On Normative and Liberal Pension Policy in Model Economy with Genetic Learning

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Abstract

A one-country two-generation model is developed which includes three prototypes of pension savings: a continuous pension system balanced by the means of tax and fiscal policy, a global pension fund system, and the individual pension savings. The formalism is a modification of two-overlapping generation model by Arifovic Genkay suitable for evaluation of long term policies. Unlike from their previous model, the government attempts to optimally redistribute endowments between agent life periods, possibly from various goal target viewpoints. The agents investment decisions are driven by a genetic algorithm which determines the savings level and the savings allocation ratio into open pension fund shares. Stability of price levels, asset per pension share, and inequality in consumption level across the society are discussed. The optimization problem includes both the micro level (society members aiming at their maximal lifetime utility, driving thus the system dynamics) and macro level (government optimizing redistribution policy and evaluated by the long-term system dynamics). The general three-pillar pension savings case and its extreme variants of normative and liberal pension savings policy are analyzed along with the origin of stylized facts arising in the time series of the model. From a broader point of view, this work provides a framework for assessment of some stylized long-term policy implications in GA driven societies.

Keywords: genetic algorithm, policy optimization, pension system, allocation of savings.

1 Introduction

Finding and pursuing an optimal social and pension policy is not only a practical key problem in most economies, but also one of the important test beds of differences between normative and positive economics theory [1]. Since such policies apply and are evaluated on the scale of human generations, i.e. at large exceed the usual governmental terms of office, the existence of long scale policies (and the more their evaluation) remains a complicated issue. Most frequently, it is the temporary fiscal surplus or deficit that serves as an economic criterion for social policy success or failure, omitting social utility and other non-accounting points of view. Due to the extreme time span of the problem, a computational study on social policy may only capture the very basic features of the problem. These are especially the three universal pillars of the social system as defined by the World Bank (non-contributory, contributory, and voluntary savings), and the evolutionary dynamics of the society including trends in individual savings and pension investments. We therefore opt for a genetic-algorithm driven model of the society as a useful tool to investigate this problem. The genetic algorithm applied in the present work represents the fundamental exchange of generations in human society; it is staged into pension-saving period (youth) and pension utilization period (old age). This is different from other applications of this method, which either target static optimization problems (based on mutation of parameters) or ad-hoc simulate information exchange (based on cross-over of parameter strings).

Computationally intelligent models of markets and societies have in general proved successful in dealing with certain aspects of real markets and real societies, and became a powerful experimental tool that complements the old-fashioned economic theory [2]. Dealing with the pension policy problem, we can elucidate the response of individual saving actions of society members to income redistribution by the government on the level of the society. Some of the artificial intelligence (AI) tools for society modelling are the genetic algorithm, GA [3], genetic programming, GP [4], neural networks, NN [5], reinforcement learning, RL [6] and so forth. The AI field field is vast and the number of applications to social sciences is expected to keep growing.

In this paper, we draw a computer simulation framework for social impacts of pension policy, building on the Kareken and Wallace economy model [7], extended later by Arifovic and Gencay [8]. Fundamental features of this model were recently discovered and analyzed thoroughly by Lux and Schornstein [9]. The major departures in this work are (1) one country formulation, (2) incorporation of fiscal policy as a redistribution of endowments, and (3) the genetic algorithm that describes the evolution of the society rather than an agent learning mechanism.

The paper is organized as follows. In Section 2, the model of the society and the pension saving system is postulated and fundamental equations are given. Section 3 briefly describes the implementation of genetic algorithm with real and binary encoding and states the optimal pension policies. In Section 4, the results of numerical experiments and analysis of the stylized facts are given. Concluding remarks and suggestions for future research close the paper in Section 5.

