An Application of Kohonen's SOM for the management of benchmarking policies

Raquel Florez-Lopez

University of Leon (Spain) – Department of Economics and Business Admin. Email: dderfl@unileon.es

Abstract

Efficiency is one of the most strategic information for managers. The DEA model provides scores about firms' efficiency, but it does not obtain an overall map about each unit's position, in order to identify competitive clusters and to improve the design of complete benchmarking policies. This lack difficulties the interpretation of DEA results, together to its real applications for the administration of firms. To overcome this problem, this paper proposes to combine both DEA and SOM models to reach a graphical representation of the situation of units in terms of competitive strategies. This hybrid proposal is evaluated through a well-known public dataset about insurance agencies.

Keywords: Benchmarking, Self Organising Map, DEA, efficiency.

1. Efficiency, benchmarking and DEA model

In current competitive environments, the evaluation of efficiency is a topic of increasing interest for firms' survival and profitability.

Data Envelopment Analysis (DEA) is one of the most useful methods for the evaluation of the relative efficiency of 'decision making units' (DMU´s). Besides, in the last years DEA has been extensively applied in performance evaluation and benchmarking analysis of universities, financial firms, or schools, among others [13], [17], [20].

Considering a set of homogeneous decision making units, characterized by multiple inputs and factors, the efficiency score for the *i-th* individual is:

$$Efficiency_{DMU\_i} = \left(\frac{weighted\_sum\_of\_outputs}{weighted\_sum\_of\_inputs} \right)_{DMU\_i}$$

According this score, the DEA model classifies each DMU as 'efficient' or 'inefficient', through its comparison to all possible lineal combinations of the rest of DMUs in the sample. The group of efficient DMUs forms the *efficient frontier*, such that the inefficiency of each unit is measured through its distance to the frontier.

For each inefficient DMU, DEA identifies a set of efficient individuals that can be used as benchmarking references; also, a measure is provided about the proportional input reduction (input orientation) or output increment (output orientation) that the unit needs to become efficient.

To obtain previous scores, different approaches can be followed. [4] proposed the following model (CCR), solved through a dual problem, which assumes constant returns to scale (CRS):

Let each DMU i in the set on n DMUs be characterized by input-output data collected in the row vector (X_i, Y_i). Let (X,Y) denote the matrix of input-output data for all DMUs in the sample. The relative efficiency score of a test DMU i is obtained by solving the following model:

'Input orientation'	'Output orientation'
$\text{Min } \theta \quad \text{s.t.:}$	$\text{Max } \varphi \quad \text{s.t.:}$
$\lambda Y - s^{+} = y_0$	$\lambda Y - s^{+} = \varphi\, y_0$
$\lambda X + s^{-} = \theta X_0$	$\lambda X + s^{-} = X_0$
$\lambda, s^{+}, s^{-} \geq 0;$	$\lambda, s^{+}, s^{-} \geq 0;$
$\theta \in (0,1]$	$\varphi \in [1,\infty)$

where X is the inputs' matrix; Y is the outputs' matrix; θ is a scalar; λ is the weights' vector.

In the 'input orientation', θ provides the index of efficiency of each DMU, which must be interpreted as the maximum level we could decrease inputs without changes in the mix (in the 'output orientation', φ represents the maximum growth in outputs).

Later on, [3] suggested an extension of the model (BCC) toward variable returns to scale (VRS), which considers that diverse circumstances (imperfect competition, access restriction to finance resources, etc.) can cause that units don't operate in the efficient scale; thus, they modified the lineal program to introduce a restriction of convexity, such that:

$$e'\lambda = 1$$

The BCC-efficiency scores are also called 'pure technical efficiency scores', since they eliminate the 'scale component' of the efficiency measure.

