

Ranking Stocks Using FMCDM

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Abstract

A FMCDM approach is applied to rank alternative stocks. It can deal with qualitative information additionally to quantitative information. A hierarchy of major-sub criteria is established. The ratings of alternatives versus qualitative sub-criteria and the weights of major- and sub-criteria are assessed in linguistic terms represented by fuzzy numbers. Each sub-criterion is in a benefit, cost, or balance nature. New standardization methods for sub-criteria in the cost and balanced nature are presented. The backpropagation aggregation is performed by using α -cut and interval arithmetic of fuzzy numbers. The algorithms of membership functions of the final aggregation are completely developed, not approximated. The final aggregations in fuzzy numbers are then defuzzified into crisp values to rank the performance of alternatives. Moreover, the ratio of market price to performance (*PP*) is suggested to filter the over/under-pricing of alternatives. Finally, a set of buying/selling rules based on the performance and *PP* are recommended.

Keywords: stock, FMCDM, membership function, α -cut, interval arithmetic

1. Introduction

The way to make profits from stock investments is to purchase stocks at prices below their values and sell them at prices beyond their values. Therefore the key to making profits is to identify the values of stocks. Almost all valuation methods in traditional finance used quantitative variables only. The qualitative variables, though generally acknowledged as important, are not included for measurement difficulties. The quantitative variables in crisp number do not consider the imprecision of measurement.[3] For example, the profit/loss of a business can be certified only when the operation ends and all assets/liabilities are liquidated/paid. However, GAAP requires a business to cut its duration into many accounting periods in order to provide users of financial statements with timely information. Another example is that the depreciation expenses of a machine may vary with different combinations of duration estimates and depreciation methods. The figure shown on the financial statement is just the outcome from one of

the combinations. The insufficiencies of excluding qualitative variables and imprecise quantitative figures now can be improved by using the disciplines out of traditional finance and accounting – the fuzzy set theory [6] and MCDM (Multiple Criteria Decision Making).

Fuzzy set theory catches the vagueness existing in available information, and MCDM determines the ranking order of alternatives versus the available information. This work establishes a two-level hierarchy of FMCDM -- the major-sub criteria structure, to overcome the problem existing in a single-level criteria structure, the possibility of over-score on the aspect represented by some dependent criteria. The algorithms of operations of fuzzy numbers are therefore extended from quadratic equations in the single-level structure to cubic equations in the two-level structure.

The proposed approach begins by selecting the factors for evaluating a firm's performance, labeled sub-criteria. They are grouped into quantitative and qualitative ones, labeled major-criteria. Each sub-criterion is identified to be in a benefit, cost, or balanced nature. The ratings of alternatives versus qualitative sub-criteria and the weights of major- and sub-criteria are assessed in linguistic terms [7] represented by fuzzy numbers; the ratings of alternatives versus quantitative sub-criteria are assessed in crisp numbers.

The assessments are then standardized to ensure compatibility. The standardizations of ratings versus qualitative sub-criteria in cost and balanced nature are initial ones. The algorithms of the final aggregation and membership functions are developed using α -cut and interval arithmetic of fuzzy numbers [2]. Then a ranking method, the average of relative regions [5], is applied to defuzzify the fuzzy final aggregations for ordering the performance of the alternatives. Since the membership functions are developed, the shape of the fuzzy final aggregation can be accurately depicted and the error possibly occurring in defuzzification from approximating shapes can be avoided. The "performance" of alternatives is treated as the "value." Then the ratio of price to performance (*PP*) is suggested to detect the over/under-pricing of alternatives. A set of decision rules are recommended. An empirical example then tests the proposed approach.

