The Value of Cooperation Within a Profit-Maximizing Organization

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***Abstract*— This paper proposes a measure to quantify the value of cooperation experienced by a firm. Using a "reverse" merger simulation approach, the percentage of a firm's profits due to cooperation can be precisely determined. This is accomplished by considering the profit maximizing dynamics of firms in the market as defining a value function of a coalition game. A simple example illustrates the ideas.**

I. Introduction

This paper derives a method for quantifying the value of cooperation (VC) and the relative value of cooperation (RVC) experienced by a firm. The idea is to let the profit maximizing dynamics of a given market structure define the value function for a particular coalition game.

With this idea, we may aid anyone who needs to know about a product's place in the product network. For business managers, this means they can know the products their business offers which contribute to a greater whole, as opposed to those product lines which may be sold off with minimal impact. They may also discover which product lines would be most advantageous for their business. Given any set of products inside or outside the business, we may calculate the value of this set (with profit maximization the objective).

Our method is also useful for the antitrust division of the Justice Department. They are interested maximizing total social welfare in a market by protecting market competition. To do this, they attempt to measure the control a particular company has on the market and take appropriate measures. Their preferred measure of the market power of the company is the Herfindahl-Hirshman index (HHI) [7], the sum of the squares of each firm's market share, given by

$$I_{HH} \equiv \sum_{i=1}^{N}(s_i)^2 \tag{1}$$

This measure, however, relies on legal definitions of particular markets and focuses on a computation of market share. Market share, however, has been shown to be a weak indicator of market power [3]. A more direct measure of market power that is insensitive to legal definitions of market boundaries but highly sensitive to the economics of the underlying product network would make a significant impact on antitrust efforts. The value of cooperation a firm is able to realize within a given economic environment is a step in the direction of computing market power directly.

This work draws heavily from the theory of industrial organization and coalition games[7], [1], [5], [6], [4]. The most closely related work to our study is recent work on merger simulations. One paper [2] describes how the impact of a proposed merger can be computed by evaluating the post-merger equilibrium prices. The paper considers common functional forms of demand functions, and indicates how to conduct the merger simulation in each case. The value of cooperation proposed here is found through a kind of "reverse" merger simulation that explores the impact of splitting the firm into its constituent economic units to determine the value it is realizing by unifying the objectives of these basic units.

The next section introduces the dynamic framework motivating the profit gained at equilibrium as a viable value function. A coalition game is then formulated using this value function, and the Value of Cooperation and Relative Value of Cooperation are then introduced as measures on this game. A simple example is then provided to illustrate the ideas, and the conclusion and future work summarizing the work follows.

II. Profit Maximizing Dynamics

Consider a market, $\mathcal{M}$, of N products. Without loss of generality, give these products an arbitrary order and integer label so that $\mathcal{M} = \{1, 2, ..., n\}$. Let $p \in \mathbb{R}^N$ be the vector of (non-negative) prices for these N products, and let $q : \mathbb{R}^N \to \mathbb{R}^N$ be the (non-negative) demand for these products at prices p.

A firm, F is a subset of the N products in the market, $F \in 2^{\mathcal{M}}$. This implies that the firm controls the production and distribution of the products assigned to it. Most importantly for our analysis, since we consider a Bertrand market

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model, this implies that the firm may set the prices of the $n = |F|$ products assigned to it.

We suppose that the products of the market are partitioned between m firms. This implies that no two firms control the same product, $F_i \bigcap F_j = \emptyset \quad \forall i \neq j$, and that the union of all products assigned to the m firms compose the entire market, $\bigcup_{i=1}^{m} F_i = \mathcal{M}$.

Let $c_j(q_j),\ j = 1, ..., N$ be the cost of production of q_j units of product j. The profit of the i^{th} firm then is given by

$$\pi_i = \sum_{j \in F_i} [q_j(p)p_j - c_j(q_j(p))]$$

A profit-maximizing firm under the Bertrand model of market behavior will tend to change its prices to maximize its short-term profit. We model this behavior by assuming that the firm will evolve the prices of its products in the direction of maximally improving its profits. That is, if product j belongs to firm i, then we expect the firm to evolve the price of product j as

$$\frac{dp_j(t)}{dt} = \left.\frac{\partial \pi_i(p)}{\partial p_j}\right|_{p(t)}$$

where $p(t)$ is the pricing vector for the entire market at time t.

Notice that these dynamics suggest that if the partial derivative of profits is negative with respect to the price of product j, that the firm should *decrease* the price of product j. This is in the direction of improving profits. Likewise, if the partial derivative were positive, the firm would *increase* the price of product j to improve profits. When the partial derivative is zero, the motivation is to hold the price at this locally profit-maximizing position.