Considering both proposals, [7] define the 'scale efficiency' as the ratio of the 'overall technical efficiency' score (measured by the CCR model) and the 'pure technical efficiency' score (measured by the BCC model). Finally, the comparison of both BCC-scores and NIRS-scores (DEA analysis through BCC, with the restriction of *non-growing returns to scale*) allows to discover the presence of increasing returns to scale (IRS) or decreasing ones (DRS):

If $VRS_{technical\text{-}efficiency} = NIRS_{technical\ efficiency}$ then DRS.

If $VRS_{technical\text{-}efficiency} \neq NIRS_{technical\text{-}efficiency}$ then IRS.

Additionally, [2] developed a 'superefficiency' index, which provides an additional ranking for efficient units; to get it, each efficient DMU is compared to a lineal combination of the rest of efficient individuals in the sample. If that DMU is able to increase its inputs (respectively, decrease its outputs) staying efficient, it would obtain an efficiency score higher than "1".

As conclusion, DEA scores are a useful tool that provides benchmarking references for inefficient units, which could orientate future improvement strategies.

Nevertheless, one of the main limitations of DEA is the lack of a graphical representation of each unit position related to the group, if multiple inputs and outputs are presented, which could be very interesting for the detection of competitive clusters[1]; additionally, for each *ith* inefficient unit, DEA provides a list of efficient DMUs of reference, but the combination of strategies from these benchmarks usually approaches the *ith* unit to other firms not been initially identified but which are its final competitive references. A graphical representation of firms' position could inform about these movements but DEA is only able to represent the *efficient frontier* in presence of one input and two outputs (or one output and two inputs, it depends on the model orientation), see Figure 1.

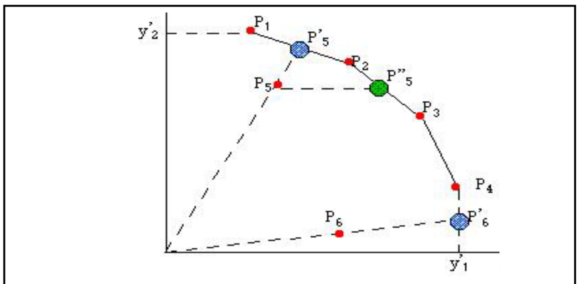

Figure 1. Efficient frontier representation in DEA

To overcome this problem, we propose to combine DEA results and self-organised neural nets. Thus way, the Kohonen's Self Organising Map (SOM) provides a topographical representation of a group of individuals [14], [15], allowing to detect clusters among them, which eases the analysis of DEA results.

[1] [9] propose two-dimensional graphs for the representation of scores, but no distances among units are obtained. Additionally, [16] considers a gradient approach for a single unit.

2. The Self Organising Map for the efficiency representation

Professor Teuvo Kohonen proposes the Self-Organising Map (SOM), a *competitive, unsupervised and self-organizing* neural net, based on the establishment of competitions and mutual links between neighbouring cells (neurons).

The SOM net is organised in two different layers (Figure 2), and performs two different stages:

1.- *Learning or Training Stage:* In this phase each output cell is trained to identify some typical patterns, such that neighbour cells represent near individuals. To get it, each time a unit $x(t)$ is presented to the input layer, the net obtains a "winner output cell" c (the nearest output neuron to the pattern); c modifies its typical parameters m (weights) together to other neighbouring neurones:

$$m_{ijk}(t+1) = m_{ijk}(t) + \alpha(t)\cdot h(|i-c|,t)\cdot(x_k(t) - m_{ijk}(t)) \quad \text{if } i \in N_c(t)$$
$$m_{ijk}(t+1) = m_{ijk}(t) \quad \text{if } i \notin N_c(t)$$

where k is an input variable, i and j are cells, $h(|i\text{-}c|,t)$ is a function related to the 'neighbourhood zone' or $N_c(t)$ and $\alpha(t)$ is a parameter known as "learning rate", such that $0<\alpha<1$.

