What Causes Persistence of Stock Return Volatility? One Possible Explanation with an Artificial Stock Market

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Abstract

This paper explores the mechanism on how the persistence of the stock return volatility is created using a model of an agent-based stock market. First, artificial stock markets with different learning mechanisms, i.e., individual and social learning are examined. The simulation result shows that an economy with social learning produces persistence of return volatility while an individual learning economy does not. Then, more importantly, the relation between agents' behavior and return volatility dynamics is investigated.

Keywords: Asset Pricing; Learning; Evolution; Volatility Clustering

1. Introduction

It is a well-known feature in financial markets that the volatility of asset returns appears to be serially correlated. This phenomenon could be identified if the ARCH process exists (Engle (1982)). While many of the papers detected the evidence on the volatility clustering of the return series (Pagan (1996)[1]), it has not been investigated much about the mechanism on what causes the volatility persistence.[2] This paper explores the mechanism with a model of an agent-based stock market.

The agent-based artificial stock market is a useful tool to show a relation between stock return dynamics and agents' behavior. It analyzes evolving systems of autonomous interacting agents. The computational model evolves over time as each agent repeatedly changes her trading strategy through interactions and learning to improve her performance in the market. The analysis of agent-based models is a bottom-up approach to the market. So, the simulation of the artificial stock market can track not only the evolution of the agents' strategies but also the dynamics of the return series with a relation to the interaction of the heterogeneous investors, which gives an important framework to analyze how the volatility clustering is created.

Several recent papers with agent-based modeling for the stock market focus on a question if the artificial markets are able to replicate the real market phenomena. For example, papers related to the Santa Fe Artificial Stock Market Model[3] demonstrate great performance to replicate the actual time series phenomena in the stock market including the ARCH process of the returns series.[4] But they are not concerned with how behavior of heterogeneous interacting agents is related to the aggregate market behavior like ARCH phenomena, which is the main focus in this paper.

How is the volatility persistence of the return series explained with an agent-based stock market? This paper takes the following steps. First, agent-based stock markets, which can and cannot produce the volatility clustering, are described. Then the relation between agents' behavior and return volatility dynamics is investigated in those markets. This paper finds such a stock market generating the volatility persistence by looking at different learning mechanisms, i.e., individual and social learning. In an agent-based stock market, learning and adaptive

[1] Mantegna and Stanley (1995, 1997) also show evidence on the time dependence of the standard deviation of the S&P 500 index.

[2] McQueen and Vorkink (2004) develops a preference-based equilibrium asset pricing model and shows that the state-dependent sensitivity to news creates the type of volatility clustering in low frequency returns.

[3] The original paper on the Santa Fe Artificial Stock Market Model is Arthur, Holland, Lebaron, Palmer, and Tayler (1996). The first paper to analyze the time series properties is LeBaron et al. (1999).

[4] Other phenomena include fat tails and nonlinear dependence for the asset return series, and for the trading volume dynamics, non-zero volume, autocorrelations, and positive contemporary cross-correlations between the volume and the squared returns.

behavior of the agents is usually modeled with the individual and social learning. In individual learning, agents update their behavioral rules from their own past performance. A social learning mechanism allows agents to update their rules through directly interacting with other agents. The simulation result shows that a social learning economy produces persistence of return volatility while an economy with individual learning does not.

Then, more importantly, the relation between the persistence and agents' expectation is investigated. It concludes that the sequences of periods with very large increases or decreases in the variance of the expectations on future prices across agents possibly cause the serially correlated return volatility. The variations of the expectation diversity are important determinants for the volatility persistence of the return series. The expectation diversity is measured by how much agents expect future prices differently at each period. In a social learning economy, the volatility clustering is observed since the variations of the expectation diversity are positively correlated over time. There are some periods when agents' expectations are so different and fluctuate much while the expectations are similar and stable for some other periods. But in an individual learning economy, the expectations across agents differ and almost always fluctuate much so that there is no convergence in their expectations. Therefore, we don't see the volatility clustering of the return series in an individual learning economy.

Section 2 presents the market structure. Section 3 gives the results from the computer experiments, and the last section concludes.

2. Market Structure

This section describes an artificial stock market based on the one outlined in LeBaron et al. (1999). The experiments are conducted on an artificial stock market with different styles of learning. But the market structure is exactly identical for all experiments. It is presented as follows.