2. Fuzzy Numbers and Interval Arithmetic

The fuzzy set theory was introduced by Zadeh [6] to deal with problems existing in fuzzy phenomena. A real fuzzy number, denoted by A , is described as any fuzzy subset of the real line R with a membership function, denoted by f_A . Assume that A is a trapezoidal fuzzy number. Its membership function is expressed as: [1,2]

$$f_A(x) = \begin{cases} f_A^L(x) = (x-a)/(b-a), & a \leq x \leq b, \\ 1, & b \leq x \leq c, \\ f_A^R(x) = (x-d)/(c-d), & c \leq x \leq d, \\ 0, & \text{otherwise,} \end{cases} \quad (1)$$

where $f_A^L(x)$ and $f_A^R(x)$ are respectively the left and right membership functions of fuzzy number A . It can be denoted by $A=(a,b,c,d)$, $a,b,c,d \in R$.

The α -cut of fuzzy number A is defined as : [2]

$$A^\alpha = \{x | f_A(x) \geq \alpha\}, \quad (2)$$

where $x \in R$, $\alpha \in [0,1]$. Here, A^α is a non-empty bounded closed interval contained in R . The α -cut of trapezoidal fuzzy number $A=(a,b,c,d)$ is:

$$A^\alpha = [A_l^\alpha, A_u^\alpha] = [(b-a)\alpha + a, (c-d)\alpha + d], \quad (3)$$

where A_l^α and A_u^α are the lower and upper bounds of the closed interval, respectively.

3. Ranking Fuzzy Numbers by the Average of the Relative Regions

Suppose that a set of fuzzy numbers, $A_i = (a_i, b_i, c_i, d_i)$, $i = 1 \sim n$, are compared to put in their orders. The left and right relative regions of fuzzy number A_i , respectively denoted as $S_L(A_i)$ and $S_R(A_i)$, are defined as

$$S_L(A_i) = b_i - x_{\min} - \int_{a_i}^{b_i} f_{A_i}^L(x) dx, \quad (4)$$

$$S_R(A_i) = c_i - x_{\min} + \int_{c_i}^{d_i} f_{A_i}^R(x) dx, \quad (5)$$

where $x_{\min} = \inf P$, $x_{\max} = \sup P$, $P = \bigcup_{i=1}^n P_i$, $P_i = \{x | f_{A_i}(x) > 0\}$. The average of the relative regions for A_i is defined as

$$S(A_i) = \frac{1}{2} [S_L(A_i) + S_R(A_i)]. \quad (6)$$

A larger $S(A_i)$ means a larger A_i . Therefore, if $S(A_i) > (=, <) S(A_j)$, then $A_i > (=, <) A_j$.

4. The FMCDM Approach

Assume that there are several alternative stocks ($i=1 \sim m$) to be evaluated versus both quantitative ($j=1$, $k=1 \sim n_1$, $n_1 \in N$) and qualitative ($j=2$, $k=1 \sim n_2$, $n_2 \in N$) criteria. The detailed evaluating steps are developed as follows.

4.1. Establishing the Hierarchy

A hierarchy of major-sub criteria is established as

$$C_j = \{c_{jk} | j=1,2, k=1 \sim n_j, n_j \in N\}. \quad (7)$$

where C_j denotes major-criteria and c_{jk} denotes sub-criteria of C_j . Here, all sub-criteria are identified to be in benefit (B), cost (C), or balance nature (M). For the benefit nature a greater value is better, e.g., earnings per share; for cost nature a smaller value is better, e.g., bankruptcy probability and debt ratio; for balance nature a mean value is the best whereas the value farther away from the mean value is worse. e.g., current asset to current liability.

Let x_{ijk} present the assessed rating of alternative i versus sub-criterion k under major-criterion j ; x_{i1k} and x_{i2k} respectively denote the ratings versus quantitative and qualitative sub-criteria. Let W_j and w_{jk} respectively represent the assigned weight of major-criterion j and sub-criterion k under major-criterion j . x_{i1k} , are crisp numbers. Terms W_j , w_{jk} and x_{i2k} , are assessed in linguistic terms [7] represented by fuzzy numbers.

