Prospect Theory and the Irrational Herding in Stock Markets

Len-Kuo Hu[1] Ching-Mann Huang[2] Yu-Shiu Lin[3]

[1]**National Chengchi University**

[2]**National Cheng Kung University**

[3]**National Kaohsiung University of Applied Sciences**

Abstract

We study investors' herding behavior in the stock market by examining their dynamic portfolio choices with the prospect-type utility. The idea that people care about changes in the financial wealth rather than final wealth positions and that they are risk averse over gains and risk loving over losses can substantially affect their investing strategies and herding attitude over time. Using the concept of value function suggested by Kahneman and Tversky (1979,1992), we are able to track informed as well as uninformed investors' investment decision and predict the occurrence of herding where informed traders disregard their private information. Finally we investigate how the existence of institutional investors who are more likely to be risk neutral than noisy traders affects the likelihood of herding in the stock market.

Keywords: Herding Behavior, Prospect Theory, Informed Traders

1. Introduction

In history there are many times financial markets experience investors' herding behavior. When people herd, their decision is independent of their private information, in other words they disregard their private information and herd on the public information. Since Bikhchandani, Hirshleifer, and Welch (1992) this phenomena is rationalized by the Bayes approach.

Herds in a financial market must occur as a result of the Martingale convergence theorem which implies that the public belief converges almost surely. In the limit, the support of the distribution of public belief must be included in one of the two cascade sets (either all buy or all sell). The more pertinent and interesting issue about herding is when and how frequent it will occur. The latter has much to do with the speed of learning from the observation of a private signal. In a binary model, the convergence of the public belief is exponential. However, if the private signals are Gaussian distributed, the convergence is significantly slower.

Our study reexamines the financial herding behavior from another perspective, i.e., an investor is more concerned with her gain or loss from investing than the sheer magnitude of her investment wealth. Drawing upon the prospect theory developed by Kahneman and Tversky (1979,1992) and others, we assume that an investor's utility is defined over gains and losses rather than over final wealth. Furthermore, the shape of value function is concave in gains and convex in losses. After formulating investors' behavior as the prospect-type, we then consider their optimal investment decisions and the resulting optimum expected utility.

We assume that there are two kinds of investors, informed as well as uninformed (or noisy) traders in the financial market. Informed traders are endowed with private information while noisy traders have public information only. By Bayes rule under the Gauss model, we update informed traders belief and find that it is quite likely that informed traders disregard their private information and herd on the public information behaving like uninformed traders. The risk taking attitude toward loss as emphasized by the prospect theory has greatly lessened the merits of private information, which is supposed to enhance the precision of unknown investment return, and increases the likelihood of herding. We finally discuss what factors and how they affect the occurrence of herding.

The remainder of this paper is as follows. In section 2 we propose the model depicting the value function of investors with different information sets and derive their objective functions. Section 3 implements the numerical analysis of the model.

2. The Model

We consider the market for a balanced fund that is an investment portfolio including both riskfree and risky assets. Let S_t represents the price of this asset at time *t*. Specifically, S_t is assumed to follow a log normal distribution

$$S_t = \exp((\mu - \tfrac{1}{2}\sigma^2)t + \sigma\varepsilon\sqrt{t}\ ),$$

where ε is a standard normal distribution with variance 1. Equivalently $\ln S_t$ follows a normal distribution

$$\ln S_t \sim N(\ln S_0 + (\mu - \tfrac{1}{2}\sigma^2)\,t,\ \sigma^2 t\ ),$$

where μ is expected rate of return and σ is the standard deviation of the rate of return.

