Forecasting Exchange Rates with Nonlinear Models

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Abstract

This work investigates the hypothesis that the nonlinear models of feedforward and radial basis function neural networks and the Takagi-Sugeno (TS) fuzzy system are able to provide a more accurate out-of-sample forecast than the traditional ARMA and ARMA-GARCH linear models. Using series of Brazilian exchange rate (R$/US$) returns with 15 min., 60 min., 120 min., daily and weekly basis, the one-step-ahead forecast performance is compared. Results indicate that forecast performance is strongly related to the series' frequency and the forecasting evaluation shows that nonlinear models perform better than their linear counterparts. In the trade strategy based on forecasts, nonlinear models achieve higher returns when compared to a buy-and-hold strategy and to the linear models.

Keywords: Forecasting, nonlinear, linear models.

1. Introduction

The literature related to financial time series has registered since the 1990's important advances with the incorporation of newly developed methods that attempt to determine patterns of relationships in financial market data. These approaches are, in general, computationally intensive and characterized by the capacity of modeling nonlinear dynamic systems, i.e., systems in which the variables of the environment possess complex patterns of interrelationships that alter throughout time.

Given the growing report of presence of nonlinear structures in financial time series, the use of deterministic linear models to describe and forecast financial prices movements has been criticized [1]. This aspect has stimulated researchers of diverse academic backgrounds to apply modern techniques of system identification to various problems in Finance [2,3].

Two relevant approaches, among many others, used for forecasting financial series are fuzzy systems and neural networks. In recent years, researchers [4,5] have proposed a varied spectrum of methodologies for identification and nonlinear forecasting based upon fuzzy systems to deal with nonlinear systems. On the other hand, neural networks have received attention in last decade due to their abilities to perform learning, thus applied in a great number of situations. This article is organized as follows. The next section describes the linear and nonlinear models. In section 3 and 4 we present the methodology of the study and the results, respectively. And finally, in section 5, we bring concluding remarks.

2. Forecast models

2.1. Linear models

Following the Box-Jenkins methodology, an auto-regressive moving average process ARMA(*p,q*) can be written as

$$\begin{aligned} y_t = \phi_0 + \phi_1 y_{t-1} + \ldots + \phi_p y_{t-p} + \varepsilon_t \\ + \theta_1 \varepsilon_{t-1} + \ldots + \theta_q \varepsilon_{t-q} \end{aligned} \quad (1)$$

where $\{e_t\}$ is a white noise process with zero mean and a constant variance of σ^2. The terms $\phi_0, \phi_1, \ldots, \phi_p$ and $\theta_1, \ldots \theta_q$ are parameters that must be estimated. Also considering the stylized fact of non-constant residual variance, we use ARMA models with an auto-regressive conditional heteroskedasticity process – ARMA(*p,q*)-GARCH(*p.q*) models [6]. From the equation 1, we consider the generator process of $\{e_t\}$ as given by

$$\varepsilon_t = v_t \sqrt{h_t}\ ,\ v_t \sim \text{IID}(0,1) \quad (2)$$

$$h_t = \alpha_0 + \sum_{i=1}^{q} \alpha_i \varepsilon_{t-i}^2 + \sum_{i=1}^{p} \beta_i h_{t-1}\,. \quad (3)$$

2.2. Nonlinear models

The output of a single layer feedforward neural network model (NN-FF) with n_1 hidden units (and a linear component) can be defined as

$$y_{t+h}^{h} = w_0^{'} Z_t + \sum_{i=1}^{n} w_{1i}^{'} g(Z_t) + \varepsilon_{t+h} \quad (4)$$

where $g(x) = 1/(1+e^{x})$ and $Z_t = (1, y_t, y_{t-1}, \dots, y_{t-p-1})$. The Levenberg-Marquardt [7] algorithm was used for training the network.

The nonlinear approximation of a radial basis function (NN-RBF) neural network with *m* hidden units is specified by

$$y^h_{t+h} = \sum_{i=0}^{m} w_i \varphi(\|Z_t - \mu_i\|) \quad (5)$$

where $\varphi(.)$ is the basis function and μ_i is the centre of the *i*-th basis function. $\varphi(.)$ is usually defined as a gaussian function $\varphi(x) = \exp(-x^2/2\sigma^2)$, where σ define the width of each function. The least squares solution of the weights w_i satisfies $(A^T A)w = A^T b$ where *A* is the matrix with element A_{ij} representing the output element of the *j*-th hidden neuron for the *i*-th input.

