A short overview on DSmT for Information Fusion

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Abstract

This short paper introduces the recent theory of plausible and paradoxical reasoning, known as DSmT (Dezert-Smarandache Theory) in literature, which deals with imprecise, uncertain and potentially highly conflicting sources of information. Recent publications have shown the interest and the potential ability of DSmT to solve fusion problems where the Dempster-Shafer Theory (DST) provides counter-intuitive results, especially when conflict between sources becomes high. This presentation is focused on the foundations of DSmT, and its main rules of combination. The Shafer's model on which is based DST is only a specific DSm hybrid model and can be easily handled by DSmT.

Keywords: Information Fusion, Plausible Reasoning, DSmT, Dezert-Smarandache Theory.

1 Introduction

The development of the DSmT [6] arises from the necessity to overcome the inherent limitations of the DST [5] which are closely related with the acceptance of Shafer's model (i.e. working with an *homogeneous*[1] frame of discernment Θ defined as a finite set of *exhaustive* and *exclusive* hypotheses θ_i, $i = 1, \ldots, n$), the third middle excluded principle, and the Dempster's rule for the combination of independent sources of evidence. Limitations of DST are well reported in literature [10, 8] and several alternative rules to the Dempster's rule of combination can be found in [1, 9, 2, 3, 4, 6]. DSmT provides a new mathematical framework for information fusion which appears less restrictive and more general than the basis and constraints of DST. The basis of DSmT is the refutation of the principle of the third excluded middle and Shafer's model in general, since for a wide class of fusion problems the hypotheses one has to deal with, can have different intrinsic nature[2] and also appear only vague and imprecise in such a way that precise refinement is just impossible to obtain in reality so that the exclusive elements θ_i cannot be properly identified and defined. Many problems involving fuzzy/vague continuous and relative[3] concepts described in natural language with different semantic contents and having no absolute interpretation enter in this category. We claim that in general, the negation/complement is not accessible, but DSmT offers the possibility to deal with negation and Shafer's model as well. When the model of the problem fits with these constraints (negation follows from exclusivity constraints), we include them in the frame and then one forms the hyper-power set in the normal way. Thus DSmT deals naturally with negations/complements when necessary. DSmT starts with the notion of *free DSm model* and considers Θ only as a frame of exhaustive elements which can potentially overlap and have different intrinsic semantic natures and which also can change with time with new information and evidences received on the model itself. DSmT offers a flexibility on the structure of the model one has to deal with. When the free DSm model holds, the conjunctive consensus is performed. If the free model does not fit the reality because it is known that some subsets of Θ contain elements truly exclusive but also possibly truly non

[1]Although the homogeneity of Θ is not explicitly mentioned in the DST, it is a strong implicit assumption inherent to the Shafer's model. When working with DST, one implicitly assumes that all finite and exclusive elements of Θ have somehow the same semantic nature, otherwise the complement defined over the power-set becomes just a non-sense. The Shafer's model can't deal directly with non-homogeneous elements (carrying different semantics) of Θ. This property however is necessary in many applications where the information given by the sources can't be expressed with same semantic due to the potentially different intrinsic nature of information carried by the sources/experts/sensors.

[2]By example, in some target tracking and classification applications, one has to deal both with imprecise and uncertain information like radar-cross section, as well as Doppler/velocity measurements

[3]The notion of relativity comes from the own interpretation of the elements of the frame Θ by each sources of evidences involved in the fusion process.

existing at all at a given time (in dynamic[4] fusion), new fusion rules must be performed to take into account these integrity constraints. The constraints can be explicitly introduced into the free DSm model to fit it adequately with our current knowledge of the reality; we actually construct a *hybrid DSm model* on which the combination will be efficiently performed. Shafer's model, which is the basis of DST, corresponds to a very specific hybrid DSm (and homogeneous) model including all possible exclusivity constraints. DSmT has been developed to work with any kind of model, to combine imprecise, uncertain and potentially high conflicting sources for static and dynamic information fusion. DSmT refutes the idea that sources provide their beliefs with the same absolute interpretation of elements of Θ; what is considered as good for somebody can be considered as bad for somebody else. Advances and first applications of DSmT are detailed in [6].

2 Notion of hyper-power set

Let $\Theta = \{\theta_1, \dots, \theta_n\}$ be a finite set (called frame) of n exhaustive elements[5]. The free Dedekind's lattice denoted *hyper-power set* D^Θ [6] is defined as

1. $\emptyset, \theta_1, \dots, \theta_n \in D^\Theta$.
2. If $A, B \in D^\Theta$, then $A \cap B$ and $A \cup B$ belong to D^Θ.
3. No other elements belong to D^Θ, except those obtained by using rules 1 or 2.

