Almost perfect conservative optical logic gates

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Abstract

Conservative logic can be perfect in principle: no energy consumption, no bandwidth slowdown, constructive completeness, and full cascadability.

Keywords: conservative logic, bandwidth, energy, optical logic

1. In search of perfection

The work of Landauer [1}, Bennett [2}, Fredkin and Toffoli [3} began the search for logic devices that would consume no energy. Your computer needs to be cooled, because the logic gates it uses dissipate information and therefore energy. A Boolean logic gates takes in two bits and produces one bit. Obviously, one bit of information is destroyed in that process. Landauer showed that this has an energy cost of

$$E = kT\ln 2,$$

where k is the Boltzmann constant and T is the operating temperature. Physicists will recognize this as roughly the expected kinetic energy of a particle at temperature T. It is a very tiny energy not worth worrying about, except that we now routinely do more than 10^9 of those operations per second, so the minimum power that must be dissipated is now very large. In addition, current Boolean logic gates do not dissipate kT per operation; they dissipate about 10^6 kT or more per operation. The modern computer dissipate a huge amount of power, and that causes many other problems.

Obviously, as well; Boolean logic gates are irreversible. There is no general way to recover the two input bits from the one output bit. Physicists will again note the similarity to quantum physics, where the act of getting useful information out (measurement) produces an irreversible loss of information. Such considerations raise the interesting observation that if reversible (information conserving) logic gates could be made, it might be possible to avoid that energy dissipation altogether! From an energy viewpoint, such logic gates would be perfect.

On the surface, the idea of a logic gate that conserves information and is therefore reversible seems silly. Let me illustrate with a related concept – dissipative and conservative arithmetic. Conventional arithmetic is dissipative, e.g.

$$2 + 3 = 5$$

Obviously, this is irreversible. Given 5, we cannot supply unique values of x and y in the formula

$$x + y = 5.$$

We might have x = 2, y = 3, but we might also have x = 5, y = 0; x = 16, y = -11; or any of infinitely many other pairs. But

$$2 + 3 = 3 + 2$$

Is an example of conservative arithmetic. It has conserved all of the information and is fully reversible. But it does not seem to accomplish anything.

Actually, however, it does not differ from any computer in that regard. Consider the computation of

$$1 + 2 + 3 + \ldots + 100 = 5050.$$

The answer was implicit in the input data and the procedure. Had we been smart enough, we would not have needed a computer to find the answer. A computer is equivalent to a giant lookup table. Given the instructions and the data, the result is fully determined. Because the lookup table would be absurdly big, we ‘work out” the answer from the instructions and data, much as your calculator works out entries in what would be a huge sine table as they are needed. In a mathematical sense, they accomplish nothing.

What Fredkin and Toffoli accomplished was the exhibition of conservative logic gates that were constructively complete. That is, stringing them together appropriately can produce any of the 16 Boolean logic gates. Their approach was very much like that of the reversible arithmetic example shown earlier. The output signals contained the input signals rearranged in a signal-dependent fashions.

To date, noone has made a conservative logic gate that works at or below kTln2.

Another kind of perfection sought is unlimited bandwidth [4]. I must be careful in stating that goal in two respects. First, all physical gates will suffer from latency. Small, passive optical gates will obviously have near-minimum latency, but it will never be 0. Second, “unlimited bandwidth” applies to the logic, not to the input and output to that logic. It simply means that such logic will operate at the bandwidth of the input and output, whatever that is.

Implicit in everything I have written above is the assumption that these logic gates can be cascaded without penalty in energy or bandwidth.

I mean by “almost perfect” logic gates, gates that meet the zero-energy, constructive completeness, and bandwidth unlimited goals but fail to meet the cascadability-without-loss goal.

2. Interferometric Quantum Optical Logic

Consider the famous Mach-Zehnder interferometer as sketched in Fig. 1..

