Market Efficiency and Rational Expectation under Asymmetric Information and Uncertainty in Price Prediction

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Abstract

This paper presents an agent-based simulation model of stock markets characterized by the existence of private information. Through numerical simulations, we investigate the market efficiency and the rationality of traders' expectation under the influence of asymmetric information and uncertainty in price prediction. As a result, the two factors have great impact on fundamentals, causing decreases both in market efficiency and traders' rationality.

Keywords: Asymmetric Information, Uncertainty, Market Efficiency, Rational Expectation, Artificial Stock Markets

1. Introduction

In the traditional economics or financial engineering, it is assumed that stock price reflects all the information existing in the stock market, and traders have all the information, interpret them completely and expect future stock price rationally. These assumptions are called the Efficient Market Hypothesis and the Rational Expectation Hypothesis [1][2]. According to Fama, who advocates the Market Efficiency Hypothesis, the market efficiency can be classified into three forms as follows.

- The weak form
  Stock price reflects only the past information.
- The semi-strong form
  Stock price reflects all the public information.
- The strong form
  Stock price reflects all the information, both public and private.

In the real stock market, jump or fall of stock price may often be caused by the existence of private information, the asymmetric information among traders and the uncertainty in price prediction. However, the strong form of market efficiency is very difficult to be studied using the traditional theories.

The purpose of this study is to evaluate the strong form of the market efficiency through the mulit-agent simulation of stock markets. In particular, we investigate, in a quantitative way, the market efficiency and the traders' rationality in a market where asymmetric information and uncertainty in price prediction are taken into account.

2. Related Studies

There are some studies about the simulation of overall stock market using agent-based approach [3] [4]. In addition, in the field of market microstructure that analyzes stock trading system mathematically, there are also some studies about the market efficiency of the strong form market [5] [6]. In these studies, it is assumed that private information is spread into all the traders through the trading so that the market price will still converge to the rational expectations.

However, a more realistic market involving a particular trading system and the process of opinion formation of traders is almost impossible to be studied with a mathematical model, because it becomes extremely complicated and difficult to get holomorphic equilibrium solution.

Chan et al [7], which concerns with the effect between public information and market efficiency. We extend their model by introducing private information.

3. Model

3.1. Stock

There is only one stock in this model, and its fundamentals is determined by "dividend" as public information (D: Integer from 1 to 4) and "insider information" as private information (I: Integer form 1 to 3). We analyze three patterns: 1) Insider information has no effect to fundamentals (Pattern 1), namely, there is no private information in the market,

2) Insider information has little effect (Pattern 2) and 3) Insider information has great effect (Pattern 3).

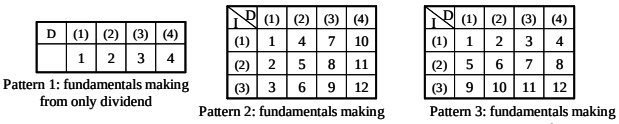

Fig. 1: Three patterns of fundamentals making

3.2. Traders

There are three types of traders in this model. First type is "informed trader" that has both dividend and insider information and make order using both. Second type is "fundamental trader" that has only dividend and make order using it. Third type is "noise trader" that has dividend but make order randomly without using any information.

In addition, all traders received prior probability that possible dividend and insider information against defined dividend and insider information. We set three cases of prior probability to observe the process of distributing information among traders through market (See Fig. 2). It represents uncertainty that traders have against stock market or whole economy.

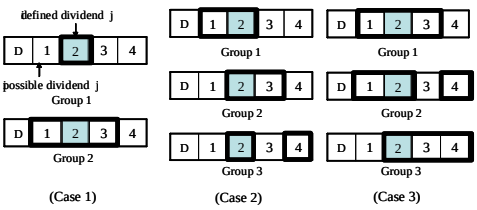

Fig. 2: Three cases of prior probability

3.3. Trading System

We adopt "double auction system" as the trading system. It used in the relatively large stock exchanges that have adequate liquidity.

It is the way that gathers into board that separated buy and sell orders from traders and contract orders that match conditions under the supply-demand situation.

We explain more concrete algorithm about double auction system. Board A in Fig. 3 represents all orders that have already received. If a trader sends sell order in \$98 and 10 stocks (Board B), firstly 6 out of 10 stocks are contracted with the buy order in \$ 100 and 6 stocks that is the highest buy order (Board C). Next, rest 4 stocks are contracted with the buy order in \$ 99 and 8 stocks. Ultimately, board becomes like Board D and waits the next order.

