An Empirical Approach toward Realistic Modeling of Capital Market Volatility

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Abstract

This paper examines the realized integrated variance of the daily return rate series of S&P 500, Nasdaq and FTSE 100 respectively. The realized integrated variance is defined to be the statistical variance of the market index return rates within a certain time window. In stead of regarding the realized integrated variance as an empirical approximate of the latent spot volatility as formally defined in a mathematical framework, such as GARCH, we treat it as a direct "observable" volatility measure and try to model its dynamics straightforward. First, we find the volatility series is a stochastic jump-decay process rather than being all-over-the-time stationary in each market. Also, under switching bull and bear market conditions, the volatility series exhibits significantly different dynamic characteristics. Secondly, some major market structures, related to volatility mechanism, might have changed due to the hefty usage of options for hedging volatility risk after 1997.

Keywords: Volatility, Realized integrated variance, CEV model, Wavelet

1. Introduction

Since Hull & White(1987) proposed the stochastic volatility model in 1987, the dynamics of volatility has become central in the research of financial engineering. However the major concern of most studies, such as GARCH, is spot volatility. Spot volatility is latent. We can only approximate it by short-windowed standard deviation of return rate. Barndorff-Nielsen et al (2001) and Meddahi(2002) show that high frequency realized volatility converges to integrated volatility.

However, as the variance swap took off as a financial product, the realized integrated variance becomes a tradable underlying. To most of the traders the realized integrated variance, instead of latent spot volatility, is an observable measure of volatility. Thus it is essential to model its dynamics straightforward in order to check how traders make their investment decisions according to their observations on this measure.

In this paper we examine the realized integrated variance of daily return rate series of S&P500, Nasdaq and FTSE 100 respectively. Our research starts from the parametric framework of generalized CEV modeling to find the dynamic characteristics of realized integrated variance of in the three major stock markets - that is, following each jump, the realized integrated variance will stochastically decay to the local equilibrium level by the CEV law.

Along the time horizon of each market, it is clear that there are many spikes (jumps with fast decays) in the volatility process. It is questionable that the spikes (jumps with fast decays) are generated endogenously from a simple CEV model. Wavelet technique is introduced to verify that the volatility jumps are not natural to CEV modeling. Therefore, we find the volatility series is a stochastic jump-decay process rather than being all-over-the-time stationary in each market.

Also, with direct estimation to parameters of the CEV model in each time window, under switching bull and bear market conditions, the volatility series exhibits significantly different dynamic characteristics. Some major market structures might have changed due to the hefty usage of options for hedging volatility risk after 1997.

2. Literature Review

There are extensive literature in the study of volatility since Black and Scholes (1973, also in reference) introduce their option pricing formula based on the geometric Brownian motion with constant volatility. Hull and White (1987) propose the stochastic volatility model, of which squared volatility is also assumed to follow the geometric Brownian motion. Stein and Stein (1991) assume volatility an Ornstein-Uhlenbeck process with a mean-reverting property. Heston (1991) and Heston & Nandi (2000) both use the square-root process for volatility modeling. These studies aim to derive an option pricing formula under various assumptions. Normally a simple modeling framework is used for both the observable return rate and the latent spot volatility, but it is usually more complicated to calculate the integrated variance. Even if the realized integrated variance follows a simple model, the return rate process might be very complicated contrast to the simple framework assumed in the past. Therefore, it is our purpose to directly model the realized integrated variance from a more realistic prospect.

3. Methodology

3.1. Generalized CEV Model and Realized Integrated Variance

Our direct volatility measure - realized integrated variance is defined to be the statistical variance of daily return rates of consecutive 20 days,

$$V_i = \sum_{k=0}^{19} (r_{i-k} - \bar{r})^2$$

The generalized constant elasticity of variance (CEV) model that assumes the volatility process with a mean reverting level is

$$dV_t = -k(V_t - \theta)dt + \sigma V_t^{\beta} dz, \qquad (1)$$

where k >0. For the beginning works, we simply assume $\beta = 1/2$ for making data-fitting more consistent before the basic framework of modeling in (1) is further refined.

3.2. Jump-Decay and Wavelet Analysis

According to Daubechies (1992), the process with Hölder continuity α leaves the maximum absolute value of wavelet coefficient of order $O(2^{-j(\alpha+\frac{1}{2})})$ in level j . The CEV process has larger Hölder continuity than Brownian motion ($\alpha = 1/2$). In order to tell the jump effect, consider in Fig. 1 the fifth-order Daubechies basis wavelet filtration of S&P 500 realized integrated variance series from Mar. 1988 to Dec. 2004. We can find that the spikes (jumps) leave significant coefficients in every level with decaying rate much slower than 2^{-j} . The simulation of a CEV process of length 10000 with $k = 0.01$, $\theta = 0.02$, $\sigma = 0.01$ is shown in Fig. 2. There is hardly any standing-out coefficient in high-resolution levels to match the result in Fig. 1. It is then obvious that the real spikes (jumps) are beyond the explanation of CEV model.