4.2. Standardizing Quantitative Ratings

The ratings versus different sub-criteria may be in different nature or units, e.g. \$ or % and need to be standardized to assure compatibility with each other. [4]

$$x_{i1k} = \begin{cases} 0, & x_{i1k} \leq \mu_{i1k} - 2\sigma_{i1k}, \\ \frac{x_{i1k} - (\mu_{i1k} - 2\sigma_{i1k})}{4\sigma_{i1k}}, & \mu_{i1k} - 2\sigma_{i1k} < x_{i1k} < \mu_{i1k} + 2\sigma_{i1k}, \quad x_{i1k} \in B, \\ 1, & x_{i1k} \geq \mu_{i1k} + 2\sigma_{i1k} \end{cases} \quad (8)$$

$$x_{i1k} = \begin{cases} 1, & x_{i1k} \leq \mu_{i1k} - 2\sigma_{i1k}, \\ \frac{(\mu_{i1k} + 2\sigma_{i1k}) - x_{i1k}}{4\sigma_{i1k}}, & \mu_{i1k} - 2\sigma_{i1k} < x_{i1k} < \mu_{i1k} + 2\sigma_{i1k}, \quad x_{i1k} \in C, \\ 0, & x_{i1k} \geq \mu_{i1k} + 2\sigma_{i1k}, \end{cases} \quad (9)$$

$$x_{i1k} = \begin{cases} 0, & x_{i1k} \leq \mu_{i1k} - 2\sigma_{i1k}, \\ \frac{x_{i1k} - (\mu_{i1k} - 2\sigma_{i1k})}{2\sigma_{i1k}}, & \mu_{i1k} - 2\sigma_{i1k} < x_{i1k} < \mu_{i1k}, \\ 1, & x_{i1k} = \mu_{i1k}, \quad x_{i1k} \in M, \\ \frac{(\mu_{i1k} + 2\sigma_{i1k}) - x_{i1k}}{2\sigma_{i1k}}, & \mu_{i1k} < x_{i1k} < \mu_{i1k} + 2\sigma_{i1k}, \\ 0, & x_{i1k} \geq \mu_{i1k} + 2\sigma_{i1k}, \end{cases} \quad (10)$$

where x_{i1k} denotes the standardized value of x'_{i1k} , the original assessed rating, which is assumed to have a population of m observations with mean μ'_{1k} and standard deviation σ'_{1k} , $0 \leq x_{i1k} \leq 1$.

To make crisp x_{i1k} compatible with fuzzy x_{i2k} in later aggregation, x_{i1k} is treated as a special form of a fuzzy number, denoted as $x_{i1k} = (o_{i1k}, p_{i1k}, q_{i1k}, r_{i1k})$, where $o_{i1k} = p_{i1k} = q_{i1k} = r_{i1k} = x_{i1k}$.

4.3. Determining the Linguistic Rating Set and Standardizing Fuzzy Qualitative Ratings

Assume that the decision maker prefers the linguistic rating set $L = \{VL, L, M, H, VH\}$ (Very Low, Low, Middle, High, Very High) to measure the ratings of alternatives versus qualitative sub-criteria.

Let $x'_{i2k} = (o'_{i2k}, p'_{i2k}, q'_{i2k}, r'_{i2k})$ denote the fuzzy ratings representing the linguistic terms, where $o'_{i2k}, p'_{i2k}, q'_{i2k}, r'_{i2k} \geq 0$. The standardizations $x_{i2k} = (o_{i2k}, p_{i2k}, q_{i2k}, r_{i2k})$ are obtained as:

$$x_{i2k} = \left(o'_{i2k} / \max_i o'_{i2k}, p'_{i2k} / \max_i p'_{i2k}, q'_{i2k} / \max_i q'_{i2k}, r'_{i2k} / \max_i r'_{i2k} \right), \quad x_{i2k} \in B, \quad (11)$$

$$x_{i2k} = \left(1 - (r'_{i2k} / \max_i r'_{i2k}), 1 - (q'_{i2k} / \max_i q'_{i2k}), 1 - (p'_{i2k} / \max_i p'_{i2k}), 1 - (o'_{i2k} / \max_i o'_{i2k}) \right), \quad x_{i2k} \in C. \quad (12)$$