If $|\Theta| = n$, then $|D^\Theta| \leq 2^{2^n}$. The generation of D^Θ is presented in [6]. Since for any given finite set Θ, $|D^\Theta| \geq |2^\Theta|$, we call D^Θ the *hyper-power set* of Θ. $|D^\Theta|$ for $n \geq 1$ follows the sequence of Dedekind's numbers:1,2,5,19,167,7580,... An analytical expression of Dedekind's numbers obtained by Tombak and al. can be found in [6].

3 Free and hybrid DSm models

$\Theta = \{\theta_1, \dots, \theta_n\}$ denotes the finite set of hypotheses/concepts characterizing the fusion problem. D^Θ constitutes the *free DSm model* $\mathcal{M}^f(\Theta)$ and allows to work with fuzzy concepts which depict a continuous and relative intrinsic nature. Such kinds of concepts cannot be precisely refined in an absolute interpretation because of the unapproachable universal truth. When all θ_i are truly exclusive discrete elements, D^Θ reduces naturally to the classical power set 2^Θ. This is what we call the Shafer's model. We deonote it $\mathcal{M}^0(\Theta)$. Between the free DSm model and the Shafer's model, there exists a wide class of fusion problems represented in term of DSm hybrid models where Θ involves both fuzzy continuous concepts and discrete hypotheses. In such class, some exclusivity constraints and possibly some non-existential constraints (especially when working on dynamic fusion) have to be taken into account. Each hybrid fusion problem is then characterized by a proper hybrid DSm model $\mathcal{M}(\Theta)$ with $\mathcal{M}(\Theta) \neq \mathcal{M}^f(\Theta)$ and $\mathcal{M}(\Theta) \neq \mathcal{M}^0(\Theta)$. From a general frame Θ, we define a map $m(.) : D^\Theta \to [0, 1]$ associated to a given body of evidence $\mathcal{B}$ as

$$m(\emptyset) = 0 \qquad \text{and} \qquad \sum_{A \in D^\Theta} m(A) = 1 \tag{1}$$

$m(A)$ is the *generalized basic belief assignment/mass* (gbba) of A. The *generalized belief and plausibility functions* are defined as:

$$\mathrm{Bel}(A) \triangleq \sum_{\substack{B \subseteq A \\ B \in D^\Theta}} m(B) \quad \mathrm{Pl}(A) \triangleq \sum_{\substack{B \cap A \neq \emptyset \\ B \in D^\Theta}} m(B) \tag{2}$$

These definitions are compatible with the Bel and Pl definitions given in DST when $\mathcal{M}^0(\Theta)$ holds.

4 Classic DSm fusion rule

When the free DSm model holds, the conjunctive consensus, called DSm classic rule (DSmC), is performed on D^Θ. DSmC of two independent[6] sources associated with gbba $m_1(.)$ and $m_2(.)$ is thus given $\forall C \in D^\Theta$ by [6]:

$$m_{\mathcal{M}^f(\Theta)}(C) \equiv m(C) = \sum_{\substack{A, B \in D^\Theta \\ A \cap B = C}} m_1(A) m_2(B) \tag{3}$$

Since D^Θ is closed under $\cup$ and $\cap$ set operators, DSmC guarantees that $m(.)$ is a proper generalized belief assignment, i.e. $m(.) : D^\Theta \to [0, 1]$. DSmC is commutative and associative and can always be used for the fusion of sources involving fuzzy concepts whenever the free DSm model $\mathcal{M}^f(\Theta)$ holds. This rule can be directly and easily extended for the combination of $k > 2$ independent sources [6].

[4]i.e. when the frame Θ and/or the model $\mathcal{M}$ is changing with time.

[5]We do not assume here that elements θ_i have the same intrinsic nature and are necessary exclusive. There is no restriction on θ_i but the exhaustivity.

[6]While independence is a difficult concept to define in all theories managing epistemic uncertainty, we consider that two sources of evidence are independent (i.e. distinct and noninteracting) if each leaves one totally ignorant about the particular value the other will take.

