Avalanche dynamics of the financial market*

Pei-Ling Zhou[1] Chun-Xia Yang[1] Tao Zhou[1,2] Min Xu[1] Jun Liu[1] Bing-Hong Wang[2]
[1]Department of Electronic Science and Technology
[2]Department of Modern Physics
University of Science and Technology of China, Hefei Anhui, 230026, PR China

Abstract

A parsimonious percolation model for stock market is proposed, of which the avalanche dynamics agree with the real-life one well. We have also investigated how the interaction parameter p affects the price dynamics. Simulation results about the formation of bullish/bearish market and corresponding avalanche taking place in the market indicate that the magnified "herd behavior" resulting from the evolution of p may be the origin of the observed avalanche phenomena.

Keywords: complex system, financial market, percolation, nonlinear dynamics, avalanche

1 Introduction

The Science of Complexity is helping us perceive the economy as a collection of nonlinear interacting units. This collection is complex; everything depends on everything else. Physicists are looking for empirical laws that will describe, and theories that will help understand, this complex interaction[1, 2, 3, 4]. Bubbles and crashes are the key properties of market prices since centuries. Do they arise from external perturbation such as government decisions on war and peace, the interest rate by the central bank, for stocks of single companies the introduction of new (un)successful products and so on, or, arise from intrinsic market mechanisms like the October 1987 crash on Wall Street[5]? While we cannot make a clear distinction between external and intrinsic reasons for a crash in reality, we can demonstrate the empirical laws of bubbles and crashes and do so at least in computer simulations to probe the relationship between them and complex interaction of units consisting the economic system. Because bubbles and crashes result from a possible amplification or "flight of fear" where previous gains(losses) lead to further buying(selling), strengthening the upward(downward) trend, we call them "avalanches", whose distribution is our focus. Previous studies pay attention to the Probability Density Function(PDF) of the fixed time-scale return that may depict the price fluctuation. Calculating it, physicists simply dump all of the returns "on the floor", then pick the data points up off the floor and make a histogram [6, 7, 8, 9]. This research neglects the relative positions of the return as they unravel themselves as a function of time by only counting their frequency, so can't capture the persistence and correlation existing in the market. Contrarily, they are embodied by the distribution of "avalanches". In this paper, some empirical results about avalanche size distribution are shown, and a model is established which may interpret the underlying mechanism of avalanche dynamics.

2 Avalanche size distribution

Following Johansen and Sornette, a positive/negative avalanche (drawup/drawdown) is defined as a persistent increase/decrease in the price over consecutive days[10]. The size of it is thus the cumulative gain/loss from the last minimum/maximum at time t_1 to the next maximum/minimum at time t_2 of the price, i.e., we can use the "elastic" time-scale return to quantify it.

$$R(t_2) = \mathtt{ln}(p_r(t_2)) - \mathtt{ln}(p_r(t_1)) \qquad (1)$$

Avalanches embody a rather subtle dependence since they are constructed from runs of same sign variations. In particular, an avalanche embodies the interplay between a series of gains/losses and hence measures a "memory" of the market. Avalanches illustrate the effect of correlation in price variations when they appear. Thus the distribution of avalanche size captures the way successive ups/drops can influence each other and construct in this way a (quasi)persistent process.The information contained in it is quite different from that in the distribution of return over a fixed time scale. This information

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is relevant to identify the possible burst of local dependence leading to possibly extraordinary large cumulation gains/losses. Previous results have shown that the vast majority of negative avalanches(drawdowns) occurring on the major financial markets e.g., the US stock markets, the Hong-Kong stock market, have a distribution which are well parameterized by a stretched exponential distribution[10]. While the tail of the distribution possibly reveal the persistence and correlation existing in the financial market preferably. Furthermore the empirical law avalanche obeys would help us understand the dynamics of the financial market. The demonstration about avalanche size distribution of Dow Jones Industrial Average(DJIA) indicates that there is a power law distribution with exponent close to 3.

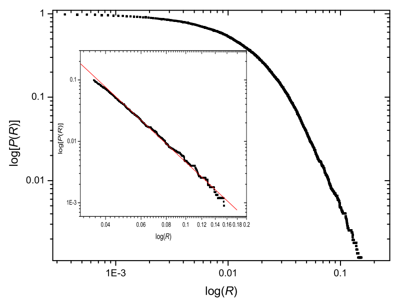

Figure 1: The cumulative distribution of positive avalanche size in the DJIA since 26.5.1896 until 31.5.2000. The insert is it's tail, the slope of fitting line is -3.07 ± 0.006.