2.- *Operation or Working Stage:* In this phase each cells acts as an specific feature detector. Each time a new individual is presented to the net, the model assigns it to an only-one cell, obtained through the calculus of the distance between the input data and the weight vector m for each output neurone; the neurone with minimum distance gets the new unit [14], [15].

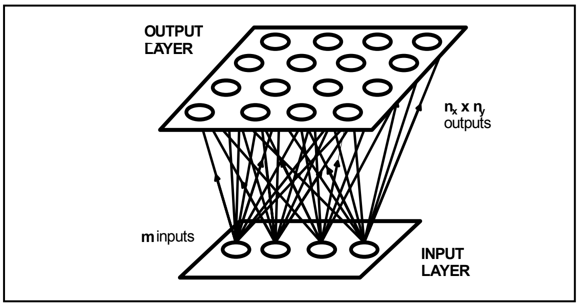

Figure 2: The SOM Model

In this paper, SOM is proposed for the topographical representation of both efficient and inefficient DMUs. To get it, some analogies must be done:

- DMUs act as $x(t)$ units in the SOM map.
- To measure efficiency, all combinations of outputs and inputs must be considered.
- Each SOM's input variable will be obtained as the quotient between one output and one input; both previously normalised to reduce bias.

Once the SOM is developed, the relative weights for outputs cell will be analysed to obtain a final score for each DMU, which will be compared to DEA results. Finally, we will study the benchmarking strategies proposed by DEA, and their consequences on inefficient DMUs positions inside the map.

3. Empirical Application

To test the SOM's performance for the representation of benchmarking strategies, a well-known dataset[2] has been evaluated [21]; it considers the efficiency of 63 agencies of an insurance company relative to the launch of a new contract (Table 1).

Inputs	Outputs
No. of clients 'type A'	No. new contracts
No. of clients 'type B'	Sum of new premiums (DM)
No. of clients 'type C'	
Potential new premiums (DM)	

Table 1. Inputs and outputs of insurance agencies

Inputs refer to several types of clients, related to the current insurance coverage; besides, the potential new premiums have been included, which depends on the clients' current coverage too. Outputs consider the aim of the insurance agencies: to sell as many contracts as possible, and to get premiums as high as possible[3].

According [21], a DEA model has been performed, using the BCC alternative (variable returns to scale), joined to the input orientation and the 'superefficiency' factor (Table 2).

	(*Super*)efficient	**Inefficient**
No. of DMUs	11	52
Min DEA score	1.029	0.292
Max DEA score	∞	0.976
Mean DEA score	-	0.651
Std. deviation	-	0.209

Table 2. DEA Results

Even if this information is valuable, due to both efficient and inefficient units are identified, next to benchmarking references, nevertheless the insurance company is not able to solve some problems, such as:

- *How many different efficient policies could be identified from these agencies?*
- *Which agencies are developing a similar competitive strategy (input-output mix)?*
- *How managers should be moved from efficient to inefficient agencies to improve the firm's efficiency?*

To solve these and other similar questions, it is necessary to get information about relative distances and neighborhoods between DMUs. We propose to obtain a 2-dimensional Kohonen's map which represents the main relationships between agencies, considering the presence of 63 individuals, defined by four inputs and two outputs.

Nevertheless, the efficiency of each agency is not directly measured from these gross numbers, but from the relationships among outputs and inputs. Thus way, if the map tries to represent the relative firms' efficiency, original data may be transformed in 8 different quotients (normalised) [23]:

V1= O1/I1; V2= O1/I2; V3=O1/I3; V4=O1/I4;
V5=O2/I1; V6=O2/I2; V7= O2/I3; V8= O2/I4;

Each variable shows the capacity of each input for obtaining each particular output; the greater the ratio, the higher the firm's efficiency.

About the map's shape, a 8x12 lattice has been selected, using Sammon's Mapping (Sammon, 1969). A decay learning rate has been used, from α=0.9 (early training) to α=0.1 (last training). The 'neighborhood radio' has been adjusted too, from 13.04 (initial maximum Euclidean distance) to 1 (actualisation of the winner cell and its four nearest neighbors) and a two-stages learning was applied (rough and fine learning).

