The Self-Referential Construction for Computational Intelligence Processing in Economics and Finance

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Abstract: In the economical and financial fields, a given object often has many indices related to it. This frequently results in some complications and confusion in our understanding of the issues, but we nonetheless use all of these indices hoping to discover new knowledge or just to show that some predictions can be made about the object's properties. By analyzing an actual example, this paper proposes a self-referential construction that acts in a manner similar to two parallel mirrors such that a person can see both the front and the back image in this mirror combination. The self-referential construction is not only mathematically described, but also practically realized with an artificial intelligence algorithm. Thus, we hope that it will be widely used in many fields in addition to economics and finance.

Keywords: self-referential construction; embedding theorem; neural network; Jackknife error estimation.

1. Introduction

In economics and finance, for an object, there are usually many indices given that are related to its properties. Typically, if the object is for example the GDP, then the first, second and third industrial indices are related to it. In general, these indices have the following characteristics: every pair is not the same, each with respect to the other but they are not independent mathematically either. Different indices reflect different aspects of the object; and all of them together can show the all aspects of the object, but no single index does so alone. In general, we frequently are deluded by these numerous indices. However, we can derive more implications based on accounting for all these indices. Then with these implications, we can explain the various phenomena that seem to be out-of-order at first glance but which are actually naturally in accord with one another.

Making an investment decision or making a significant economic plan requires us to consider all aspects as much as possible and any oversight will result in a huge loss of the potential wealth. In contrast, if we can reliably predict the trend in the future based on these indices, then it will be very useful to shepherd investment accordingly. It is possible to realize this idea theoretically and in practice as we show below.

For example, in April 2003, we initially aimed to predict the trends for the prices of copper, gold and silver in the London Future Market. At the same time, we casually gave predictions for the rate of exchange for the US dollar versus the euro, the rate for the US dollar versus the Japanese yen, the index for the US dollar, the S&P index and DJ30 index. (These trends are shown in the 8 figures provided in the appendix that will be explained individually latter). Combining these results, we believe everybody would be able to conclude that the US dollar will be devaluated if we assume that index for the US dollar is the object and the other seven indices are related to it. This example reveals an insight telling us that we may construct an algorithm in the form similar to a pair of mirrors that show whether the prediction made is true or false.

2. The self-referential model

Self-reference is a most ubiquitous word in philosophy. Self-reference in formal arithmetic can be compared to a formal way of self-recognition in the mirror. The basis for this comparison is the role of the Gödel code in arithmetical self-reference [1] (Baumgartner *et al* (ed.)). There exist theories of cognition that assume the importance of self-referentiality and/or self-modification. Certain theories of cognition assume that selected aspects of mental activity can be characterized via self-referential or self-modifying constructs [2] (Luis Rocha ed.). Typically, in economics and financial fields, every single index has the possibility to misrepresent reality. So we need to construct a self-referential construction for proving the authenticity of information.

Mathematically, every index represents a stochastic process. For an object, it is also a stochastic process. In general, there are many indices that can be related to the same object. Amongst these indices, some may correlate with each other. We have no way to prove that a certain conclusion regarding the object is true or false if we only depend on the object. So we need to produce a construction based on these related indices to prove that prediction of the object whether true or false. How to produce such a construction and how many indices are sufficient for it are the key questions that we intend to answer.

Let $\eta(t)$ be the stochastic process that represents the object and $\xi_1(t), \xi_2(t), ..., \xi_M(t)$ be all of the indices that related to the object. Let

$$\xi_{n_1}(t), \xi_{n_2}(t), ..., \xi_{n_K}(t)$$

be a subsequence, that we say is good enough to construct the mirrors of object $\eta(t)$, if the mutual information between $\eta(t)$ and $(\xi_{n_1}(t), \xi_{n_2}(t), ..., \xi_{n_K}(t))$ is the same as the entropy of $\eta(t)$, that is,

$$\begin{aligned} &I(\eta(t); \xi_{n_1}(t), \xi_{n_2}(t), ..., \xi_{n_K}(t)) = \\ &I(\eta(t); \eta(t)) = H(\eta(t)) \end{aligned},$$