Therefore an all-over-the-time stationary CEV model will mismatch the mean reverting level θ as well as ignore the jump phenomenon. In Section 4 we will propose a jump-decaying CEV model to depict the realized integrated volatility process.

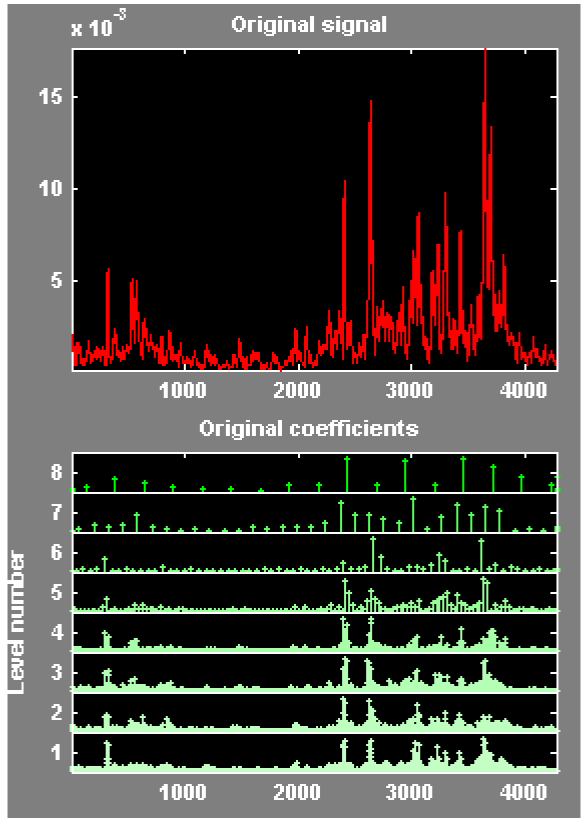

Fig. 1, Realized integrated variance series of S&P 500 and its wavelet filtrations

3.3. Regime Switching and Parameter Estimation

As we can visually check that the volatility series seems quite following a CEV decay-path right after each jump (compared to the simulation in Fig. 2), with parameters varying from time to time, it is suggestive to segment the time horizon into various time windows to check the locally stationary CEV characteristics.

Besides the jumps, we first assume that the volatility series as a piecewise CEV process with the parameters k, θ and σ varying by a simple regime-switching model to attack nonstationarity. 12-month moving average of monthly return rate of each index is used to segment the horizon into bull and bear market times as regime-switching conditions. As shown in Fig. 3, bear-market period is associated with the down-trend of MA(12) and bull-market period the up-trend.