Reordering the N market products so that each firm's products are grouped together, and letting n_i be the number of products controlled by firm i, we then can partition the pricing vector into components associated with each firm. If every firm in the market is assumed to be profit maximizing, this yields the following market dyanmics:

$$\left[\begin{array}{c} \dot{p}_1(t) \\ \vdots \\ \dot{p}_{n_1}(t) \\ \hline \dot{p}_{n_1+1}(t) \\ \vdots \\ \dot{p}_{n_1+n_2}(t) \\ \hline \vdots \\ \hline \dot{p}_{\sum_{i=1}^{m-1} n_i+1}(t) \\ \vdots \\ \dot{p}_{\sum_N}(t) \end{array}\right] = \left[\begin{array}{c} (\partial\pi_1/\partial p_1)(p(t)) \\ \vdots \\ (\partial\pi_1/\partial p_{n_1})(p(t)) \\ \hline (\partial\pi_2/\partial p_{n_1+1})(p(t)) \\ \vdots \\ (\partial\pi_2/\partial p_{n_1+n_2})(p(t)) \\ \hline \vdots \\ \hline (\partial\pi_m/\partial p_{\sum_{i=1}^{m-1} n_i+1})(p(t)) \\ \vdots \\ (\partial\pi_m/\partial p_N)(p(t)) \end{array}\right] \tag{2}$$

where the dot notation $\dot{p}(t)$ is used to represent $dp(t)/dt$.

Notice that if the market system (2) has an equilibrium, such a pricing vector p_{eq} would represent prices from which no firm can improve its profits by unilaterally changing the prices over which it has control. Under certain technical conditions such an equilibrium can be shown to exist. Moreover, this equilibrium can often be shown to be asymptotically stable, in the sense that any pricing vector $p(0)$ will converge to the equilibrium p_{eq} as $t \to \infty$.

III. The Firm as a Coalition in a Multi-Coalition Environment

Under the assumption that the market dynamics are stabilizing, we expect price perturbations to re-equilibriate. In this context, it is convenient to simplify the problem by only considering the profits of the firms at equilibrium. These profits define a payoff function reminiscent of those used to define coalition games.

Let $v(F_i) = \pi_i|_{p=p_{eq}}$ be the payoff or profit of firm i at the market equilibrium prices p_{eq}. In this way the firm may be thought of as a coalition of n_i players in an N-player cooperative game. Each player is a one-product company that completely manages the production, distribution, and pricing decisions for its product. The firm, then, is a confederacy of these one-product companies that works together to maximize their combined profits or payoffs.

The theory of coalition games studies the behavior of such coalitions once the payoff function is defined for every possible coalition. The idea is that any given coalition F_i yields a well-defined payoff $v(F_i)$, and then a number of questions can be explored regarding how to distribute the payoff among the members of the coalition, etc.

Our situation is different because the payoff to a given firm doesn't just depend on the products it controls, but also on the market structure of the products outside the firm. For example, consider a 10-product market and a three product firm in the market. The payoff to the firm does not just depend on the prices of the three products it controls, but also on the prices of the other seven products. The profit-maximizing equilibrium prices of these other seven products, however, may be set differently depending on whether they belong to a single firm or whether they are controlled by seven different companies. Thus, the payoff to the three-product firm depends on the entire market structure.

Coalition game theory addresses such situations by considering partition systems and restricted games. For our purposes, it is sufficient to partition the N products of $\mathcal{M}$ into m firms and then assume that this structure is fixed outside of the particular firm that we are studying. This enables us to work with a well defined payoff function induced by the profit-maximizing dynamics of firms within the market without eliminating the multiple-coalition (i.e. multiple firm) cases of interest.

IV. Value of Cooperation

To quantify the value of organizing a group of one-product companies into a single firm, we need to compare the profits the firm receives if it sets its prices as if each of its products were independent companies with those it realizes by fully capitalizing on cooperation between the products. More precisely, let p_{eq} be the profit-maximizing equilibrium prices for the given market structure. In contrast, consider the new profit maximizing equilibrium prices achieved without cooperation if F_i were divided into its constituent one-product companies and each independently optimized their prices. Let this second set of equilibrium prices serve as a basis for comparison, or reference, and be denoted p_{ref}. The value of cooperation (VC) of a given firm F_i in market $\mathcal{M}$ with structure $S = F_1, F_2, ..., F_m$ is then given by

$$\mathrm{VC}_{\mathrm{ref}}(F_i, S) = \pi_i|_{p_{\mathrm{eq}}} - \pi_i|_{p_{\mathrm{ref}}}. \tag{3}$$

This measure captures precisely the value realized by the firm due to cooperation within its organization. Nevertheless, the measure carries units of dollars and reflects a kind of absolute dollar-value of cooperation within the firm. Moreover, note that the measure is always non-negative since the cooperating firm can always recover at least the non-cooperating, or reference, profits by simply setting the prices it controls in p_{eq} to those of p_{ref}.