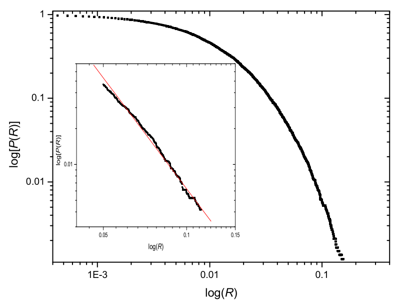

Figure 2: The cumulative distribution of negative avalanche size in the DJIA since 26.5.1896 until 31.5.2000. The insert is it's tail, the slope of fitting line is -3.09 ± 0.009.

3 Model

Cont and Bouchaud[11] successfully applied percolation theory to modelling the financial market(CB model), which is one of the simplest models able to account for the main stylized fact of financial markets, e.g. fat tails of the histogram of log-returns[7, 8]. Based on it, our model incorporates the following components different from the original CB model: (1)The cluster, defined as a set of interconnected investors, grows in a self-organized process. (2)The effect of "herd behavior" on the trade-volume is magnified step by step during the cluster's self-organized accumulating process rather than instantaneously formed like EZ model[12]. (3)Some encountering smaller clusters will form a bigger cluster through cooperating or one defeating the rivals. (4)An infinite cluster maybe exist without the need to tune p to p_c and it's trade activity influence price fluctuation. (5)The number of investors participating in trading will vary dynamically. Now let's see the model.

3.1 Dynamic of investor groups

Initially, M investors randomly take up the sites of an $L * L$ lattice. Then for each cluster, a strategy is given: buying, selling, or sleeping, which are denoted by 1, -1 and 0 respectively. In reality, the circle of professionals and colleagues to whom a trader is typically connected evolves as a function of time: in some cases, traders are following strong herding behavior and the effective connectivity parameter p is high; in other cases, investors are more individualistic and smaller values of p seem more reasonable. In order to take the complex dynamics of interactions between traders into account, we assume that it undergoes the following evolution repeatedly:

(1) Growth: most of investors would like to imitate the strategies which have been adopted by many others, which induces "herd behavior" occurring. In this sense the herd behavior is amplified. Specially, the affection of the herd behavior will be magnified gradually with the increase of the number of investors adopting this strategy, i.e., with the growth of the clusters. During cluster's growth, a number of new investors will be attracted by it and become its members. In other words, every cluster will absorb new investors with the probability

$$P_d(\tau) = P_d(\tau - 1) + k(N_T - N(\tau - 1)) \quad (2)$$

where k is a kinetic coefficient controlling the growth speed and N_T is a threshold parameter (It has been validated that the value of the parameters k and N_T could be any value. These two parameters have no effects on the self-organization process of the clusters[13]). $N(\tau - 1)$is the number of the agents along it's boundary, defined as a set made up of agents which belong to a cluster and at least border on a site which isn't part of this cluster, at the last time step $\tau - 1$. The new participating investor will take up the empty sites around the old clusters and imitate the same strategy as that of it. The probability P_d is obviously limited to the range $[0, 1]$ so that we have to impose $P_d = 0$ and $P_d = 1$ if the recurrence relationship Equ.(2) gives values for $P_d < 0$ or $P_d > 1$.

(2) New cluster's birth: some investors will randomly and independently enter the market with the probability P_n. These investors don't belong to an arbitrary existing cluster and will take up the empty sites.

(3) Cooperation: encountering clusters will operate cooperation and confliction between them. When their strategies are same, they are thought to cluster together to form a new group of influence. Or there would be confliction between them. The consequence of confliction is that losers would be annexed by the winner and that a new and bigger cluster whose strategy inherent the winner's will be formed. The probability of cooperation or confliction is as follow, i.e., some a cluster will cooperate with or defeat others with the probability

$$P_m(k) \sim |s_\tau^k| \tag{3}$$

where $|s_\tau^k|$ is the size of k-th cluster at time τ.

(4) Metabolism: in reality, no matter how huge has the size of a group ever been it would collapse due to different influences such as government decision on war and peace. Some new clusters will come into the world in the wake of aging clusters' death. The probability with which an aging clusters will die is:

$$P_o = \frac{x+y}{2L} \tag{4}$$

where x and y is the width of this cluster occurring on the lattice in the x and y direction. Equ.(4) indicates that the probability with which a cluster disbands would increase with the cluster growth. Once a spanning cluster exists, it would surely die. When a cluster disbands, all its members would leave the market and the sites where the death cluster ever occupied will be taken up by new investors with the probability P_n. Such occupied sites form a few new clusters. Every new cluster would be given a strategy randomly.

Although each cluster could trade with others at every trading step, the evolution frequency of the network topology should not be so often. Thus, we assume that the network structure of the market composed by investor groups would evolve every N trading steps. With the evolution of this artificial stock market, the number of investors participating in trading isn't constant. The network will take on different structure; the affection of the herd behavior on the trade-volume is gradually magnified. Without any artificial adjustment of the connectivity probability p to p_c, spanning cluster may exist, whose activity would influence the price fluctuation.