$$x_{i2k} = \left(\max_i o'_{i2k} - \max_i o'_{i2k}, \max_i p'_{i2k} - \max_i p'_{i2k}, \max_i q'_{i2k} - \max_i q'_{i2k}, \max_i r'_{i2k} - \max_i r'_{i2k} \right), \quad x_{i2k} \in M, \quad (13)$$

where

$$D' = \left(\left| o' - \frac{\max_i r'}{2} \right|, \left| p' - \frac{\max_i r'}{2} \right|, \left| q' - \frac{\max_i r'}{2} \right|, \left| r' - \frac{\max_i r'}{2} \right| \right).$$

4.4. Determining the Linguistic Weighting Set and Fuzzy Weights of Criteria

Assume that the decision maker prefers the linguistic weighting set $W = \{UI, C, I, VI, EI\}$ (Unimportant, Common, Important, Very Important, Extremely Important) to measure weights of sub-criteria and major-criteria. Let $w_{jk} = (a_{jk}, b_{jk}, c_{jk}, d_{jk})$ be the weight given to sub-criterion k under major-criterion j , where $a_{jk}, b_{jk}, c_{jk}, d_{jk} \geq 0$. Let $W_j = (w_j, x_j, y_j, z_j)$, be the weight assigned to major-criterion j , where $w_{jl}, x_{jl}, y_{jl}, z_{jl} \geq 0$.

4.5. Obtaining the Final Aggregation

Let G_i denotes the final aggregation for alternative

i . The α -cut of G_i is obtained as:

$$\begin{aligned} G_i^\alpha &= \sum_{j=1}^2 (W_j^\alpha \otimes G_{ij}^\alpha) \\ &= \left[\left(\sum_{j=1}^2 \left((x_j - w_j) \cdot \left(\sum_{k=1}^{n_j} [(b_{jk} - a_{jk})(p_{ijk} - o_{ijk})] \right) \right) \right) \alpha^3 \right. \\ &\quad + \left(\sum_{j=1}^2 \left(w_j \cdot \left(\sum_{k=1}^{n_j} [(b_{jk} - a_{jk})(p_{ijk} - o_{ijk})] \right) + (x_j - w_j) \cdot \left(\sum_{k=1}^{n_j} [a_{jk}(p_{ijk} - o_{ijk}) + o_{ijk}(b_{jk} - a_{jk})] \right) \right) \right) \alpha^2 \\ &\quad + \left(\sum_{j=1}^2 \left(w_j \cdot \left(\sum_{k=1}^{n_j} [a_{jk}(p_{ijk} - o_{ijk}) + o_{ijk}(b_{jk} - a_{jk})] \right) + (x_j - w_j) \cdot \left(\sum_{k=1}^{n_j} (a_{jk} o_{ijk}) \right) \right) \right) \alpha \\ &\quad + \left. \sum_{j=1}^2 \left(w_j \cdot \left(\sum_{k=1}^{n_j} (a_{jk} o_{ijk}) \right) \right) \right] \\ &\quad \left(\sum_{j=1}^2 \left((y_j - z_j) \cdot \left(\sum_{k=1}^{n_j} [(c_{jk} - d_{jk})(q_{ijk} - r_{ijk})] \right) \right) \right) \alpha^3 \\ &\quad + \left(\sum_{j=1}^2 \left(z_j \cdot \left(\sum_{k=1}^{n_j} [(c_{jk} - d_{jk})(q_{ijk} - r_{ijk})] \right) + (y_j - z_j) \cdot \left(\sum_{k=1}^{n_j} [d_{jk}(q_{ijk} - r_{ijk}) + r_{ijk}(c_{jk} - d_{jk})] \right) \right) \right) \alpha^2 \\ &\quad + \left(\sum_{j=1}^2 \left(z_j \cdot \left(\sum_{k=1}^{n_j} [d_{jk}(q_{ijk} - r_{ijk}) + r_{ijk}(c_{jk} - d_{jk})] \right) + (y_j - z_j) \cdot \left(\sum_{k=1}^{n_j} (d_{jk} r_{ijk}) \right) \right) \right) \alpha \\ &\quad + \left. \sum_{j=1}^2 \left(z_j \cdot \left(\sum_{k=1}^{n_j} (d_{jk} r_{ijk}) \right) \right) \right] \\ &= [E_{1i} \alpha^3 + F_{1i} \alpha^2 + H_{1i} \alpha + Q_i, E_{2i} \alpha^3 + F_{2i} \alpha^2 + H_{2i} \alpha + Z_i], \quad (14) \end{aligned}$$