Figure 3 shows the map obtained for the 63 agencies (in bold, red and bigger, efficient units). Below, there is a contour graph about each output cell's average score, obtained from its 8 weights.

The analysis of previous map allows to observe that efficient firms obtain a higher average-weight (0.47) that inefficient ones (0.23). Besides, efficient units are mainly located at the borders of the map, and inefficient ones are nearer to the centre, not far from the usual image of the efficient frontier (Figure 1).

Also, it is possible to observe that efficient firms have different strategies between them; for example, n.6 is very far from entity n. 47, and the inputs and outputs mix shows the reason: n. 6 employs minimum inputs, and its outputs are quite limited too, such that only input 1 and output 2 have modest values. Nevertheless, agency n.47 employs many resources and gets high outputs too (it is focused on input 2 and output 1). Even if DEA identifies both units as efficient, the common sense says that n.6 is getting a worse performance than n.47; the SOM model allows to identify this situation, if weights are compared.

[2] Free disposable at http://www.wiso.uni-dortmund.de/lsfg/or/scheel/data/scheel1.txt
[3] A discussion on the effect of the number of variables in DEA can be consulted in [11], [13].

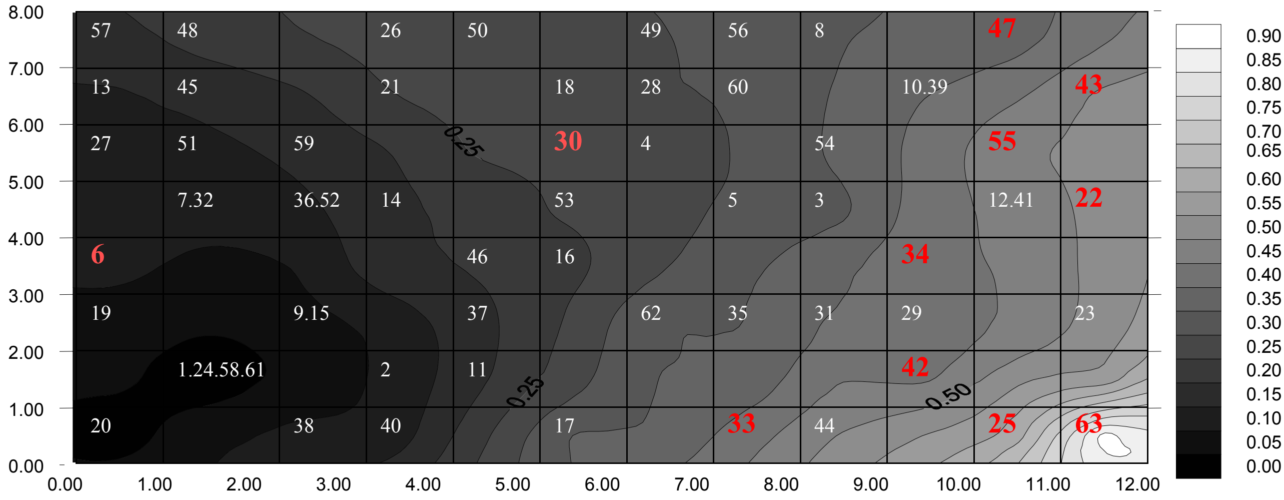

Figure 3. SOM for efficient and inefficient DMUs

Additionally, SOM identifies n.63 as the most efficient unit, and the analysis of inputs and outputs confirms it: from minimum inputs (especially resources 1 and 3), it reaches some of the highest levels of outputs, that is to say, it has developed the best competitive strategy of all agencies; nevertheless, DEA could not provide this sort of information.

Other interesting efficient agencies are n. 43, 47 and 55, which are nearly located in the map. DEA simply identifies them as efficient units, but the neural net is able to verify that their competitive positions are quite similar (focused on inputs 1, 2 and 4, and on output 2).