A related measure captures the "relative" value of cooperation (RVC) by normalizing $\mathrm{VC}_{\mathrm{ref}}$ by the equilibrium profits as:

$$\mathrm{RVC}_{\mathrm{ref}}(F_i, S) = \frac{\pi_i|_{p_{\mathrm{eq}}} - \pi_i|_{p_{\mathrm{ref}}}}{\pi_i|_{p_{\mathrm{eq}}}} \tag{4}$$

This measure is interpreted as the percentage of profits due to cooperation within the organization. It is bounded between zero and one, and it facilitates direct comparison between firms of different sizes.

Sometimes we may be interested in measuring the value of cooperation between structures other than the current market structure and the reference structure. This could be the case when considering mergers between firms, or when management is considering selling off a piece of the firm. In such cases it is easy to extend the definitions of VC and RVC by simply replacing the equilibrium and reference prices with the equilibriated profit-maximizing prices of the two market structures being compared.

It is instructive to contrast the VC and RVC with other measures used to characterize cooperative games. Hart and Mas-Colell defined a measure, called the potential, P, that computes the expected normalized worth of the game i.e. the per-capita potential, P/N, equals the average per-capita worth $(1/m)\sum_i(\pi_i)/(|F_i|)$. Given a market structure, this measure characterizes the expected profit of an average-sized firm (where size is measured with respect to the number of products the firm controls) in the market, even if such a firm does not actually exist.

Moreover, the potential has been connected to another measure, called the Shapley value,Φ_j, which yields the marginal contribution of each product in the market. This measure characterizes how the payoff of a coalition should be divided between members of the team. In both cases, the potential and Shapley value do not suggest anything about the intrinsic benefit of forming coalitions in the first place.

The Value of Cooperation, VC, and Relative Value of Cooperation, RVC, on the other hand, capture the natural significance for organizing production into multi-product firms. Nevertheless, these measures do not yield any information about how the profit of a firm should be efficiently invested into each of the firm's constituent production lines. Thus, the measures are inherently different from the potential or shapely value of the coalition game that focus more on the value of a member of a coalition to the group rather than the value of the coalition as a whole.

V. Example

To illustrate the point, consider a two product economy with linear demand given by

$$\begin{bmatrix} q_1(t) \\ q_2(t) \end{bmatrix} = \begin{bmatrix} -3.5 & -1 \\ -3 & -2 \end{bmatrix} \begin{bmatrix} p_1(t) \\ p_2(t) \end{bmatrix} + \begin{bmatrix} 100 \\ 100 \end{bmatrix} \tag{5}$$

Suppose that the unit production cost of each product is $c_1 = 10$, $c_2 = 10$. If we consider a market structure where each product is produced by an independent company, the profit function for each company becomes

$$\begin{aligned} \pi_1(t) &= q_1(t)\,(p_1(t) - c_1) \\ &= -3.5p_1^2 - p_1p_2 + 135p_1 + 10p_2 - 1000 \end{aligned} \tag{6}$$

$$\begin{aligned} \pi_2(t) &= q_2(t)\,(p_2(t) - c_2) \\ &= -2p_2^2 - 3p_1p_2 + 30p_1 + 120p_2 - 1000 \end{aligned} \tag{7}$$

Taking the partial derivatives of each profit function with respect to the appropriate pricing variable, we find the profit-maximizing market dynamics to be:

$$\begin{bmatrix} \frac{dp_1(t)}{dt} \\ \frac{dp_2(t)}{dt} \end{bmatrix} = \begin{bmatrix} -7 & -1 \\ -3 & -4 \end{bmatrix} \begin{bmatrix} p_1(t) \\ p_2(t) \end{bmatrix} + \begin{bmatrix} 135 \\ 120 \end{bmatrix} \tag{8}$$