3.2 Trading rules

Each cluster trades with probability a (called activity); if it trades, it gives equal probability to a buying or selling with demand proportional to the cluster size. The excess demand is then the difference between the sum of all buying orders and selling orders received within one time interval. The price changes from one time step to the next by an amount proportional to the excess demand. To explain the "volatility", Stauffer introduces the feedback mechanism between the difference of the "supply and demand" and activity of the investors[14]. Whereas in our model, the difference of the "supply and demand" not only affects the activity probability but also the probability with which the active clusters choose to buy or sell. The probability a evolves following the Equ.(5):

$$a(t) = a(t-1) + lr(t-1) + \alpha \tag{5}$$

where r is the difference between the demand and supply, l denotes the sensitivity of a to r and α measures the degree of impact of external information on the activity. Each active cluster choose to buy or sell with probabilities $\frac{1}{2}a(t)(1-p_s(t))$ and $\frac{1}{2}a(t)p_s(t)$ respectively. For $r>0$, $p_s(t) = 0.5 + d_1 r(t-1)$, while for $r<0$, $p_s(t) = 0.5 + d_2 r(t-1)$. According to Kahneman and Tversky[15], it is asymmetry that agents make their decisions when they are in face of gain or loss. When referring to gain, most of the agents are risk adverse. On the contrary, they are risk preference. These determine the parameters d_1 and d_2, representing the sensitivity of the agent's mentality to the price fluctuations and differing from each other. In our model we assume $d_2 = 2d_1$. The difference between the demand and supply is:

$$r(t) = \sum_{j=1}^{m} \mathtt{sign}(s_t^j)(|s_t^j|)^\gamma \tag{6}$$

where m is the total number of clusters occurring on the market. γ measures the degree of impact of each cluster's trade-volume on the price, $0<\gamma<1$ allowing for a nonlinear dependence of the change of (the logarithm of the) price as a function of the difference between supply and demand[16]. So the evolution of the price is:

$$P_r(t) = P_r(t-1)\exp(\lambda r(t)) \tag{7}$$

4 Simulation result

Here we set $a(0) = 0.09, r(0) = 0, p_r(0) = 1, P_d(0) = 0.4,\ k = 0.0001, N_T = 50,\ l = \lambda = \frac{1}{L^2}, L = 100,\ d_1 = 0.00005, \gamma = 0.8, P_n = 0.6, M = 100, N = 50$. Figure 3 shows the price time series which is rather similar to that of real market. More simulations have been done indicating that this model not only spontaneously exhibits reasonable statistics for the probability distribution of price returns with centered Lévy distribution and fat-tail, which are similar to the stylized facts observed in financial time series; but also has the power-law relationship in agreement with the empirical studies on the Hang Seng Index between the peak value of the probability distribution and the time scales[17]. The distribution of the positive avalanche size is given in Fig.4, with its tail obeying power-law form. In addition, the power law exponent is 2.93, agreeing with the reality very well. The negative case is almost the same and is thus omitted.

5 Concluding remarks

A parsimonious percolation model for stock market is proposed, of which the avalanche dynamics agree with the reality well. Unlike the microstructure literature based on the perspective that price movements are caused primarily by the arrival of new information[18], our model absorbs the widely accepted idea that price fluctuations are due to the nonlinear interaction among market players and the trading activity, and exhibits that how the interaction parameter p affects the price dynamics. Simulation results about the formation of bullish/bearish market and corresponding avalanche taking place in the market indicate that the magnified herd behavior resulting from the evolution of p may be the origin of the observed avalanche phenomena.

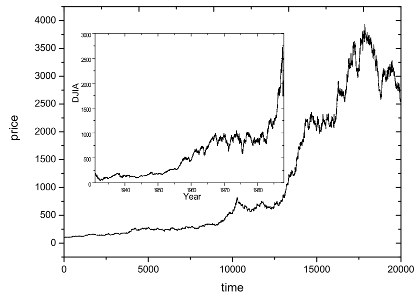

Figure 3: Time series of the typical evolution of the stock price. The insect is the Dow Jones Industrial Average(DJIA) from 2.1.1931 to 31.12.1987.

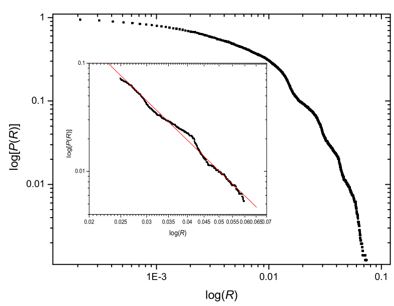

Figure 4: The cumulative distribution of positive avalanche in our model. Its tail is shown in the inset.

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