Finally, to check the relative effects of the benchmarking proposals on distances among agencies, we have developed another additional map, using the data proposed for the benchmark coefficients, without slacks. These new individuals have been positioned on the original map, to discover the projected movements of firms' positions (Figure 4).

This new map shows that benchmarking proposals really improve the inefficient agencies' scores, as DEA affirms (new value: 0.46), but additionally the SOM informs about the real units of reference after movements, and the novel competitive strategies.

For example, previous information is relevant for the staff management: manager from agency n.47 could supervise future decisions of agencies n.56 and n.49; manager from agency n.33 could direct future competitive policies for n.40 and n.11, and so on.

4. Conclusions

From the analysis of previous results, we can obtain three main conclusions:

- The DEA's benchmarking proposals allow to guide the internal management of entities, identifying their strong and weak points.

- Nevertheless, the presence of multiple inputs and outputs avoids obtaining a direct graphical representation of DEA scores, which reduces a lot the interpretation of results and the efficient execution of improvement actions.

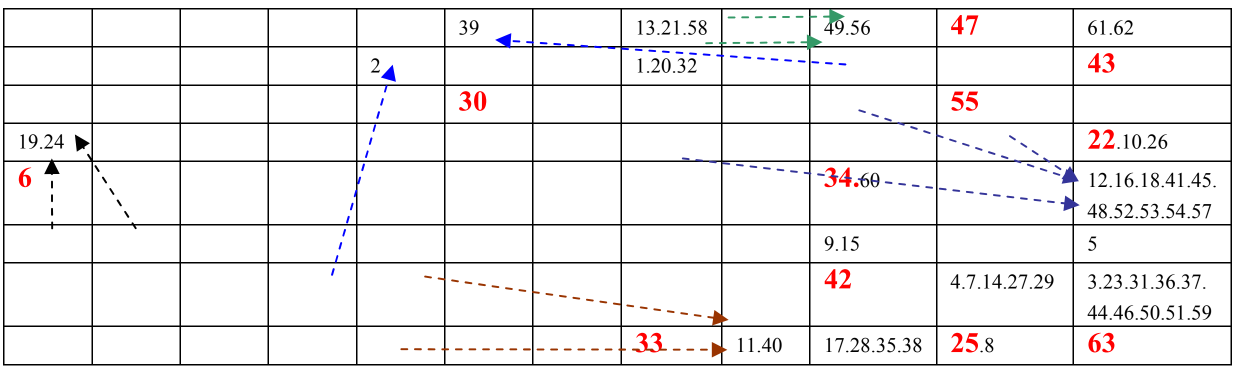

Figure 4. SOM after benchmarking proposals.

- To overcome this problem, the application of a SOM model, using quotients between outputs and inputs as explanative attributes, complete DEA results and allows to identify similarities among efficient and inefficient units, and additionally recognizes individuals that, being characterized as efficient by the DEA model, have very particular behaviours, getting a 'theoretical' but not practical efficiency.

As future developments, the effect of weights restrictions on inputs and outputs will be included in the SOM model [1], [11], [17], [22]; moreover, the influence of both input and output orientations will be analysed, together to other DEA extensions as 'window analysis', additive models or fuzzy measures of efficiency [5], [7], [8]. Finally, a sensitivity analysis should be performed [10].

5. References

[1] R. Allen, A. Athanassopoulos, R.G. Dyson, E. Thanassoulis, "Weights restrictions and value judgments in DEA: evolution, development and future directions", *Annals of Operations Research,* 73, pp. 13-34, 1997.

[2] P. Andersen and N.C. Petersen, "A procedure for ranking efficient units in data envelopment analysis", *Management Science,* 39(10), pp. 1261-1264, 1993.