then using standard notions in informatics [3], we have a function f such that

$$\eta(t) = f(\xi_{n_1}(t), \xi_{n_2}(t), ..., \xi_{n_K}(t)).$$

Typically, if $\eta(t)$ is the GDP index, and $\xi_1(t), \xi_2(t), \xi_3(t)$ are the contribution rates of the first, second and third industrial, then

$$\begin{aligned} \eta(t) &= f(\xi_1(t), \xi_2(t), \xi_3(t)) \\ &= \xi_1(t) + \xi_2(t) + \xi_3(t) \end{aligned}.$$

In general, let

$$X(t) = (\xi_{n_1}(t), \xi_{n_2}(t), ..., \xi_{n_K}(t)),$$

then we have $\eta(t) = f(X(t))$. It is easy to understand that $X(t)$ is a K-dimensional stochastic process, and we regard it as a flow. In practice, the range of $X(t)$ is a bounded set as the time t has a range of all positive integers. Hence, it represents a flow in a K-dimensional compact manifold. On the other hand, we can embed $X(t)$ into a D_E-dimensional Euclidean space R^{D_e}, here, D_E is the embedding dimension. This method is developed in [4] (Ruan and Shen) based on the original idea of Takens F. [5]. Here, we can regard f as an observer such that $\eta(0), \eta(1), ..., \eta(N)$ are the observed data by f at the equidistance time on the flow $X(t)$, that is, $\eta(i) = f(X(i))$. Let

$$Y(t) = (\eta(t), \eta(t-1), ..., \eta(t - D_E + 1)),$$
$$t = D_E, D_E + 1, ..., N.$$

Then $Y(t)$ is a D_E-dimensional flow whose range is R^{D_e}. Following reference [4], we have functions:

$$F(Y(t)) = X(t) \text{ and } G(Y(t)) = X(t - D_E)$$

for $t \geq D_E$.

If $\eta(N+1)$ is given (by any method), then we will have the new vector

$$\begin{aligned} &Y(N+1) = \\ &(\eta(N - D_E + 2), ..., \eta(N), \eta(N+1)) \end{aligned}.$$

Then $\hat{X}(N+1) = F(Y(N+1))$ is found to be the predicted value of $X(t)$ at $t = N+1$, and $\hat{X}(N - D_E + 1) = G(Y(N+1))$ is obtained as the checkout value of $X(t)$ at $t = N - D_E + 1$.

We have the following self-referential construction:

1. Prediction value $\eta(N+1)$ based on $\eta(0), \eta(1), ..., \eta(N)$ is true if the following conditions hold:

$$\begin{cases} \hat{X}(N+1) = F(Y(N+1)), & (1) \\ \eta(N+1) = f(\hat{X}(N+1)), & (2) \\ X(N - D_E + 1) = & \\ \hat{X}(N - D_E + 1) = G(Y(N+1)), & (3) \end{cases}$$

2. If $\hat{X}(N+1)$ is given, then $\eta(N+1)$ and $Y(N+1)$ are known, and if $\hat{X}(N+1)$ is true, then (1), (2) and (3) hold.
3. If $Y(N+1)$ is given, then $\hat{X}(N+1)$ and $\eta(N+1)$ are known, and if $Y(N+1)$ is true, then (1), (2) and (3) hold.

3. The neural network Method for realizing the self-referential construction

It is easy to understand that we may get the prediction $\eta(N+1)$ based on the observed data

$$\eta(0), \eta(1), ..., \eta(N),$$

and we may get the prediction $X(N+1)$ based on the observed data $X(0), X(1), ..., X(N)$. If both $\eta(N+1)$ and $X(N+1)$ are true, then we will have

$$\eta(N+1) = f(X(N+1)).$$

Thus for getting the self-referential or self-modifying construction as that in section 2, we need to train three neural networks NN1, NN2 and NN3 to replace functions f, F, G based on the dataset

$$\begin{pmatrix} \eta(0), \eta(1), ..., \eta(N) \\ X(0), X(1), ..., X(N) \end{pmatrix}$$

and its derivative data.