The artificial stock market has two tradable assets, a risky stock and risk free bond. The bond is in infinite supply and it pays a constant interest rate, r_f=10%. The stock pays a highly persistent and stochastic dividend which follows:

(1) $$d_t = \bar{d} + \rho(d_{t-1} - \bar{d}) + \mu_t$$

$$\textit{with} \quad \bar{d} = 10,\ \rho = 0.95,\ \textit{and}\ \mu_t \sim N(0, \sigma_\mu^2)\ .$$

The number of shares of the stock is 150, which equals the number of agents in the market.

The market consists of many heterogeneous interacting agents who have different methods of prediction on the stock price and dividend. The steps of how events in this artificial market proceed are as follows.

1. Information set:

At time t, agents observe the past price and dividend, and calculate technical indicators. They form a set of information, 'z_t'. Here the technical rules are based on simple moving averages formed as

(2) $$m_{k,t} = MA(k) \quad \text{where } k\text{=5 and 10.}$$

$MA(k)$ denotes the moving average calculated with prices of past k periods. The information set, 'z_t', includes, $r_{t-1} = \dfrac{p_{t-1} + d_{t-1} - p_{t-2}}{p_{t-2}}$, d_{t-1}, $\log\left(\dfrac{d_{t-1}}{p_{t-1}}\right)$, $\log\left(\dfrac{p_{t-1}}{m_{5,t-1}}\right)$, and $\log\left(\dfrac{p_{t-1}}{m_{10,t-1}}\right)$.

2. Prediction:

Each agent i process the past information and make predictions on the future price and dividend according to:

(3) $$\hat{E}_t^i(p_{t+1} + d_{t+1}) = a_t^i(p_t + d_t) + b_t^i$$

where a_t^i and b_t^i are produced through a neural network consisting of past information'z_t', which is defined as in LeBaron (2002) as follows:

(4) $$h_k = g(\omega_{1,k} z_{t,k} + \omega_{0,k})$$

(5) $$\lambda_l(z_t) = 0.5*(1 + g(\omega_2 + \sum_{k=1}^{5} \omega_{3,k} h_k)) \qquad l = a, b$$

(6) $$g(u) = \tanh(u)$$

The a set of information, 'z_t'is combined with weights and transformed to produce a signal, λ, which lies between 0 and 1 by construction.

Permitting to range 'λ' with the allowable bounds for 'a_t^i' and 'b_t^i' in LeBaron et al. (1999), that is, $a \in [0.7, 1.2]$ and $b \in [-10, 19]$ [5], the forecast parameters 'a_t^i' and 'b_t^i'are given by

(7) $$a_t^i = 1.2 * \lambda_a(z_t) + 0.7 * (1 - \lambda_a(z_t))$$

(8) $$b_t^i = 19 * \lambda_b(z_t) + (-10) * (1 - \lambda_b(z_t))\ .$$

Agents are heterogeneous in terms of their expectation since each has different values of weights in their own neural nets. Here agents build their forecast using the neural network. This is an extension from the financial market in LeBaron et al. (1999).[6]

3. Strategy making:

[5] Those ranges are given to be centered around the rational expectation equilibrium values.

[6] The agents in LeBaron et al. (1999) forecast using what are called 'condition-forecast' rules.

Based on the prediction, each agent i sets his demand for share as:

$$(9)\quad s^{i}* = \frac{\hat{E}_t^i(p_{t+1}+d_{t+1})-(1+r_f)p_t}{\gamma\hat{\sigma}^2_{p+d,i}}.$$

4. Price determination:

The new equilibrium price, p_t, is determined according to the market equilibrium condition as:

$$(10)\quad \sum_{i=1}^{N} s_t^i(p_t) = \sum_{i=1}^{N}\frac{a_i(p_t+d_t)+b_i-(1+r_f)p_t}{\gamma\hat{\sigma}_i^2} = 150.$$

5. Volume determination:

After revealing the price, forecasting parameters a_t^i and b_t^i are updated according to the neural network, (4)-(6) to get a_{t+1}^i and b_{t+1}^i, and trading volume is recorded.

Wealth, w, for agent i is evolved according to:

$$(11)\quad w_{t+1}^i = s_t^i(p_{t+1}+d_{t+1}) + (1+r_f)(w_t^i - p_t s_t^i)$$

Each agent is initially allocated 20,000 units of cash.

6. Genetic Algorithm:

Steps 1-5 are repeated for S (=25) periods. Then genetic algorithm (hereafter GA) is invoked to update their forecasting parameters. The steps 1-5 with GA are repeated 500 times every 25 periods.

The GA manipulates the parameters, 'ω', in the neural network, equation (4) to (6), to improve the performance according to a fitness criterion, which is given as:

$$(12)\quad V_i = \sum_{t=1}^{S=25} U_i(w_{t+1})$$

$$where \quad U(w_{t+1}) = -\exp(-\gamma w_{t+1})$$ [7]

γ is a constant absolute risk aversion coefficient and assumed to be 0.5.