where $W_j^\alpha = [(x_j - w_j)\alpha + w_j, (y_j - z_j)\alpha + z_j]$ and

$$\begin{aligned} G_{ij}^\alpha &= \sum_{k=1}^{n_j} (w_{jk}^\alpha \otimes x_{ijk}^\alpha) \\ &= \left[\left(\sum_{k=1}^{n_j} [(b_{jk} - a_{jk})(p_{ijk} - o_{ijk})] \right) \alpha^2 + \left(\sum_{k=1}^{n_j} [a_{jk}(p_{ijk} - o_{ijk}) + o_{ijk}(b_{jk} - a_{jk})] \right) \alpha + \sum_{k=1}^{n_j} a_{jk} o_{ijk}, \right. \\ &\quad \left. \left(\sum_{k=1}^{n_j} [(c_{jk} - d_{jk})(q_{ijk} - r_{ijk})] \right) \alpha^2 + \left(\sum_{k=1}^{n_j} [d_{jk}(q_{ijk} - r_{ijk}) + r_{ijk}(c_{jk} - d_{jk})] \right) \alpha + \sum_{k=1}^{n_j} d_{jk} r_{ijk} \right] \end{aligned}$$

The membership functions of the final aggregation are derived from the root α of the following two equations:

$$E_{1i} \alpha^3 + F_{1i} \alpha^2 + H_{1i} \alpha + Q_i - x = 0, \quad (15)$$

$$E_{2i} \alpha^3 + F_{2i} \alpha^2 + H_{2i} \alpha + Z_i - x = 0. \quad (16)$$

The left and right membership functions of G_i are:

$$f_{G_i}^L(x) = A_{1i}^{1/3} + B_{1i}^{1/3} - F_{1i}/3E_{1i}, \quad E_{1i} \neq 0, \quad Q_i \leq x \leq R_i, \quad (17)$$

$$f_{G_i}^R(x) = A_{2i}^{1/3} + B_{2i}^{1/3} - F_{2i}/3E_{2i}, \quad F_{2i} \neq 0, \quad Y_i \leq x \leq Z_i, \quad (18)$$

where $A_{1i} = C_{1i} + D_{1i}$, $B_{1i} = C_{1i} - D_{1i}$,

$$C_{1i} = -\frac{1}{2} \left(\frac{Q_i - x}{E_{1i}} - \frac{1}{3} \left(\frac{F_{1i}}{E_{1i}} \cdot \frac{H_{1i}}{E_{1i}} \right) + \frac{2}{27} \left(\frac{F_{1i}}{E_{1i}} \right)^3 \right),$$

$$D_{1i} = \left(\frac{1}{4} \left(\frac{Q_i - x}{E_{1i}} - \frac{1}{3} \left(\frac{F_{1i}}{E_{1i}} \cdot \frac{H_{1i}}{E_{1i}} \right) + \frac{2}{27} \left(\frac{F_{1i}}{E_{1i}} \right)^3 \right)^2 + \frac{1}{27} \left(\frac{H_{1i}}{E_{1i}} - \frac{1}{3} \left(\frac{F_{1i}}{E_{1i}} \right)^2 \right)^3 \right)^{1/2}$$

$$Q_i = \sum_{t=1}^k \left(w_t \cdot \left(\sum_{j=1}^n (a_{jt} o_{ijt}) \right) \right), \quad R_i = \sum_{t=1}^k \left(x_t \cdot \sum_{j=1}^n (b_{jt} p_{ijt}) \right),$$