We train a 3-layer perceptron with K input nodes, 2k+1 hidden nodes and 1 output node based on the observed data

$$\begin{pmatrix} \eta(0), \eta(1), ..., \eta(N) \\ X(0), X(1), ..., X(N) \end{pmatrix}$$

denoting it by NN1. Based on the observed data

$$\{X(t) \mid t = D_E, D_E + 1, ..., N\},$$
$$\{X(t) \mid t = 1, 2, ..., N - D_E\}$$

and the derivative data

$$\{Y(t) \mid t = D_E, D_E + 1, ..., N\},$$

we can train two 3-layer perceptrons NN2 and NN3 that have D_E input nodes, $2D_E + 1$ hidden nodes and K output nodes to replace the function $X(t) = F(Y(t))$ and the function $X(t - D_E) = G(Y(t))$ for all t, respectively.

Making the Jackknife error estimation [6], we will get the average error μ_i and the square root of variance, σ_i, $i = 1,2,3,$ for NN1, NN2 and NN3, respectively.

After the above preparations are made, we can use three NNs to realize the self-reference and self-modification as follows:

Using the method used in the example given in the introduction section (the 8 figures are based on the principle that given in [7].) to get the prediction values of $\eta(t)$ for a period, say,

$$\eta(N+1), ..., \eta(N+P), \text{ where } P > D_E,$$

we can get new points of the flow $Y(t)$ at

$$t = N+1, ..., N + D_E - 1$$

defined by

$$Y(t) = (\eta(t), \eta(t-1), ..., \eta(t - D_E + 1).$$

Putting them into NN2 and NN3 orderly, we have the prediction values and the checkout values of $X(t)$ denoted by

$$\hat{X}(t) \text{ and } \hat{X}(t - D_E) \text{ for}$$
$$t = N+1, ..., N + D_E - 1.$$

They should satisfy in the ideal case the following relationships:

$$\eta(N+i) = f(\hat{X}(N+i)), X(N - D_E + i)$$
$$= \hat{X}(N - D_E + i), \quad for \quad i = 1,2,..., D_E - 1$$

In practice, we use a relaxed rule to replace the above ideal case, namely:

If the following inequalities hold:

$$||\eta(N+i) - f(\hat{X}(N+i)| - \mu_1| < k\sigma_1 \quad \text{and}$$
$$|dis(X(N - D_E + i), \hat{X}(N+i)) - \mu_3| < k\sigma_3$$

for $i = 1,2,..., D_E - 2$, then we consider $\eta(N+1), ..., \eta(N + D_E - 1)$ of $\eta(t)$ to be true at a k-level, where

$$dis(X(N - D_E + i), \hat{X}(N+i))$$

means the Euclidean distance between $X(N - D_E + i)$ and $\hat{X}(N+i)$, $k = 1,2,3$. For $k = 1$, the achieved reliability is much higher.

4. Discussions

We believe the presented model giving the self-referential construction can be widely used in economic and financial fields. For example, we can use this model to study the GDP or GNP with the industrial structure vector (1^{st} industrial, 2^{nd} industrial, 3^{rd} industrial), the interest rates or the inflation rates with the construction of money and so on, are also well captured this model. In 2001, 2002, and 2003, many students were supervised by us to check this theory partly with the actual data from the US and China. The results obtained in these studies provide strong support for this model and have been summarized in their theses (unpublished).

5. Appendix

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Figure 1: Index of Copper in London Future Market. On the right hand side of the black line, four curves constructed by 206 actual end price weekly, the black line is the deadline: 7th of April of 2003. Left side of the black line is the prediction result. It means the price of the Copper will arrive at new peak after April of 2003.