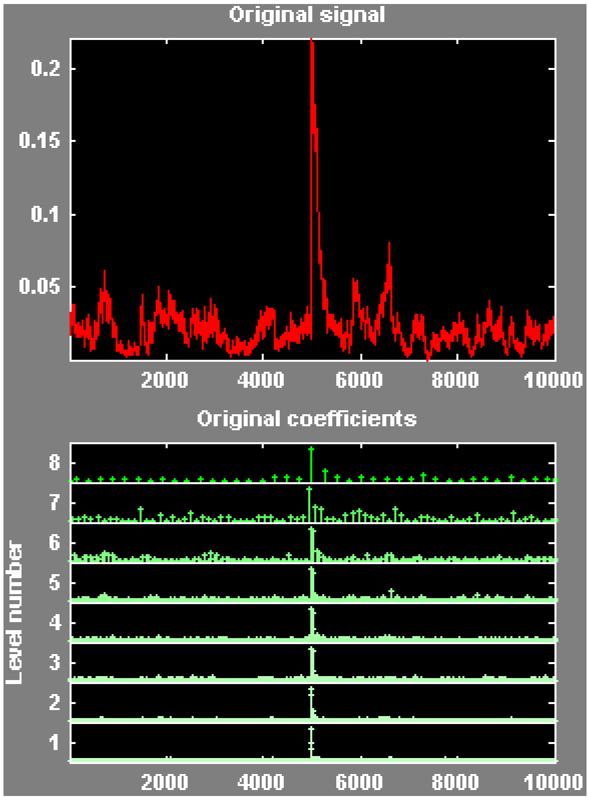

Fig. 2, Simulated CEV process with jump and wavelet filtration

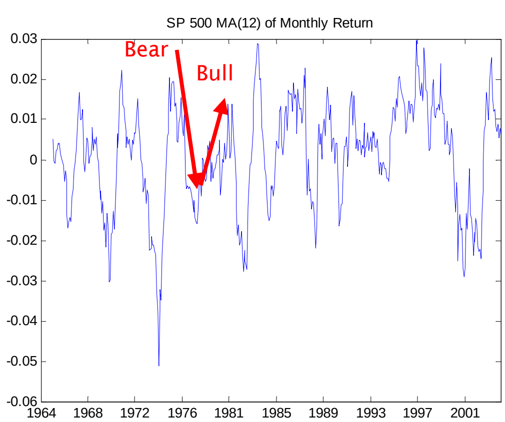

Fig. 3: S&P's 500 MA(12) of monthly return

4. Results of Data Analysis

Fig.4 - Fig. 6 show the estimate of parameter k and θ which exhibit significant difference in different time window. This shows that piecewise CEV process of market realized integrated variance is not time-stationary. In general, it is significant that the mean-reverting level of bull time is lower than the level of bear time. Moreover, it is an intriguing finding that, after 1997, k dramatically increases in bull time while decreasing in bear time for the three major markets. Interestingly, the soaring trading volume of option in CBOE historical record (Fig. 7) had a coincidence of timing. The following conjecture might provide a reason for this change of volatility dynamics:

The investors eventually realized that market volatility, on average, intends to be milder in bull markets. Thus, more traders were willing to short volatility via writing options, and the results were cheaper option premiums. With cheaper option premiums on average, investors tended to protect larger ratios of their holdings. Then, lower turn-over rates followed. This could be the key reason for the volatility to converge to their mean-reversion level by a faster rate k.

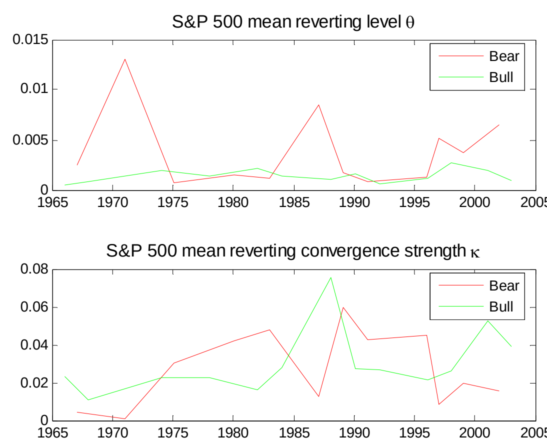

Fig. 4, S&P500's k & θ

5. Conclusion & Further Approach

Finally, we summarize our data analysis into the following framework of jump-decay piecewise CEV modeling:

$$dV_t = J_t - k(V_t - \theta)dt + \sigma V_t^{\frac{1}{2}} dz_t$$

$$k = A\chi_t + B + Cdz_t^k$$

$$\theta = E\chi_t + F + Gdz_t^{\theta}$$

J_t is a jump process. χ_t is a random variable which is 1 when t belongs to a set of disjoint random intervals Λ and -1 when t belongs to Λ^c.

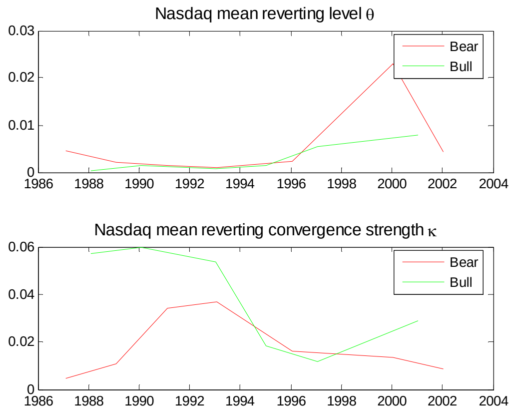

Fig. 5, Nasdaq's k & θ

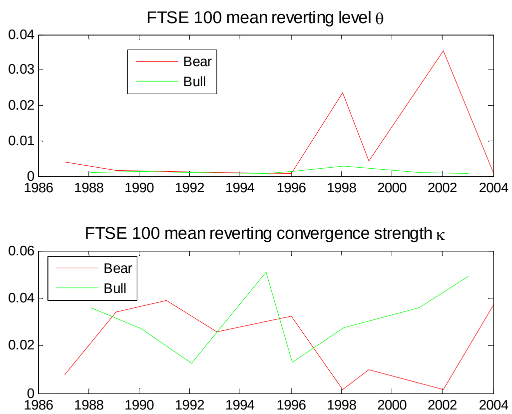

Fig. 6, FTSE's k & θ

Moreover, in order to refine the discrete 2-state regime-switching model, we try to monitor how volatility is dominated by sentiment in a continuous way. Once again, we employ wavelet technique to analyze the volatility time series and the corresponding time series of 20-day return-rate moving averages based on 8-level dmey decomposition. Here we have the striking mirror patterns (in all the three markets) between the local trends of return-rate moving average and realized integrated volatility, as shown in Fig. 8-Fig. 10.