The fuzzy system tested was a TS type with Gustafson-Kessel [8,9] fuzzy covariance matrix clustering (rule preceding) and least squares (rule consequent). The *IF-THEN* rules of this model assume the general form

$$R_i : IF\ x_1 \text{ IS } A_{i1} \text{ AND} \dots \text{AND } x_n \text{ IS } A_{1n}$$
$$\text{THEN } \hat{y}_i = \boldsymbol{a}_i \boldsymbol{x} + b_i, \quad i = 1, 2, \cdots, K \quad (6)$$

where the preceding *IF* defines the preceding portion (premise) while the rule functions *THEN* constitute the consequential part of the fuzzy system; R_i is the i^{th} rule, $\boldsymbol{x} = [x_1, \cdots, x_n]^T$ is the vector of the rule input variables (preceding), and y_i is the output of the rule.

3. Methodology

In this work, it was used series of 15min., 60 min. and 120 min. returns (first difference of log-prices) from 01/01/2002 to 01/01/2003 and daily and weekly log-returns (first difference of log-prices) from 01/01/2000 to 01/01/2004 of the Brazilian exchange rate (R$/US$). The first 80% of the data were used for model estimation while the last 20% were used for validation and one-step-ahead out-of-sample forecasting. Table 1 shows the number of observations for estimation and validation (forecasting) steps for each series.

	15 m.	60 m.	120 m.	Daily	Weekly
Estimation	4,515	1,180	730	804	180
Validation	1,128	294	183	201	20
Total	5,643	1,474	913	1,005	200

Table 1: Number of observations for estimation and validation steps.

In table 2, the results of the specification tests Augmented-Dickey-Fuller (ADF) unit root test, the ARCH LM test and the BDS test for nonlinearity are shown. Results indicate that all series are stationary and contain ARCH effects. Departing from residuals of a AR(1)-GARCH(1,1) model, the BDS test indicates the presence of nonlinearity in all series. Results shows that the higher the series' frequency, the higher the level of nonlinearity.

Series	15 min.	60 min.	120 min.	Daily	Weekly
ADF	-13.61*	-7.30*	-7.69*	-14.88*	-13.80*
ARCH LM	58.57*	23.57*	9.75*	174.9*	25.17*
BDS z-score	0.025 16.64*	0.032 11.51*	0.034 9.57*	0.036 11.98*	0.032 4.29*

Table 2: Specification tests results. * indicates statistic significance at 95% confidence level.

In the neural network models, we used a single hidden layer network for the feedforward and radial basis function networks. The number of neurons that produce smaller forecast error were defined through various simulations. In the TS fuzzy system, the number of membership functions that produce best results were also defined through simulations.

The NARX and NARMAX (*Nonlinear Auto-Regressive Moving Avegare with eXogenous inputs*) approaches were used for the nonlinear dynamic representation of the series [10]. The Akaike Information Criterion was used for choosing the number of terms and its lags in the linear and nonlinear models. In the results, only the best-fit model structure is presented.

The forecast was evaluated using the following criteria: root mean squared error (RMSE), U-Theil inequality index, percentage of corrected predicted signals (CPS) and the Pesaran-Timmermann (PT) predictive failure statistic [11].

4. Results

Tables 3 to 7 show the forecast evaluation for the financial series used in this work. The most important result is that nonlinear models performed better than linear models in all series. In the series of 15 min. returns, the best linear model obtained a U-Theil index of 0.867 and predicted correctly 44% of the returns movements (signals direction), although without statistic significance. On the other hand, the best nonlinear model obtained a U-Theil index of 0.786 and predicted correctly 54% of the returns movements, with a 99% statistic significance level, measured by the PT statistic.

It is also clear that the lower is the series' frequency, the greater is the model's ability for making good forecasts. Tables 3 to 7 show that when the frequency is decreasing, the quality of the forecast increases. A possible explanation is that high frequency time series exhibits a greater level of nonlinearity, what makes the forecast a difficult task (see Table 2). In this sense, the worst forecast results were obtained for the 15 min. returns (table 3) and the best results for the weekly returns series (table 7). In this last case, the NN-FF-NARMAX model obtained a U-Theil index of 0.469 and predicted correctly 82% of the signals direction with 99% of statistic significance, while the best linear model obtained a U-Theil index of 0.654 and predicted correctly 67% of the movements, with 95% of statistic significance.