5 Hybrid DSm fusion rule

When $\mathcal{M}^f(\Theta)$ does not hold (some integrity constraints exist), one deals with a proper DSm hybrid model $\mathcal{M}(\Theta) \neq \mathcal{M}^f(\Theta)$. The DSm hybrid rule (DSmH) for $k \geq 2$ independent sources of information is thus defined for all $A \in D^\Theta$ as [6]:

$$m_{\mathcal{M}(\Theta)}(A) \triangleq \phi(A) \cdot \Big[S_1(A) + S_2(A) + S_3(A)\Big] \quad (4)$$

where $\phi(A)$ is the *characteristic non-emptiness function* of a set A, i.e. $\phi(A) = 1$ if $A \notin \boldsymbol{\emptyset}$ and $\phi(A) = 0$ otherwise, where $\boldsymbol{\emptyset} \triangleq \{\boldsymbol{\emptyset}_{\mathcal{M}}, \emptyset\}$. $\boldsymbol{\emptyset}_{\mathcal{M}}$ is the set of all elements of D^Θ which have been forced to be empty through the constraints of the model $\mathcal{M}$ and $\emptyset$ is the classical/universal empty set. $S_1(A) \equiv m_{\mathcal{M}^f(\theta)}(A)$, $S_2(A)$, $S_3(A)$ are defined by

$$S_1(A) \triangleq \sum_{\substack{X_1,X_2,\ldots,X_k \in D^\Theta \\ (X_1 \cap X_2 \cap \ldots \cap X_k) = A}} \prod_{i=1}^{k} m_i(X_i) \quad (5)$$

$$S_2(A) \triangleq \sum_{\substack{X_1,X_2,\ldots,X_k \in \boldsymbol{\emptyset} \\ [\mathcal{U}=A] \vee [(\mathcal{U} \in \boldsymbol{\emptyset}) \wedge (A = I_t)]}} \prod_{i=1}^{k} m_i(X_i) \quad (6)$$

$$S_3(A) \triangleq \sum_{\substack{X_1,X_2,\ldots,X_k \in D^\Theta \\ u(c(X_1 \cap X_2 \cap \ldots \cap X_k)) = A \\ (X_1 \cap X_2 \cap \ldots \cap X_k) \in \boldsymbol{\emptyset}}} \prod_{i=1}^{k} m_i(X_i) \quad (7)$$

with $\mathcal{U} \triangleq u(X_1) \cup \ldots \cup u(X_k)$ where $u(X)$ is the union of all θ_i that compose X, $I_t \triangleq \theta_1 \cup \ldots \cup \theta_n$ is the total ignorance, and $c(X)$ is the canonical form[7] of X, i.e. its simplest form (for example if $X = (A \cap B) \cap (A \cup B \cup C)$, $c(X) = A \cap B$). $S_1(A)$ corresponds to DSmC rule for k independent sources based on $\mathcal{M}^f(\Theta)$; $S_2(A)$ represents the mass of all relatively and absolutely empty sets which is transferred to the total or relative ignorances associated with non existential constraints (if any, like in some dynamic problems); $S_3(A)$ transfers the sum of relatively empty sets directly onto the canonical disjunctive form of non-empty sets. DSmH generalizes DSmC and is not equivalent to Dempster's rule. It works for any models (the free DSm model, Shafer's model or any other hybrid models) when manipulating *precise* generalized (or eventually classical) basic belief functions.

[7] The canonical form is introduced here in order to improve the original formula given in [6] for preserving the neutral impact of the vacuous belief mass $m(\Theta) = 1$ within complex hybrid models. The canonical form is the conjunctive normal form, also known as conjunction of disjunctions in Boolean algebra, which is unique.

6 Fusion of imprecise beliefs

In general, it is very difficult to have sources providing precise basic belief assignments (especially when information is provided by human experts), and a more flexible plausible and paradoxical theory dealing with imprecise information becomes necessary. We extended the DSmT and its fusion rules for dealing with *admissible imprecise generalized basic belief assignments* $m^I(.)$ defined as real subunitary intervals of $[0, 1]$, or even more general as real subunitary sets (i.e. sets, not necessarily intervals). In our general case, these sets can be unions of (closed, open, or half-open/half-closed) intervals and/or scalars all in $[0, 1]$. An imprecise belief assignment $m^I(.)$ over D^Θ is said *admissible* if and only if there exists for every $X \in D^\Theta$ at least one real number $m(X) \in m^I(X)$ such that $\sum_{X \in D^\Theta} m(X) = 1$. The following simple commutative operators on sets (addition $\boxplus$ and multiplication $\boxdot$) are required [6] for fusion of imprecise beliefs:

$$\mathcal{X}_1 \boxplus \mathcal{X}_2 \triangleq \{x \mid x = x_1 + x_2, x_1 \in \mathcal{X}_1, x_2 \in \mathcal{X}_2\}$$

$$\mathcal{X}_1 \boxdot \mathcal{X}_2 \triangleq \{x \mid x = x_1 \cdot x_2, x_1 \in \mathcal{X}_1, x_2 \in \mathcal{X}_2\}$$