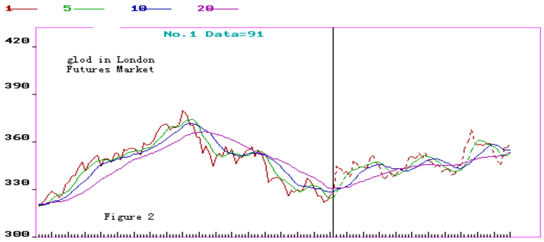

Figure 2: Index of Gold in London Future Market. On the right hand side of the black line, four curves constructed by 91 end prices weekly, the black line is the deadline: 7th of April of 2003. Left side of the black line is the prediction result. It means that price of the Gold will increase after the 7th of April of 2003.

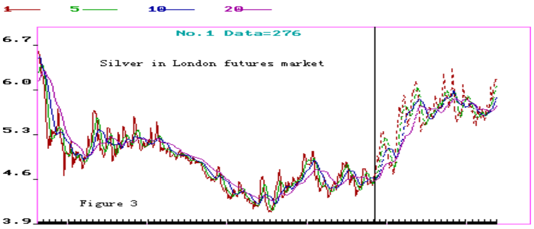

Figure 3: Silver in London Future Market. On the right hand side of the black line, four curves constructed by 276 actual end prices weekly, the black line is the deadline: 7th of April of 2003. The left side of the black line is the prediction result. It means that Silver will reach about 6.5 US dollars

Figure 4: Rate of the US dollar to the euro. On the right hand side of the black line, four curves constructed by 269 actual end prices weekly, the black line is the deadline: 7th of April of 2003. Left side of the black line is the prediction result. It means that the euro will reach at least 1.3 as the top rate.

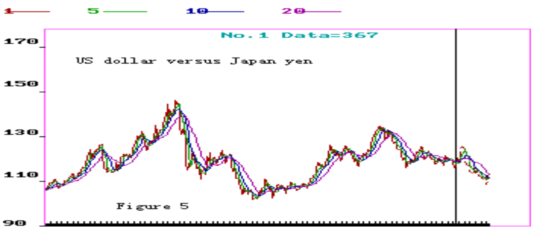

Figure 5: Rate of America dollar to Japan yen. On the right hand side of the black line are consisting of four curves constructed by 367 actual end prices weekly. The black line is the deadline: 7th of April of 2003. Left side of the black line is the prediction result. It means that the Japanese yen will arrive at the top value 105 finally.

Figure 6: Index of S&P. On the right hand side of the black line, four curves constructed by 177 actual end prices weekly, the black line is the deadline: 7th of April of 2003. Left side of the black line is the prediction part. It means that index of S&P will arrive at the top value 1500.

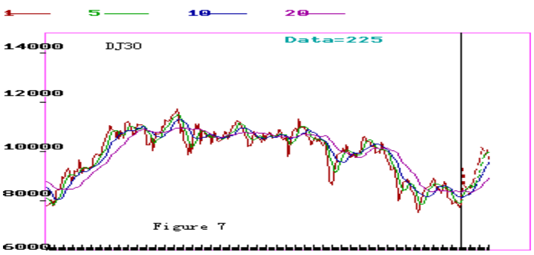

Figure 7: Index of DJ30. On the right hand side of the black line, four curves are constructed by 225 actual end prices weekly, the black line is the deadline: 7th of April of 2003. Left side of the black line is the prediction result. It means that index of DJ30 will increase after the 7th of April of 2003.

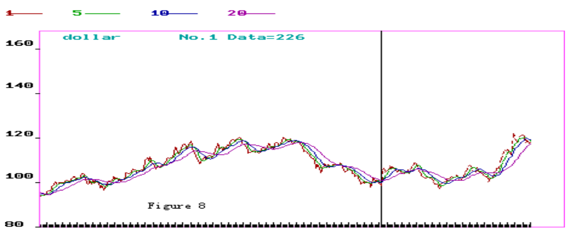

Figure 8: Index of America dollar. On the right hand side of the black line, four curves are constructed by 226 actual end prices weekly, the black line is the deadline: 7th of April of 2003. Left side of the black line is the prediction result. It means that the US dollar will be devalued and the lower value hopefully is 0.90.