Model	RMSE	U-Theil	CPS	PT
AR	0.005	0.931	44%	-3.08
MA	0.005	0.930	44%	-3.01
ARMA	0.005	0.894	42%	-5.35
ARMA-GARCH	0.005	0.867	44%	-4.74
NN-FF-NARX	0.005	0.783	53%	2.19*
NN-FF-NARMAX	**0.005**	**0.786**	**54%**	**3.62***
NN-RBF-NARX	0.005	0.828	51%	1.91
NN-RBF-NARMAX	0.005	0.815	48%	1.56
TS-NARX	0.03	0.957	54%	1.14
TS-NARMAX	0.04	0.940	51%	0.03

Table 3: Forecast results for the Brazilian 15 min. exchange rate returns. * indicates that PT statistic is significant at 95% confidence level .

Model	RMSE	U-Theil	CPS	PT
AR	0.01	0.843	51%	0.67
MA	0.01	0.830	50%	0.19
ARMA	0.01	0.817	51%	0.62
ARMA-GARCH	0.01	0.877	49%	-0.47
NN-FF-NARX	0.001	0.652	59%	3.56*
NN-FF-NARMAX	**0.001**	**0.640**	**60%**	**3.31***
NN-RBF-NARX	0.001	0.740	59%	3.10*
NN-RBF-NARMAX	0.001	0.725	60%	3.21*
TS-NARX	0.07	0.953	54%	2.69*
TS-NARMAX	0.07	0.900	55%	2.03*

Table 4: Forecast results for the Brazilian 60 min. exchange rate returns. * indicates that PT statistic is significant at 95% confidence level .

It should be emphasized that the forecast evaluation based on the size of the forecast error (RMSE) was not useful, in some cases, to distinguish between the best and the worst models. In the case of the 15 min. and weekly returns, the RMSE of the nonlinear models was equal or even greater than the RMSE of the linear models.

Model	RMSE	U-Theil	CPS	PT
AR	0.013	0.782	55%	1.46
MA	0.013	0.762	54%	1.01
ARMA	0.013	0.735	49%	-0.20
ARMA-GARCH	0.013	0.844	53%	0.70
NN-FF-NARX	0.01	0.637	63%	3.74*
NN-FF-NARMAX	**0.01**	**0.617**	**61%**	**3.19***
NN-RBF-NARX	0.01	0.753	58%	2.42*
NN-RBF-NARMAX	0.01	0.714	58%	2.32*
TS-NARX	0.09	0.938	60%	3.79*
TS-NARMAX	0.09	0.837	55%	2.17*

Table 5: Forecast results for the Brazilian 120 min. exchange rate returns. * indicates that PT statistic is significant at 95% confidence level .

Model	RMSE	U-Theil	CPS	PT
AR	0.007	0.781	52%	1.96*
MA	0.008	0.787	51%	1.49
ARMA	0.008	0.771	47%	0.19
ARMA-GARCH	0.008	0.867	54%	1.55
NN-FF-NARX	**0.001**	**0.700**	**61%**	**3.85***
NN-FF-NARMAX	0.001	0.701	58%	3.21*
NN-RBF-NARX	0.001	0.821	60%	3.14*
NN-RBF-NARMAX	0.01	0.732	58%	2.42*
TS-NARX	0.10	0.953	57%	1.17
TS-NARMAX	0.10	0.772	59%	2.34*

Table 6: Forecast results for the Brazilian daily exchange rate returns. * indicates that PT statistic is significant at 95% confidence level .

Model	RMSE	U-Theil	CPS	PT
AR	0.013	0.681	57%	1.78
MA	0.013	0.654	67%	2.33*
ARMA	0.013	0.624	62%	2.06*
ARMA-GARCH	0.013	0.658	57%	0.74
NN-FF-NARX	0.02	0.458	75%	3.27*
NN-FF-NARMAX	**0.02**	**0.469**	**82%**	**4.32***
NN-RBF-NARX	0.02	0.698	62%	2.11*
NN-RBF-NARMAX	0.13	0.796	65%	1.50
TS-NARX	0.13	0.910	62%	2.48*
TS-NARMAX	0.13	0.904	60%	2.00*

Table 7: Forecast results for the Brazilian weekly exchange rate returns. * indicates that PT statistic is significant at 95% confidence level .

Table 8 summarizes the forecast results of the best performance model for each series. The NN-FF model generated the best forecasts for all of the analyzed series. The NARMAX structure was the best mathematical representation, with exception in the daily returns series.