From these operators, one can generalize DSmC from scalars to sets as follows (see [6] chap. 6): $\forall A \neq \emptyset \in D^\Theta$,

$$m^I_{\mathcal{M}^f(\Theta)}(A) = \boxed{\sum_{\substack{X_1,X_2,\ldots,X_k \in D^\Theta \\ (X_1 \cap X_2 \cap \ldots \cap X_k) = A}}} \boxed{\prod_{i=1,\ldots,k}} m^I_i(X_i) \quad (8)$$

where $\boxed{\sum}$ and $\boxed{\prod}$ represent the summation, and respectively product, of sets. Similarly, one can generalize DSmH from scalars to sets by:

$$m^I_{\mathcal{M}(\Theta)}(A) \triangleq \phi(A) \boxdot \Big[S^I_1(A) \boxplus S^I_2(A) \boxplus S^I_3(A)\Big] \quad (9)$$

with

$$S^I_1(A) \triangleq \boxed{\sum_{\substack{X_1,X_2,\ldots,X_k \in D^\Theta \\ (X_1 \cap X_2 \cap \ldots \cap X_k) = A}}} \boxed{\prod_{i=1,\ldots,k}} m^I_i(X_i) \quad (10)$$

$$S^I_2(A) \triangleq \boxed{\sum_{\substack{X_1,X_2,\ldots,X_k \in \boldsymbol{\emptyset} \\ [\mathcal{U}=A] \vee [(\mathcal{U} \in \boldsymbol{\emptyset}) \wedge (A = I_t)]}}} \boxed{\prod_{i=1,\ldots,k}} m^I_i(X_i) \quad (11)$$

$$S^I_3(A) \triangleq \boxed{\sum_{\substack{X_1,X_2,\ldots,X_k \in D^\Theta \\ u(c(X_1 \cap X_2 \cap \ldots \cap X_k)) = A \\ (X_1 \cap X_2 \cap \ldots \cap X_k) \in \boldsymbol{\emptyset}}}} \boxed{\prod_{i=1,\ldots,k}} m^I_i(X_i) \quad (12)$$

7 Zadeh's example

We compare here the solutions for well-known Zadeh's example [10] provided by several fusion rules. A detailed presentation with more comparisons can be found in [6, 7]. Let's consider the frame $\Theta = \{A, B, C\}$, the Shafer's model and the two following belief assignments

$$m_1(A) = 0.9 \qquad m_1(B) = 0 \qquad m_1(C) = 0.1$$
$$m_2(A) = 0 \qquad m_2(B) = 0.9 \qquad m_2(C) = 0.1$$

The total conflicting mass is high since it is

$$m_1(A)m_2(B) + m_1(A)m_2(C) + m_2(B)m_1(C) = 0.99$$

- with Dempster's rule and Shafer's model (DS), one gets the counter-intuitive result (see justifications in [10, 1, 9, 8, 6]): $m_{DS}(C) = 1$
- with Yager's rule [9] and Shafer's model: $m_Y(A \cup B \cup C) = 0.99$ and $m_{DS}(C) = 0.01$
- with DSmC and the *free-DSm model*:

  $$m_{DSmC}(A \cap B) = 0.81 \qquad m_{DSmC}(C) = 0.01$$
  $$m_{DSmC}(A \cap C) = m_{DSmC}(B \cap C) = 0.09$$

- with DSmH and Shafer's model:

  $$m_{DSmH}(A \cup B) = 0.81 \qquad m_{DSmH}(C) = 0.01$$
  $$m_{DSmH}(A \cup C) = m_{DSmH}(B \cup C) = 0.09$$

- The Dubois & Prade's rule (DP) [1] based on Shafer's model provides in this Zadeh's example the same result as DSmH, because DP and DSmH coincide in all static fusion problems[8].

One sees that when the total conflict between sources becomes high, the DSmT is able (upon authors opinion) to manage more adequately the combination of information than the Dempster's rule, even when working with Shafer's model - which is only a specific hybrid model. DSmH rule is in agreement with DP rule for the static fusion, but DSmH and DP rules differ in general (for non degenerate cases) for dynamic fusion. Besides this particular example, we showed in [6, 7] that there exist several infinite classes of counter-examples to the Dempster's rule which can be solved by DSmT. More sophisticated, and what we think more precise and efficient DSm fusion rules based on new Proportional Conflict Redistribution rules can be found in [7] but cannot be reported here due to space limitation.

[8]DP rule has been developed for static fusion only while DSmH is more general since it works for any models as well as for static and dynamic fusion.

8 Conclusion

This paper brings a short overview on basis of DSmT which is not yet well known in the fuzzy set community. Only two rules of fusion have been presented here for the combination of precise and imprecise beliefs due to space limitation restrictions. More precise and complex fusion rules based on proportional conflict redistribution have been recently developed by the authors and connections of DSmT with neutro-fuzzy sets and neutrosophic logic are under investigation.

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