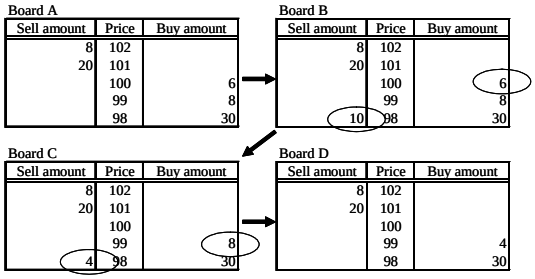

Fig. 3: Trading example of double auction system

3.4. Simulation Flow

Single simulation consists of multiple "turns" and "steps" that are executed repeatedly in one turn. Following events occur in each point.

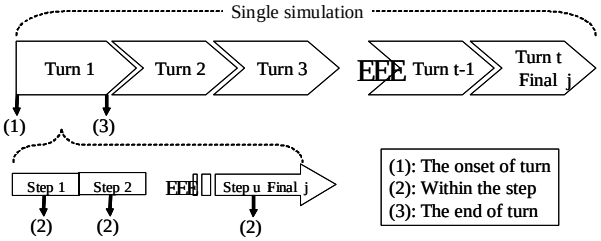

Fig. 4: Simulation flow

(1): In the onset of each turn,
- Determine stock's fundamentals(dividend and insider information)
- Distribute prior probability of dividend among informed traders and fundamental traders and insider information among informed traders

(2): Within the each step,
- All traders send orders
- Every time receive an order, contract and determine market price

(3): In the end of each turn,
- Dividend is fully informed to all traders
- Insider information is informed only to informed traders
- Market price at last step is informed to all traders as the closing price
- All traders learn dividend and closing price and informed traders learn insider information in addition

3.5. Opinion Formation

Traders expect the fundamentals as the price of order and make their orders in the simulation flow that explained in preceding section as follows. In addition, volume of order is all one scale and the decision that buys or sells is determined by looking board and easy to contract.

In the onset of each turn, all traders are separated randomly two (Case 1) or three (Case 2 and 3) groups

and get different prior probability of possible dividend. After that within the each step, informed and fundamental traders expect the dividend and insider information using prior probability of possible dividend and learning through the trading in the market and calculate fundamentals along Fig. 1. The expectation value of both values (Vt) is represented as follows using Bayes' Theorem. (In addition, following formula is about the expectation value of dividend. To expect insider information, we have to replace D by I in following formula.)

$$V_t = E[D \mid p] = \sum_{i=1}^{n} D_i \Pr(D_i \mid p)$$
$$= \sum_{i=1}^{n} D_i \frac{\Pr(D_i, p)}{\Pr(p)}$$
$$= \sum_{i=1}^{n} D_i \frac{\Pr(D_i, p)}{\sum_{j=1}^{n} \Pr(D_j, p)}$$
$$= \sum_{i=1}^{n} D_i \frac{\Pr(D_i)\Pr(p \mid D_i)}{\sum_{j=1}^{n} \Pr(D_j)\Pr(p \mid D_j)} \quad (1)$$

(where p: the market price of opinion formation point, n: the number of dividend or insider information, Pr(p|Di): learning effect)

Trader's learning effect is explained as follows. As simulation processes and go through steps, it becomes apparent the relations between market price and dividend (and insider information) and traders make and pile up those histogram like Fig. 5. On that basis when traders perceive market price, they read out Pr(p|Di) from histogram.

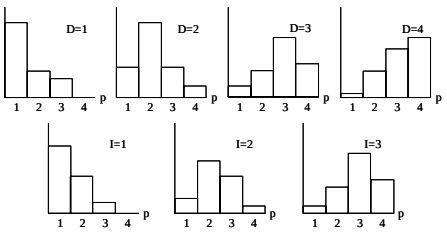

Fig. 5: Learning effect by traders

3.6. Analyze Method

As the indicator that measures the market efficiency, we use the spread between the fundamentals and closing price expressed as follows. The lower the indicator becomes, the more efficient market becomes.

$$\Delta p = \frac{1}{T} \sum_{t=1}^{T} |p_t - v_t| \quad (2)$$

(where T: the number of turn, p: the closing price, v: the fundamentals)

In addition, as the indicator that measures the trader rationality, we use the spread between the fundamentals and mean of prices on orders of all traders. The lower the indicator becomes, the more rational traders are in the same as above.

$$\Delta e = \frac{1}{N} \sum_{t=1}^{N} |E[D, I \mid p] - v| \quad (3)$$

(where N: the number of traders, E[D,I|p]: the price on order of No. t trader, v: the fundamentals)

4. Results

4.1. Simulation Setting

We run simulations under the following setting and take the average of 100 times for each pattern in Fig. 1 and case in Fig. 2.