At each time of GA, the variance estimate is updated according to a simple average of past 25 periods of squared forecast errors,

$$(13)\quad \hat{\sigma}^2_{p+d,i,t} = \sum_{t=1}^{25}\left\{(p_t+d_t)-\left[a_{t-1}^i(p_{t-1}+d_{t-1})+b_{t-1}^i\right]\right\}^2$$

This paper considers the markets with individual and social learning. A population in social learning consists of directly interacting heterogeneous agents. In social learning, investors' behavior is influenced by other investors. Investors meet, for example, at some conferences, communicate each other, and exchange their opinion about the price prediction. Then based on such interactions, they would update their trading strategies. Population evolves over time updating new parameters which are better adapted to the environment.

In a market with individual learning, each agent has a set of her private ideas. The ideas of each agent are not disclosed to other agents so that there is no role here for imitative behavior. Agents learn from their own past experience and update their behavioral rules by themselves. There are no direct exchanges of the ideas among agents in this learning. Reactions to other agents' behavior only occur indirectly through prices.

GA is run at the individual and social levels. But the steps to implement GA are exactly the same for exactly the identical market structure for social and individual learning.

A GA consists of a set of operations which manipulate a given population. [8] There is one population in a social learning economy which consists of 150 agents while an individual learning market has 150 populations each of which represents agent who has 150 ideas in her mind. In an individual (social) learning market, each idea (agent) has a set of 32 parameters in her neural network. In individual learning, "ideas" interact within each of the agent's mind while "agents" interact in social learning.

3. Experiments

[7] As simulation proceeds, the wealth is always normalized to be in 5-digit number dividing by 10 if one of the wealth exceeds 100,000. When we evaluate the utility, the wealth is divided by 1,000,000 since the utility function is negative exponential.

[8] First, GA initializes the population chosen randomly from [-1,1]. Second, at each generation, the probabilities of ideas (agents) being copied for next generation are calculated as $P_i = \frac{1/V_i}{\sum_{j=1}^{N=150} 1/V_j}$ where $V_i = \sum_{t=1}^{S=25} U_i(w_{t+1})$. The GA then introduces crossover and mutation which are used in Muhlenbein and Schlierkamp-Voosen (1993). For the crossover, a new parameter (offspring) is produced by combining two parameters (parents) as: offspring=parent 1 + α * (parent 2 – parent 1).
where α is randomly chosen in the interval [0,1]. A new 'α' is generated for each pair of parents combined together. For mutation on the real-valued GA, a parameter ω_i is selected with probability p_m (=0.08), and are added a small perturbation as: $\omega_i* = \omega_i \pm 0.5*[-1,1]\delta$.
The + or – sign is chosen with probability 0.5, and $\delta = \sum_{i=0}^{19}\eta_i 2^{-i}$. η_i =1 with probability 1/20, else 0.
Then agents do "back-testing". The fitness function (12) is calculated with updated parameters and observed prices. A set of the updated fitness is compared with the old fitness, and the algorithm chooses parameter sets for the next generation which show higher fitness.

This section shows that a social learning economy produces volatility persistence while an individual learning economy does not. It then shows that the sequences of periods with very large increases or decreases in the variance of the expectations across agents possibly cause the serially correlated return volatility. First, the volatility persistence of the return series is detected in a social learning economy not in an individual learning economy. Then, the relation between the persistence and agents' expectations is examined.

3.1. Volatility Clustering in Individual and Social Learning Economies

Simulations are conducted in each economy *separately* and repeated for 10 times each under different random seeds to collect cross-sectional statistics. The series of stock price, dividend, and point estimates on future prices made by agents (equation (3)) are recorded for the last 5,000 periods for each of the simulation. The existence of ARCH process in the returns series are tested in two markets.

For the return series statistics, the following regression is first conducted with the simulated data:

(14) $p_{t+1} + d_{t+1} = \alpha + \beta(p_t + d_t) + \varepsilon_t$.

Following LeBaron et al. (1999), the estimated residual series $\hat{\varepsilon}_t$ are analyzed, and the results are in Table 1.

Table 1: Summary Statistics

Description	Individual Learning	Social Learning
ARCH(1)	0.3407 [0]	9.7229 [0.8]

The ARCH test deals with the volatility clustering for the return series (Engle (1982)). The numbers reported are the means of the test statistics.[9] The numbers in the brackets are the fraction of runs rejecting 'no ARCH' at the 95% confidence level. The no ARCH null hypothesis is rejected 8 times in the social learning market while none in the individual learning. The result indicates the ARCH dependence in the residuals in the social learning as in the actual market but not in the individual learning.