Consider an investor who holds X_{t-1} units of this fund at the beginning of the period *t*. Thus his investments is $W_{t-1} = X_{t-1}\,S_{t-1}$ and evolves to $W_t = X_t\,S_t$ at the end of the period *t* due to the adjustment of the holding X or (and) the changes of the asset price. The investment gain (or loss) defined in the percentage term is $\frac{W_t}{W_{t-1}} = \frac{S_t}{S_{t-1}} \times \frac{X_t}{X_{t-1}}$. By taking natural logarithm at both sides, we get

$$\begin{aligned} y_t &\equiv \ln W_t - \ln W_{t-1} \\ &= \ln S_t - \ln S_{t-1} + h_t, \end{aligned}$$

where $h_t \equiv \ln X_t - \ln X_{t-1}$. Then y_t follows a normal distribution with the density function $f(y_t)$. The mean is ($\mu - \frac{1}{2}\sigma^2) + h_t \equiv \bar{y}_t$ and the variance is σ^2.

Next we define the investor's value function of the gain (or loss) in the percentage term as follows.

$$\begin{cases} g^+ = (\dfrac{W_t}{W_{t-1}})^\alpha \quad for \ \ y_t > 0 \\ g^- = -a + (\dfrac{W_t}{W_{t-1}})^\beta \quad for \ \ y_t < 0 \end{cases}$$

where α < 1 and β >1 characterizing the standard hypothesis in the prospect theory that the investor is risk averse when he gains and becomes risk loving when he loses. In addition since we define the gain or loss in a percentage form, we add a negative constant term –*a* such that the algebraic sign of g^- is guaranteed to be negative. According to the definition of y_t, these value functions can be rewrite as g^+ = $\exp(\alpha\, y_t)$ and g^- = $-a + \exp(\beta\, y_t)$. As a result, the investor's expected satisfaction from the gain is

$$G(h_t) = \int_0^{\infty} g^+ \, f(y_t)\, d\, y_t$$

$$= \int_0^{\infty} \exp(\alpha\, y_t)\, f(y_t)\, d\, y_t$$

$$= \frac{1}{\sqrt{2\pi}\,\sigma} \int_0^{\infty} \exp(\frac{-(y_t - \bar{y}_t)^2}{2\sigma^2} + \alpha\, y_t)\, dy_t \,,$$

and his expected dissatisfaction from the loss is

$$L(h_t) = \int_{-\infty}^{0} g^- \, f(y_t)\, d\, y_t$$

$$= \int_{-\infty}^{0} [-a + \exp(\beta\, y_t)]\, f(y_t)\, d\, y_t$$

$$= \frac{-a}{\sqrt{2\pi}\,\sigma} \int_{-\infty}^{0} \exp(\frac{-(y_t - \bar{y}_t)^2}{2\sigma^2})\, dy_t$$

$$+ \frac{1}{\sqrt{2\pi}\,\sigma} \int_{-\infty}^{0} \exp(\frac{-(y_t - \bar{y}_t)^2}{2\sigma^2} + \beta\, y_t)\, dy_t \,.$$

Combing the above $G(h_t)$ and $L(h_t)$, the investor chooses h_t at the beginning of period *t* to maximize his expected net satisfactions $G(h_t) + L(h_t) \equiv V(h_t)$.

So far we have shown our basic model. To discuss the possibility of herding, we then introduce two kinds of investors into the basic model, one is the informed trader who is endowed with a signal about the asset's return and the other is uninformed trader. Let r_t denotes the asset's true return at the period *t*. We assume that informed traders would receive a signal s_t at period *t* before investing

$$s_t = r_t + \theta_t$$

where θ_t is a normal distribution noise term with mean zero and variance σ_θ^2. According to the Bayes rule, after receiving the signal s_t the informed trader will update his belief about the mean return from μ to $\mu \big| s_t = \frac{\rho_\theta}{\hat{\rho}} s_t + \frac{\rho}{\hat{\rho}} \mu$ and about the variance from σ^2 to $\sigma^2 \big| s_t = \frac{\sigma_\theta^2 \sigma^2}{\sigma_\theta^2 + \sigma^2}$. In addition we define ρ_θ as the information precision of the signal or $\rho_\theta \equiv \frac{1}{\sigma_\theta^2}$, ρ as the information precision of the prior belief or $\rho \equiv \frac{1}{\sigma^2}$, and finally $\hat{\rho} \equiv \rho_\theta + \rho = \frac{\sigma_\theta^2 + \sigma^2}{\sigma_\theta^2 \sigma^2}$ as the information precision of the informed trader's updated belief. In summary, the informed trader maximizes his conditional expected net satisfactions $V(h_t \big| s_t)$ under the conditional density function $f(y_t \big| s_t)$, while the uninformed trader maximizes $V(h_t)$ under the unconditional density function $f(y_t)$.