2 Pension Savings in the Society

We start with a one-country $2N$-member society which has a constant population of two overlapping generations, each of size N. Every society member receives naturalized endowments w_1 and w_2 in the two life periods, respectively. These are either obtained as their own product diminished by obligatory pension insurance at youth, $w_1 = (1-T)W_1$, or the agent production and pension revenues of the old, $w_2 = W_2 + TW_1$. The government hereby enforces and redistributes the physical output of size NTW_1 between the two generations in a completely balanced system. All markets in the economy (product market and pension fund share market) are supposed to be completely efficient.

The physical product at disposal to each individual in youth is classified into consumption and savings, $w_1 = c_i + s_i$, $i = 1, .., N$, where c_i and s_i are the consumption and savings of individual i, respectively. The savings shall be completely consumed in the 2nd (and last) phase of the life; two forms of such savings are possible: shares of public pension fund (contributory savings) and currency notes (voluntary savings). Deflation or inflation is possible in principal for both monetary instruments between the two stages of life. Let us denote $f_i \in (0,1)$ the saving ratio into pension fund shares, $M = \sum_i m_i$ the nominal money supply, and $H = \sum_i h_i$ the nominal asset of all pension fund shares. The consumption is subject to the following restrictions,

$$c_i(t) = w_1 - \frac{m_i(t)}{p(t)} - \frac{h_i(t)}{a(t)} = w_1 - s_i(t)$$

$$c_i(t_+) \leq w_2 + \frac{m_i(t)}{p(t_+)} + \frac{h_i(t)}{a(t_+)} = w_2 + s_i(t)|_{t_+} \quad (1)$$

where $c_i(t)$ and $c_i(t_+)$ are consumption of each agent at time t and its subsequent period $t_+ = t+1$. Further, $m_i(t)$ and $h_i(t)$ are the individual holdings of the currency and pension fund shares, $p(t)$ is the price level, and $h(t)$ is the pension fund liquidity, the asset-per-share ratio. Human savings are denoted as $s_i(t)$ in the first period of life, i.e. the sum of money and pension fund shares, subject to general time changes, $s_i(t)|_{t_+}$. Note that while the personal real savings may change between t and $t+1$, as also the prices and fund liquidity do, the overall real savings are conserved. The price level $p(t)$ and pension fund liquidity $a(t)$ are calculated by dividing the nominal supply by the product traded in the market or the product savings covered in the pension fund,

$$p(t) = \frac{M}{\sum_i^N (1-f_i)(t) \cdot s_i(t)},$$

$$a(t) = \frac{H}{\sum_i^N f_i(t) \cdot s_i(t)}. \quad (2)$$

Note that $f_i(t) = h_i/(a(t)s_i(t))$. We suppose that each agent's consumption in the second period of life is maximal, that is the 2nd line in Eq. 1 turns into an equality. Therefore the only decisions for each individual to be taken are (1) the real savings $s_i \leq w_1$ (or $s_i \leq w_1 + w_2$ when loans are admitted), and the pension fund savings ratio f_i for $i = 1, .., N$.

The price of shares $p_s(t)$ is given as a ratio of the price level in the economy and the pension fund share liquidity, $p_s(t) = p(t)/a(t)$, due to the efficiency of the market. It is straightforward to obtain the consumption level of the old generation at time t_+ as a result of investment decision at time t and aggregate price and pension fund liquidity levels,

$$c_{i,t}(t_+) = w_2 + s_i(t)[f_i(t)\frac{a(t)}{a(t_+)} + (1-f_i(t))\frac{p(t)}{p(t_+)}].$$

Notice the impact of inflation $(y(t) < y(t+1))$ and deflation $(y(t) > y(t+1))$ in both monetary instruments $(y = a, p)$ on the individual consumptions values.