3.2. What Causes the Volatility Persistence?

Why is the volatility persistence in the social learning economy but not in the individual learning economy? The difference of agents' behavior in each economy is analyzed to provide one possible explanation for the persistence.

The difference is based on how the diversity of the expectations on future prices across agents changes over time. The important phenomena in social learning is the *"following the herd"*. The agents update their strategies based on ideas of someone who performed well in the past. The better ideas disseminate across agents. Such an imitative behavior by agents would possibly cause the expectations to converge for some periods. However, once some agents take different predictions, the expectations by the agents possibly start diverging away. So, for some periods all agents would expect future prices in a similar way, but after those periods, they start expecting the prices in different ways. In individual learning, the sets of ideas for each agent are separately distributed, and there are no direct exchanges of the ideas among agents. So, there is no idea dissemination in this economy. Since the prediction methods are different across agents, it is not often that many agents make an agreement on their expectation. So, the variance of the expectation diversity doesn't converge.

When the uncertainty of the future prices goes up, the agents expect future prices so differently. When the agents' point estimates on future prices differ, some traders shift expectations up and others down. When they shift their expectation up, they are more likely to buy while some who expect down tend to sell (equation (9)). As a result, the trading volume additionally increases so that the price changes.[10] Although the direction of the change in price depends on which pressure is strong between buy and sell, a large swing of the uncertainty is related to the large change in returns. In an economy with social learning, expectations possibly converge for some periods but then diverge a lot. When the expectations move in a small range, the prices also move in a small range. The large swing of the expectation causes large changes in prices so that the returns swing a lot. In a social learning economy, there are some periods of small movements in expectations while some periods show large variations of the expectations. This would lead to the volatility clustering of the returns for some periods while there is no clustering for some other periods. In an individual learning economy, there is no convergence in the expectations. So, the return series always shows the similar variations over time.

Based on the above arguments, the following three hypotheses are examined:

[9] Numbers in parenthesis are standard errors estimated using the 10 runs.

[10] Clark (1973) gives an explanation on the relation between information uncertainty and price changes.

(1) A social learning economy is more likely to show the agreement of the point estimates on future prices than in an individual learning economy.
(2) The changes in the diversity of the expectations cause the variability of the stock returns.
(3) If the changes in the diversity of the expectations cluster, the volatility of the returns series would also show the serial correlation.

The first hypothesis is examined using the series of the point estimates on future prices, $\hat{E}_t^i(p_{t+1}+d_{t+1})$. First, the standard deviation *across agents* is calculated for each period. This standard deviation reflects the similarity of the expectation (agreement) on the future asset price and dividend among agents at a particular period. When this value is high, it means that agents predict the prices very differently meaning that the future prices are uncertain for agents. The diversity of the expectations is expected to be wider in an individual learning economy than in a social learning economy since the expectation won't converge in an individual learning economy. Table 2 shows that the mean in the individual learning economy is 15.2798 while 6.4060 in a social learning economy as expected.[11]

Table 2: Summary Statistics

Description	Individual Learning	Social Learning
Mean	15.2798 (0.3205)	6.4060 (0.2212)

The upper figures in Figure 1 and 2 show how the diversity of the expectations changes over time. These plot the changes in the diversity over time, which are calculated as:

(15) $\left(std(\hat{E}_t\,(p_{t+1}+d_{t+1})) - std(\hat{E}_{t-1}(p_t+d_t))\right)$.

In the social learning economy, the figure shows some periods of convergence in the diversity while the changes diverge away for some periods. But the individual learning economy doesn't show convergence.

The second hypothesis is investigated with Figure 1 and 2. It looks that the dynamics of the diversity might be related to the return variations, which are given by taking difference in return series as $r_{t+1} - r_t$. Moreover, the second hypothesis is tested with GMM taking the variations of the diversity of the expectations as an explanatory variable, $\left(std(\hat{E}_t\,(p_{t+1}+d_{t+1})) - std(\hat{E}_{t-1}(p_t+d_t))\right)^2$, and the stock return variability as a dependent variable. The GMM is applied to a linear model with a constant term. Since the stock return variability is related to the stock price variability, the variable, $(p_{t+1}-p_t)^2$, is used

Figure 1: Dynamics of the Variation of Expectations and Return Series in an Individual Learning Economy.