[3] R.D. Banker, A. Charnes and W.W. Cooper, "Some models for estimating technical and scale inefficiencies in data envelopment analysis", *Management Science,* 30(9), pp.1079-1092, 1984.

[4] A. Charnes, W.W. Cooper and E. Rhodes, "Measuring the efficiency of decision making units", *European Journal of Operational Research,* 2, pp. 429-444, 1978.

[5] A. Charnes, W.W. Cooper, B. Golany, L.M. Seiford, and J. Stutz, "Foundations of Data Envelopment Analysis for Pareto-Koopmans efficient empirical production function", *Journal of Econometrics,* 30, pp. 91-107, 1985.

[6] A. Charnes, W.W. Cooper, A.Y. Lewin and L.M. Seiford (eds), *"Data envelopment analysis: theory, methodology, and applications".* Kluwer, Boston, 1994.

[7] W.W. Cooper, L.M. Seiford, M. Lawrence, and K. Tone, *"Data Envelopment Analysis: A Comprehensive Text with Models, Applications, References and DEA-Solver Software",* Kluwer Academic Publishers, Norwell, 2000.

[8] M. Dia, "A model of fuzzy Data Envelopment Analysis", *INFOR,* 42(4), pp. 267-279, 2004.

[9] S. El-Mahgary and R. Lahdelma, "Data Envelopment Analysis: Visualizing the results", *European Journal of Operational Research,* 85, pp. 700-710, 1995.

[10] A. Hughes and S. Yaisawarng, "Sensitivity and dimensionality tests of DEA efficiency scores", *European Journal of Operational Research,* 154, pp. 410-422, 2004.

[11] G.R. Jahanshahloo and M. Soleimani-Damaneh, "A note on simulating weights restrictions in DEA: an improvement of Thanassoulis and Allen's method", *Computers & Operations Research,* 32, pp. 1037-1044, 2005.

[12] L. Jenkins and M. Anderson, "A multivariate statistical approach to reducing the number of variables in data envelopment analysis", *European Journal of Operational Research,* 147, pp. 51-61, 2003.

[13] C. Kao and S.-H. Liu, "Predicting bank performance with financial forecasts: A case of Taiwan commercial banks", *Journal of Banking and Finance,* 28, pp. 2353-2368, 2004.

[14] T. Kohonen, "The Self-Organising Map", *Proc. IEEE,* 78, pp. 1464-1480, 1990.

[15] T. Kohohen, *"Self-Organising Maps",* Springer-Verlag, Heidelberg, 1997.

[16] S. Maital and A. Vaninsky, "Data envelopment analysis with a single DMU: A graphic projected-gradient approach", *European Journal of Operational Research,* 115, pp. 518-528, 1999.

[17] J. Paradi and C. Schaffnit, "Commercial branch performance evaluation and results communication in a Canadian bank – a DEA application", *European Journal of Operational Research,* 156, pp. 719-735, 2004.

[18] J.W.Jr. Sammon, "A Nonlinear Mapping for Data Structure Analysis", *IEEE Transactions on Computers,* C-18, pp. 401-409, 1969.

[19] C.S. Sarrico and R.G. Dyson, "Restricting virtual weights in DEA", *European Journal of Operational Research,* 159, pp. 17-34, 2004.

[20] L.M. Seiford, "Data Envelopment Analysis: The evolution of the state of the art (1978-1995)", *J. of Productivity Analysis,* 7, pp. 99-137, 1996.

[21] H. Scheel, On Continuity of the BCC efficiency measure, Working Paper, University of Dortmund, 1998.

[22] R.G. Thompson, L.N. Langemeier, C.-T. Lee, E. Lee and R.M. Thrall, "The role of multiplier bounds in efficiency analysis with application to Kansas farming". *Journal of Econometrics,* 46, pp. 96-108, 1990.

[23] J. Zhu, "DEA vs. principal component analysis: An illustrative study of economic performance of Chinese cities", *European Journal of Operational Research,* 111, pp. 50-61, 1998.