- The number of turns…300
- The number of steps…30
- The number of informed traders…25
- The number of fundamental traders…25
- The number of noise traders…10

4.2. Pattern 1

We show the result of pattern 1 in Fig. 6. In this setting, there is no informed trader and only public information (dividend) in the market. It represents semi-strong form market. In addition, we multiply four times about the indicators in pattern 1 because to the comparison to those of pattern 2 and 3.

The values of simulation beginning are nearly 1 both and diminish to about a half as simulation processes. From here onwards, both market efficiency and trader rationality become higher in the market that there is no insider information.

Furthermore, both market efficiency and traders' rationality come down drastically (indicators keep high through turns) if there is much uncertainty.

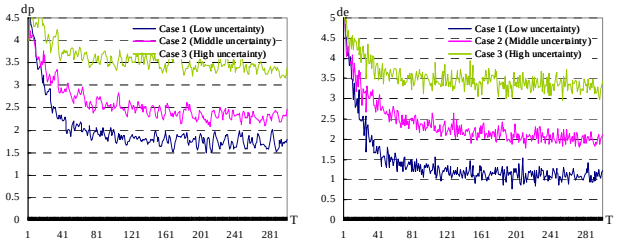

Fig. 6: The transition of delta-p and delta-e in pattern 1

Additionally, we investigate the affection of disturbing factor by trading on market efficiency by increasing the proportion of noise traders as follows. We show the result in Fig. 7.

- The number of informed traders…15
- The number of fundamental traders…15
- The number of noise traders…30

Both market efficiency and traders' rationality become low and have high variance than that of Fig. 6. It is considered that they come under the influence of disturbing factor by noise traders.

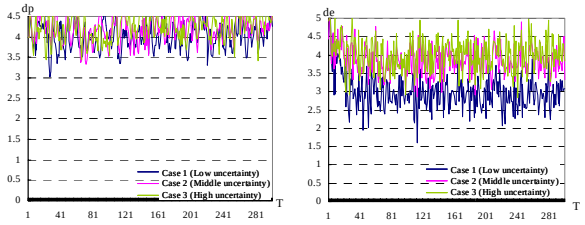

Fig. 7:The case of increase of noise traders in pattern 1

4.3. Pattern 2

We show the result of pattern 2 in Fig. 8. In pattern 2 and 3, there are both public and private information (dividend and insider information) in the market. It represents strong form market.
In addition, the impact of private information on fundamentals is little. Incidentally, both indicators are high (about four) because possible values of them are bigger than that of pattern 1.Both values slightly diminish, but its size is little as compared to pattern 1.

Furthermore, both market efficiency and traders' rationality come down slightly if there is much uncertainty.

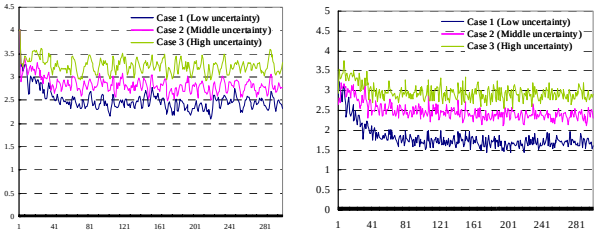

Fig. 8: The transition of delta-p and delta-e in pattern 2

4.4. Pattern 3

We show the result of pattern 3 in Fig. 9, there is insider information in the stock market and its impact on fundamentals is huge.

Both values hardly diminish and we can see both market efficiency hypothesis and rational expectation hypothesis don't achieve at all.

Furthermore, there is no difference about market efficiency and traders' rationality regardless of uncertainty.

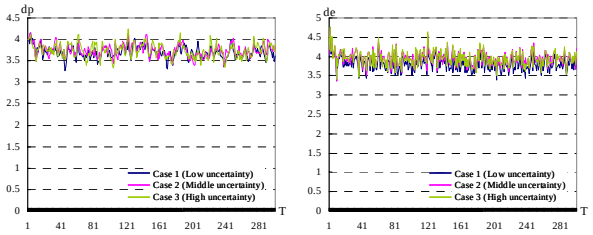

Fig. 9: The transition of delta-p and delta-e in pattern 3

5. Conclusion

The result of simulation clearly shows that both market efficiency and traders' rationality become lesser over time in strong form (pattern 2 and 3) market than in semi-strong form (pattern 1) market. Especially, the private information has an impact on fundamentals (pattern 3). Therefore, both Market Efficiency Hypothesis and Rational Expectation Hypothesis are not correct in the strong form market, and private information has great impact on the fundamentals.

In addition, the uncertainty in price prediction also has a great influence on market efficiency and traders' rationality in case that strong asymmetry of information exists in the market such as the strong form market of pattern 2 and 3.

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