach can obtain better results than MI-based method, and the 3D complex moment invariant (MI2) is better than the simple real moment invariant (MI1). The proposed approaches, especially the SH-based approach, are very efficient and provide promising measure metrics of superquadric similarity, which lays a foundation for developing SQ-based 3D object recognition systems.

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