and $A_{2i} = C_{2i} + D_{2i}, \quad B_{2i} = C_{2i} - D_{2i},$

$$C_{2i} = -\frac{1}{2} \left(\frac{Z_i - x}{E_{2i}} - \frac{1}{3} \left(\frac{F_{2i}}{E_{2i}} \cdot \frac{H_{2i}}{E_{2i}} \right) + \frac{2}{27} \left(\frac{F_{2i}}{E_{2i}} \right)^3 \right),$$

$$D_{2i} = \left(\frac{1}{4} \left(\frac{Z_i - x}{E_{2i}} - \frac{1}{3} \left(\frac{F_{2i}}{E_{2i}} \cdot \frac{H_{2i}}{E_{2i}} \right) + \frac{2}{27} \left(\frac{F_{2i}}{E_{2i}} \right)^3 \right)^2 + \frac{1}{27} \left(\frac{H_{2i}}{E_{2i}} - \frac{1}{3} \left(\frac{F_{2i}}{E_{2i}} \right)^2 \right)^3 \right)^{1/2}$$

$$Y_i = \sum_{t=1}^k \left(y_t \cdot \sum_{j=1}^n (c_{jt} q_{ijt}) \right), \quad Z_i = \sum_{t=1}^k \left(z_t \cdot \sum_{j=1}^n (d_{jt} r_{ijt}) \right).$$

4.6. Ranking the Performance

By Eqs. (4)~(6), the ranking value of G_i is:

$$S(G_i) = \frac{1}{2} \left[(R_i - \min Q_i - \int_{Q_i}^{R_i} f_{G_i}^L(x)) + (Y_i - \min Q_i + \int_{Y_i}^{Z_i} f_{G_i}^R(x)) \right]. \quad (19)$$

4.7. Computing the PP Ratio

A ratio of price to performance is suggested to detect the current price level:

$$PP_i = P_{i,t} / S(G_{i,t+1}), \quad (20)$$

where $P_{i,t}$ denotes the current market price for alternative i at the end of period t , and $S(G_{i,t+1})$ denotes the ranking value of forecast performance for alternative i for period $t+1$. The stock with a lower PP represents relatively a lower risk. The suggested buying/selling strategies are:

- (1) The stock with a good forecast performance and a low PP is expected to have a high return and is recommended as the target of buying long.
- (2) The stock with a bad forecast performance and a high PP is expected to have a poor return and is recommended as the target of selling short.
- (3) The stock with a good forecast performance and a high PP is recommended to be behind that with a good performance and a low PP and only suited to short-term holding.
- (4) The stock with a bad forecast performance and a low PP is not suggested to be purchased.

5. An Empirical Example

An empirical example of five TSE-listed electronics stocks (UMC, MII, HonHai, TSMC, ACER) illustrates the use. The correlation coefficient between forecast performance for 2003 and actual return for 2003 is 0.91. It shows that the performance aggregated by the proposed approach moves with next-year returns at the same direction and in high relevance. HonHai, the best performing alternative for 2003, has the highest PP of 42.4 and it gains

34.9% the following year. ACER, TSMC, and UMC are forecasted as the 2nd, 3rd, and 4th performing alternatives and respectively have PP s of 12, 16.7, and 8.9; their returns for 2003 are respectively 82.6%, 61%, and 43.5%, all higher than HonHai. It consists with the suggested strategies that the best performing alternative does not guarantee the highest return if its current market price is too much higher than its forecast performance.

6. Concluding Remarks

The proposed FMCDM can comprehend evaluators' fuzzy assessments with various rating attitudes and the trade-off among various criteria to ensure convincing decision making. Since the membership functions of the evaluations are clearly developed, the decision makers can easily rank alternatives and more confidently make their decisions than doing so by the approximating method. However, the best solution can be guaranteed only under the circumstance of precise forecast information. The results may vary by picking different criteria, setting different linguistic terms or fuzzy numbers, or applying different ranking methods.

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