This discovery not only sheds a light on how to model the transient behavior of the mean-reversion level along with the varying sentiment measured by return-rate moving average, but also indicates an intriguing hint to extends the stochastic volatility modeling of Hull and White into a more general framework where

$$dS_t / S_t = \mu_t dt + \sigma_t dW_t$$

with the low-frequency stochastic price return-rate fluctuation-determinant and the high-frequency stochastic fluctuation-determinant interacting each other in a well-correlated way.

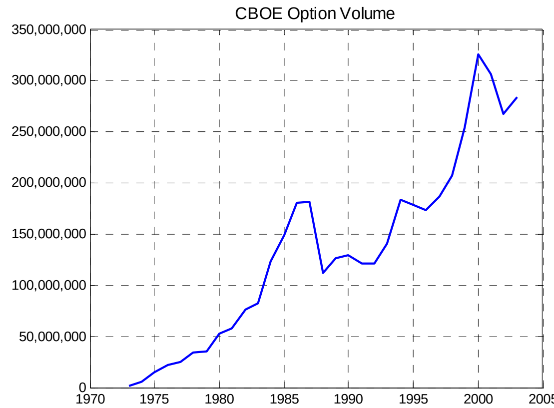

Fig. 7, CBOE option volume

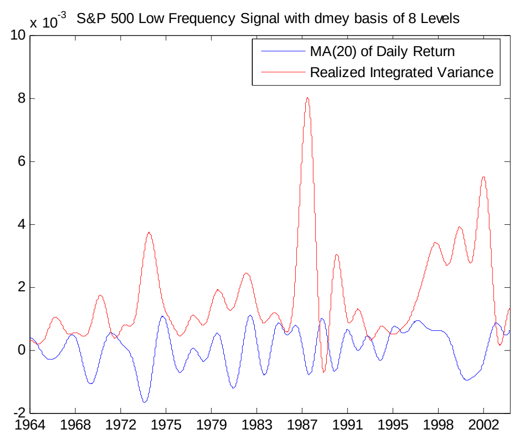

Fig.8, S&P 500's low frequency trend of realized integrated variance and moving average of return

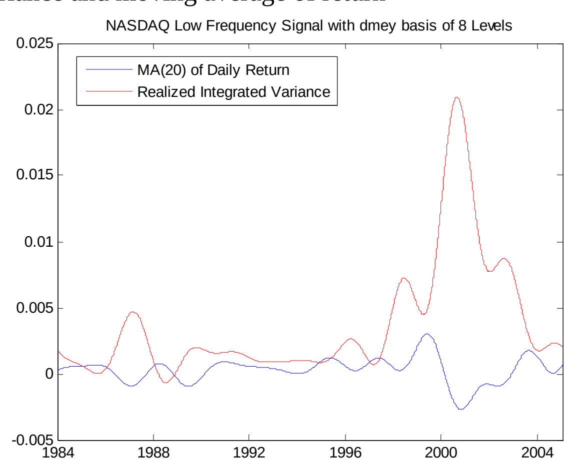

Fig.9, Nasdaq's low frequency trend of realized integrated variance and moving average of return

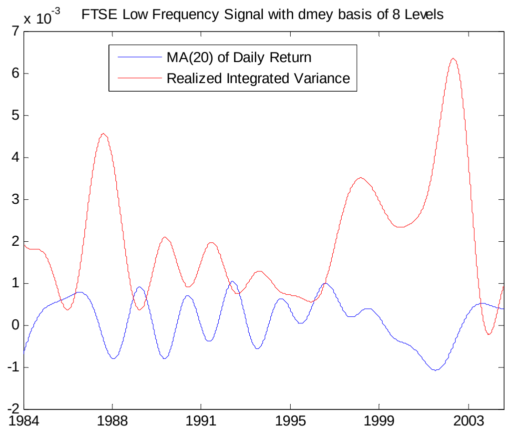

Fig.10, FTSE's low frequency trend of realized integrated variance and moving average of return

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