Series	15 min.	60 min.	120 min.	Daily	Weekly
Best model	NN-FF	NN-FF	NN-FF	NN-FF	NN-FF
RMSE	0.005	0,001	0,01	0,001	0,02
U-theil	0.786	0.640	0.617	0.700	0.469
CPS	54%	60%	61%	61%	82%
PT	3.62*	3,31*	3,19*	3.85*	4.32*

Table 8: Summary of forecast results for the best performance models. * indicates that PT statistic is significant at 95% confidence level .

Table 9 shows the results of the trade strategy based on the model's forecast. It compares the total return, standard deviation and Sharpe index of the strategy based on the best linear model, best nonlinear model and the buy-and-hold strategy, respectively. It is assumed no transaction costs, no bid-ask spreads and the possibility of uncovered short.

The trade strategy based on nonlinear models achieved greater returns in all cases when compared to the others strategies. However, when the risk of the strategy is considered (the standard deviation), the linear models achieved higher Sharpe indexes in all cases, with exception of the 15 min. and weekly returns.

Strategy / Model	Return	Std. Dev.	Sharpe
15 min.			
AR	12.5%	3.9%	3.20
NN-FF-NARMAX	48.1%	11.6%	4.13
Buy-and-hold	13%	3.9%	3.26
60 min.			
ARMA	36.5%	8.6%	4.21
NN-FF-NARMAX	92.7%	28.2%	3.28
Buy-and-hold	13.6%	5.2%	2.60
120 min.			
MA	25.1%	6.2%	4.03
NN-FF-NARMAX	80.2%	24.3%	3.30
Buy-and-hold	12.4%	4.8%	2.54
Daily			
AR	36.9%	10.2%	3.40
NN-FF-NARMAX	50%	15.7%	3.13
Buy-and-hold	-18.1%	-4.2%	-4.27
Weekly			
ARMA-GARCH	6.3%	3.2%	2.00
NN-FF-NARMAX	55.2%	17.2%	3.20
Buy-and-hold	-6.2%	1.4%	-4.44

Table 9: Trade strategy results

5. Conclusions

In this work, the forecast accuracy of the nonlinear models of NN-FF, NN-RBF and the TS fuzzy system is compared to the traditional ARMA and ARMA-GARCH linear models. Empirical evidence showed that nonlinear models performed better than linear models in forecasting Brazilian exchange rates and the accuracy of the forecasts is related to the series' frequency, possibly due to nonlinearities effects. In the trade strategy, nonlinear models achieved greater returns than other strategies.

6. References

[1] A. Aguirre, and L. Aguirre, "Time series analysis of monthly beef cattle prices with nonlinear autoregressive models", *Applied Economics*, v.32, pp. 265-277, 2000.

[2] R Malhotra, and D. Malhotra, "Differentiating between good credits and bad credits using neuro-fuzzy systems", *European Journal of Operational Research,* v. 136, pp. 190-211, 2002.

[3] R. Tsaih, and Y. Hsu, and C. Lai, "Forecasting S&P 500 stock index futures with a hybrid AI System", *Decision Support Systems,* v. 23, pp. 161-174, 1998.

[4] F.-M Tseng, and G.-H Tzeng, "A fuzzy seasonal ARIMA model for forecasting", *Fuzzy Sets and Systems*, v.126, pp. 367-376. 2002.

[5] T. León, V. Liern, and E. Vercher, "Viability of infeasible portfolio selection problems: a fuzzy approach", *European Journal of Operational Research*, v. 139, pp. 178-189, 2002.

[6] Y. Fang, and D. Xu, "The predictability of asset returns: an approach combining technical analysis and time series forecasts", *International Journal of Forecasting*. V.19, pp. 369-386, 2003.

[7] D. Marquardt, "An algorithm for least-squares estimation of nonlinear parameters". *SIAM J. of Applied Mathematics,* pp. 431-441, 1963.

[8] T. Takagi, and M. Sugeno, "Fuzzy identification of systems and its application to modeling and control", *IEEE Trans. Systems, Man, and Cybernetics*, v.15, pp. 116–132. 1985.

[9] D. Gustafson, and W. Kessel, "Fuzzy Clustering with a Fuzzy Covariance Matrix", *Proceedings of IEEE Conference on Control and Decision,* pp. 761-766. 1979.

[10] S. Chen, and S. Billings, "Representation of non-linear systems: the NARMAX model", *International Journal of Control*, v. 49, pp. 1012-1032, 1989.

[11] M. Pesaran, and A. Timmermann, "A simple nonparametric test of predictive performance", *Journal of Business & Economic Statistics,* v.10, pp. 561-565, 1992.