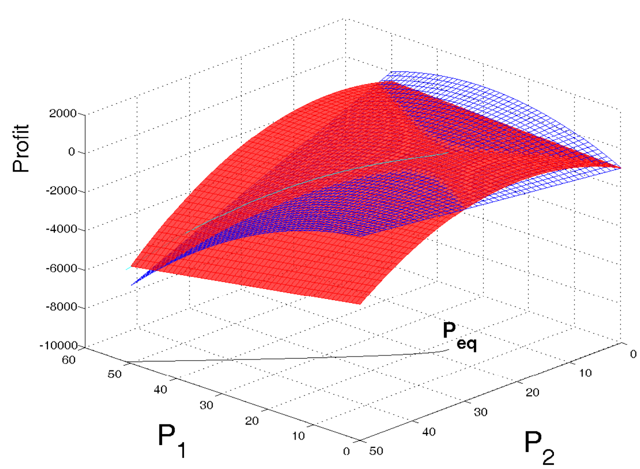

Fig. 1. Two firm price trajectory and profit function

Figure 1 shows how the two-firm dynamics drive an initial pricing vector to a profit-maximizing equilibrium. This equilibrium price is

$$p_{ref} = \begin{bmatrix} 16.8 \\ 17.4 \end{bmatrix}.$$

and the associated equilibriated profits are $\pi_1 = 161.84$, $\pi_2 = 109.52$.

Now, consider a market structure where both products are controlled by the a same firm. In this case, the firm's profit function becomes

$$\begin{aligned} \pi(t) &= q_1(t)\,(p_1(t) - c_1) + q_2(t)\,(p_2(t) - c_2)\,, \\ &= -3.5p_1^2 + 165p_1 - 4p_1p_2 + 130p_2 - 2p_2^2 - 2000. \end{aligned} \tag{9}$$

With this market structure, the firm adjusts the prices of both products to optimize the same objective. These new dynamics become:

$$\begin{bmatrix} \frac{dp_1(t)}{dt} \\ \frac{dp_2(t)}{dt} \end{bmatrix} = \begin{bmatrix} -7 & -4 \\ -4 & -4 \end{bmatrix} \begin{bmatrix} p_1(t) \\ p_2(t) \end{bmatrix} + \begin{bmatrix} 165 \\ 130 \end{bmatrix}. \tag{10}$$

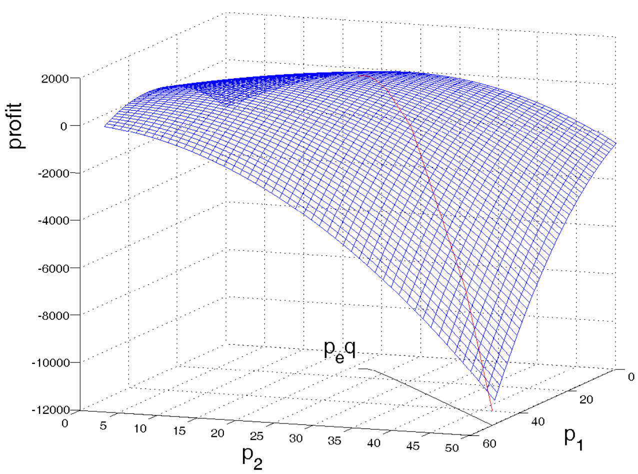

Fig. 2. One firm price trajectory and profit function

Figure 2 shows how the single-firm dynamics drive an initial pricing vector to a profit-maximizing equilibrium. The new equilibrium price is given by

$$p_{eq} = \begin{bmatrix} 11.67 \\ 20.83 \end{bmatrix}.$$

and the associated equilibriated profits are $\pi_{eq} = 316.667$. The value of cooperation in this example thus becomes

$$VC = \pi_{eq} - (\pi_1 + \pi_2) = 45.3067$$

$$RVC = 0.1431$$

This suggests that in this market, just under 15% of the profits of the two-product firm are the direct result of it's interfirm cooperation.

VI. Conclusion

This paper explored quantitative measures to calibrate the value of cooperation within a specific firm in a given market. The idea is to assume profit-maximizing dynamics among the firms within the market and compare equilibrium profits in two different scenarios. The first scenario considers the firm as it is, as a single economic entity with a unified objective and exhibiting full cooperation between its various economic units. The second scenario considers splitting the firm into its constituent economic units and computing market equilibrium prices if these units were to fail to cooperate and acted completely independently out of self interest. The difference between the cooperative profits of the first scenario and the aggregate profits of the independent units of the second scenario define a measure we call the Value of Cooperation, VC, of the firm in its current market environment. A second related measure is the Relative Value of Cooperation, RVC, which normalizes the VC measure by the cooperative profits to yield a unitless metric that reveals the percentage of profits derived from cooperation within the firm.

Quantifying the value of cooperation is a first step in understanding how firms exert market power in their respective environments. This information is important for both managers, who hope to leverage the information to better lead their organizations, and regulators, who want to monitor the impact of corporate decisions on social welfare. Future work will concretely establish the relationship between the VC and RVC measures and market power and indicate how to compute approximations to these metrics from readily available market data.

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