Finally, the success of pension saving investment and life satisfaction is measured by a certain utility function $U(c_i(t), c_i(t+1))$. It is plausible to require the low consumption to be over-proportionally suppressed in both periods of life, e.g.

$$U(c_i(t), c_i(t+1)) = c_i(t)^n * c_i(t+1)^m. \quad (3)$$

Using real savings s as an independent variable, we arrive to the requirement of equal marginal utility for consumption values in both life periods, $\partial U^*/\partial c = \partial U^*/\partial c_+$ which solves as $U^* = C(n,m)(w_1+w_2)^{n+m}$, where C is a certain scaling constant. In particular, the optimal utility U^* also depends on the pension saving factor f. Omitting the individual indices, we have for $U(s,f)$:

$$U = (w_1 - s) \cdot (w_2 + s \cdot [f\pi_1 + (1-f)\pi_2]), \quad (4)$$

with $\pi_1 = a(t)/a(t+1)$ and $\pi_2 = p(t)/p(t+1)$. The marginal point $\partial U/\partial f = 0$ implies $\pi_1 - \pi_2 = 0$, i.e. the pension fund prices $p_s(t)$ is constant and all possible values of f are optimal. Otherwise, for $\pi_1 > \pi_2$ ($\pi_1 < \pi_2$), the maximum value is obtained for $f = 1$ ($f = 0$).

3 Optimal Pension Policy

The success or failure of each individual is determined by his pension saving decisions f_i and s_i, the endowment levels w_1 and w_2 (which implicitly includes the governmental redistribution rate T), and decisions taken by all other society members (the old and the new generations). The society is driven by a standard genetic algorithm as described by Arifovic and Gencay [8], both in binary (20 bits for s, 10 bits for f)

$$c_i = \sum_{k=1}^{20} b_i^k \cdot \frac{2^{k-1}}{C_1}, f_i = \sum_{k=21}^{30} b_i^k \cdot \frac{2^{k-21}}{C_2}, \quad (5)$$

and real encoding. The standard GA roulette wheel selection with constant population, crossover and mutation operators are used. Following Lux, we also add the election operator, to avoid offspring degradation. The marginal cases of the present model are:

- Normative pension policy - there is no open pension fund, i.e. $f = 0$ and the government uses only fiscal policy tools - redistribution rate T.
- Positive pension policy - the government does not enforce solidarity among generations, $T = 0$.

Note that the marginal cases are not strictly opposite, since the third pillar of pensions, individual savings in the currency are possible (and plausible) in each case. The setting $s_i \equiv 0$ would imply a totalitarian policy and no coupling among the generations. The evolution of the society coincides with the genetic algorithm cycles, and can be tuned in terms of mutation probability, selection pressure and crossover rate. If there are no maximal restrictions on f_i values, the pension saving regulation policy has only one tool - T. The regulatory goals can be (1) maximum time-average value of total utility in the society (social criterion) or (2) stability of the price level, (3) stability of pension fund assets or (4) stability of pension fund prices (monetary criteria). Next we provide the simulation results and discuss the importance of GA coding. Unless stated otherwise, we use the values $w_1 = 6$, $w_2 = 4$, $H = 1000$, $M = 3000$, mutation probability 0.1%, crossover rate 60%, $N = 100$, and 100,000 simulation steps. To asses the effects of fiscal policy while avoiding unnecessary parameters, we renormalize the independent variable from T to $x \in (0, 1)$, $w_1 + w_2 \equiv 10$ with $w_1 = 10x$ and $w_2 = 10(1 - x)$.

4 Discussion

Based on the simulation results for $e(t)$, we have computed the normalized log returns of pension fund prices (binary-encoded GA) as

$$r_t = \log(p_s t/p_s(t-1)). \quad (6)$$

The histograms of $r_(t)$ with b bins is defined in standard way as

$$h[i] = \sum_t (r_0 + i\Delta r \leq r(t) < r_0 + (i+1)\Delta r)?1 : 0.$$

Here r_0 is the minimal observed value, $\Delta r = (r_1 - r_0)/b$, and r_1 is the maximal observed value of $r(t)$. Results are shown in Fig. 1 as a function of the variable x. It is clearly seen that the symmetric configuration $x = 0.5$ results in the lowest spread of pension fund prices p_s (top 10 values of the genome and their decoded parameters are shown in Table 1. Similarly, the optimal values