$$\begin{cases} f(y_t) \sim N(\mu - \tfrac{1}{2}\sigma^2 + h_t, \sigma^2) \\ f(y_t \big| s_t) \sim N(\mu \big| s_t - \tfrac{1}{2}\sigma^2 \big| s_t + h_t, \sigma^2 \big| s_t) \end{cases}$$

Since our model is intertemporal, we have to describe how the prior belief $f(y_t)$ evolves through time. For simplicity we assume σ^2 is time-invariant and μ is the realized return in the last period, i.e. $\mu_t = r_{t-1}$. Therefore in order to calculate the relevant density function, we need to exogenously impose two variance parameters σ^2 and σ_θ^2, and generate the sequence of the realized return r_t as well as the noise term θ_t. After that we can solve the maximization problems for the informed as well as uninformed traders numerically and see under what circumstances will the informed trader disregard his private information and herd as uninformed trader.

3. The Numerical Analysis

In this section we discuss our numerical processes and expected results. First we need to specify eight parameters including a, α, β, σ, σ_θ, X_0 and S_0. Besides we also need to construct a sequence of r_t and θ_t. Then we use the optimization tool in the Matlab to solve the maximization of $V(h_t)$ and $V(h_t|s_t)$ numerically. By comparing these two optimum values we can analyze how the exogenous parameters affect the herding behavior.

3.1 A simple example

First we consider a simple static example. Thus we directly set μ, $\mu|s_t$ and $\sigma^2|s_t$. Here is the base case: α =1, β =1, σ =0.15, μ =0.08, and a =2. Then we change each parameter and see how it affects the optimum value. Finally we set $\mu|s_t$ =0.1(receiving a good signal) and $\sigma^2|s_t$ =0.14 and compare the optimum value with the one of the uninformed case (σ =0.15 and μ =0.08). Table 1 below summaries our result.

	h	$V(h)$	
Base case	0.69584737	2.17242993	
α			
0.8	0.69584385	1.85683671	
0.5	0.69586735	1.46979104	
β			
1.5	0.69587603	2.17249218	
2	0.69587721	2.17249475	
σ			
0.1	0.47218433	0	
0.12	0.53889976	1.85688152	
0.18	0.84465452	2.52099513	
μ			
0.06	0.71587524	2.17249047	
0.1	0.67586942	2.17247782	
a			
1.5	0.69584526	2.17242533	
2.5	0.69585891	2.17245405	
3	0.69584162	2.17241743	
Informed	0.62342404	2.06147761	

In this simple example it is shown that the informed trader obtains less expected optimum value although he is endowed with a private signal. Therefore we demonstrate that there exist possibilities that the informed trader would disregard his private information and herds as uninformed traders. This is a very preliminary result. Later we will randomly construct the noise term θ and the sequence of r to derive $\mu|s_t$ and $\sigma^2|s_t$.

4. References

[1] Bikhchandani, S., D. Hirshleifer, and I. Welch, "A Theory of Fads, Fashion, Custom, and Cultural Change as Informational Cascades," *Journal of Political Economy* 100, pp992-1026, 1992.

[2] Kahneman, D., and Amos Tversky, "Prospect Theory: An Analysis of Decision under Risk," *Econometrica* XVLII, pp 263-291, 1979.

[3] Tversky, Amos, and D. Kahneman, "Advances in Prospect Theory: Cumulative Representation of Uncertainty," *Journal of Risk and Uncertainty* V, pp 297-323, 1992.