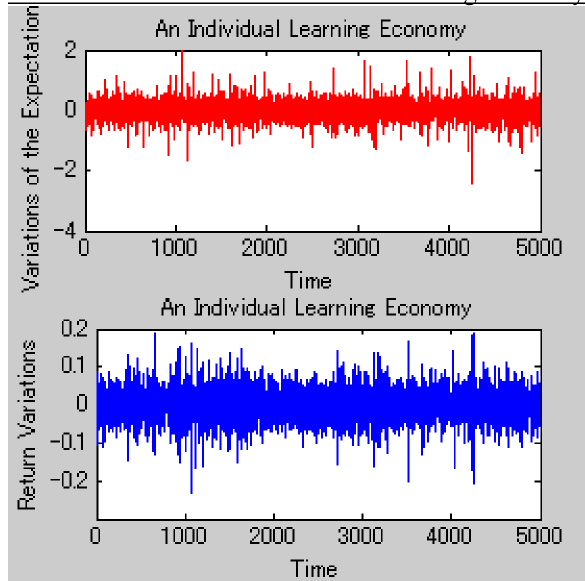
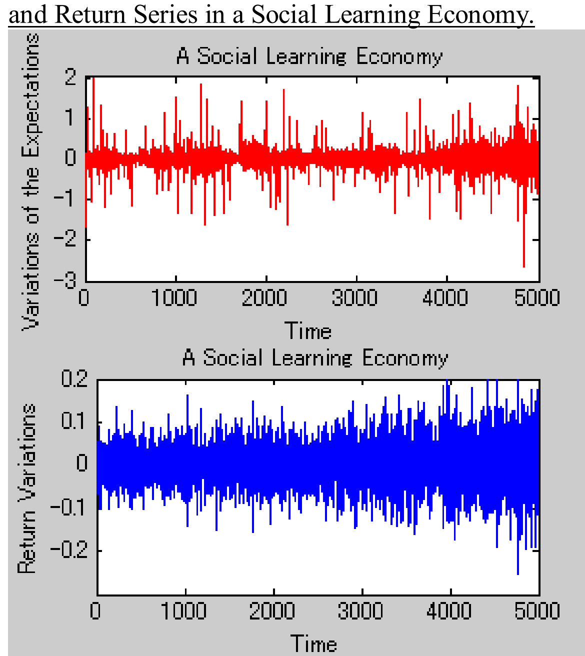


Figure 2: Dynamics of the Variation of Expectations and Return Series in a Social Learning Economy.



for the dependent variable. The results are in Table 3.[12]

The coefficients of the diversity of the expectations are highly significant in both economies (58.7225 and 23.6039, respectively). So, it concludes that the stock return volatility is closely related to the

[11] Numbers in parenthesis are standard errors estimated using the 10 runs.

[12] Estimates are means of the 10 simulations. Numbers in parenthesis are mean of the absolute values of t-statistics in 10 runs. ** means that the estimated coefficient is significant at 5% level for all 10 simulations.

variations of the expectations on future prices in both individual and social learning economies.

Table 3: GMM estimates for the stock price variability and the variables on the forecasts :		
	Individual learning	**Social Learning**
Constant	0.7718**	7.0098**
	(10.83)	(37.94)
Diversity	58.7225**	23.6039**
	(12.23)	(3.81)

The third assumption would be verified just by investigating Figure 1 and 2. Figure 1 and 2 show that in the social learning economy, the large changes in the expectation diversity of either sign are followed by the large changes of either sign while small changes are followed by the small changes. The individual learning economy doesn't show such persistence of the changes in the diversity of the expectations. The diversity changes at almost constant rate in the individual learning economy. The GMM results suggest that the expectation dynamics would explain the behavior of the stock return variations. So, when the variations of the expectation diversity cluster, the stock return volatility also clusters. This phenomenon is found only in the social learning economy.

4. Conclusion

This paper explores the mechanism on how the persistence of the stock return volatility is created using a model of an agent-based stock market. First, artificial stock markets with different learning mechanisms, i.e., individual and social learning are examined. The simulation result shows that a social learning economy produces persistence of return volatility while an individual learning economy does not. Then, more importantly, the relation between agents' behavior and return volatility dynamics is investigated.

There are three findings on the relation between the persistence and agents' expectations as follows.

(1) A social learning economy is more likely to show the agreement of the point estimates on future prices than in an individual learning economy.
(2) The changes in the diversity of the expectations cause the variability of the stock returns in both individual and social learning economies.
(3) If the changes in the diversity of the expectations cluster, the volatility of the returns series would also show the serial correlation. This phenomenon is found only in a social learning economy.

Those of the results indicate that the persistence of stock return volatility would be explained with the behavior of the heterogeneous investors.

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