Table 1: Sample of top genome creatures

gene string	c	f
1001100110111111000000	3.498538	0.186706
1001100110101010100000	3.498538	0.181818
1001100110101111000000	3.498538	0.185728
1001100110111010100000	3.498538	0.182796
1001100110111111000000	3.498538	0.186706
1100110101010100000000	3.496097	0.166178
1001100110101111000000	3.498538	0.185728
1001100110101010100000	3.498538	0.181818
1100110101111111000000	3.496097	0.123167
1001100110111111000000	3.498538	0.061584

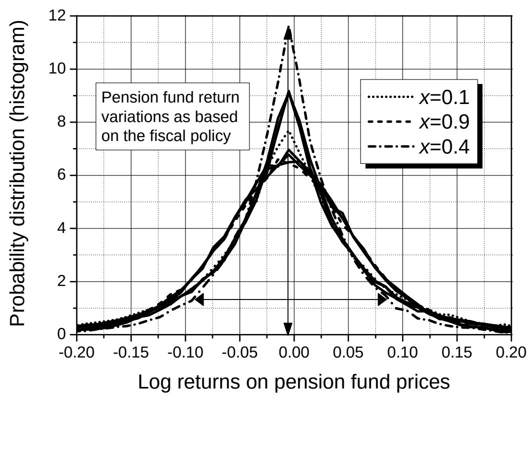

Figure 1: Histogram of normalized log returns on pension fund prices for wealth distributions $x = 0.1, .., 0.9$.

for the lowest spread of prices and the lowest spread

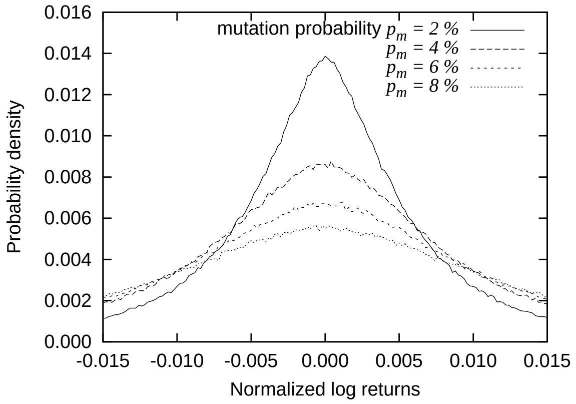

Figure 2: Histogram of normalized log returns without crossover and election, p_m = 2, 4, 6, 8 % (real-encoded GA).

of pension fund assets are found to be $x = 0.6$ and $x = 0.4$, respectively (asymmetric distributions) on the decimal grid $x = 0.1, .., 0.9$. Once any pair of the primary values W_1 and W_2 is given, the optimal values of T can be immediately derived from these results by linear transforms. Figure 2 shows representative trends in the stylized facts of returns on pension fund shares when the GA encoding changes to real parameters and different values of mutation probability are used. The flat tails are present even when the crossover and election operators are removed, unlike from binary-encoded GA which, on the other hand, shows less sensitivity to varying the values of mutation probability. The exponents fitting the power law of the probability distribution tail (cf. Figs. 1 and 2) are given in Table 2.

condition	2 %	4 %	6 %
normal	0.350	0.219	-0.030
no crossover	0.318	0.157	-0.056
no election	-0.114	-0.175	-0.181
no crossover no election	-0.112	-0.169	-0.181
binary	-0.064	-0.075	-0.075

Table 2: Power law exponent $(\alpha - 1)$ as a function of GA parameters (real and binary encoding).

5 Conclusion

We have developed a genetic-algorithm driven prototype model of mixed pension policy system in one country with two overlapping generations, including the three recommended pillars of standard pension systems. Based on monetary policy viewpoints (stability of prices and pension fund shares), we identified the optimal values of individual endowments in all life periods. Stylized features were observed in accordance with previous results by Lux [9] and their validity assessed across a range of mutation probabilities and the type of genome encoding. This work provides a framework for assessment of qualitative long-term pension plan policy implications in GA driven societies and may serve as a ground for further generalizations.

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