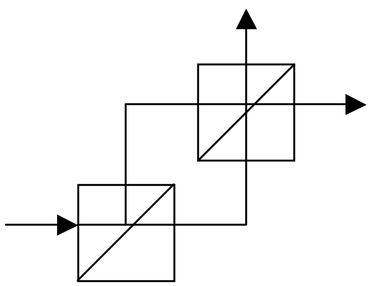

Fig. 1. Light split into two beams by the lower beamsplitter is recombined into two output beams by the second beamsplitter. Both of the output beams contain light from both of the inputs to that upper beamsplitter, but those complex wavefronts are in phase quadrature, so the whole system is reversible.

With the simple insertion of phase modulators introducing phases 0 or π as input signals A or B and a fixed phase shift of $\pi/4$ ($e^{i\pi/4}$ = i, of course) in one of those beams, see Fig. 2, we can arrange for an interferometer whose outputs C and D are given in Table 1.

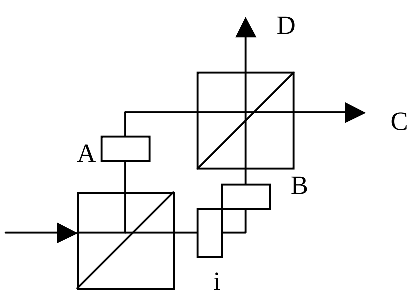

Fig. 2. Input signals A and B and a fixed factor of I result in the truth table shown below.

A	B	C	D
-1	-1	0	$-i\sqrt{2}$
-1	1	$-\sqrt{2}$	0
1	-1	$\sqrt{2}$	0
1	1	0	$i\sqrt{2}$

Table 1. here is what the interferometer of Fig. 2 does given unit amplitude inputs A and B (possibly differing in phase).

This operation is conservative and thus reversible. If we jointly phase conjugate C and D, we restore A and B traveling backward. No energy is expended, as the interferometer is passive.

3. Light switching light

Such an interferometer can also be viewed as a light switch. A can be regarded as the signal path and B can be regarded as the control path. The value set for the control phase determines which of the two output ports the light exits the system. If we have light of the proper control phase and mutually coherent with the signal beam, the signal light is routed appropriately without energy or bandwidth loss. This amounts to light switching light without material interaction. The upward going beam D can be deflected to be parallel to the transmitted beam C. All of the light is in either C or D.

4. A universal Boolean gate

Such switches can be cascaded in such a way that N of them switch light among 2^N positions in the fashion of a digital light deflector [5- 7]. Each interferometer must produce twice the beam separation of the preceding one.

As this is reversible, more interferometers and controls can restore the beam to its original position, creating what I have called elsewhere [8] a “do nothing machine.” Figs. 3 and 4 shows this schematically.

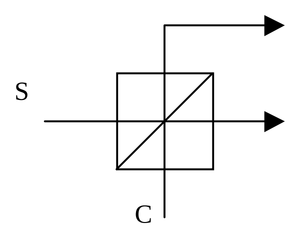

Fig. 3. This shows a simplified view of a system wherein a control C produces a signal in the lower position with one phase and a signal in the other with a different phase.

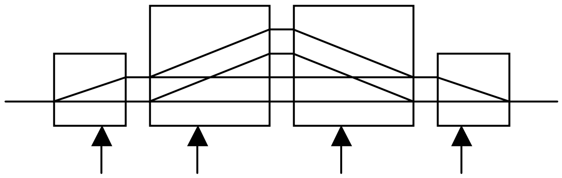

Fig. 4. This show a two-stage deflector followed by a two stage recombiner can place the input beam into any of four possible positions depending on the two control variables and then return the light to its original vertical position.

Do nothing machines are highly prized, because it is often possible to insert some sort of filter in the intermediate plane that causes the system to serve a useful purpose. For example, two successive optical Fourier transforms do nothing (except for turning the image upside down and backwards). By inserting a Fourier transform plane filter that matches some targets but not others, we can both recognize and locate objects in a scene in parallel. That is one of many examples of this "paradigm for invention" [8].

By placing the proper spatial mask in the middle of this cascade, we can block the light from all but one pair of control signal pairs, creating any one of the 16 possible two-variable Boolean gates. Figure 5 shows some sample masks.

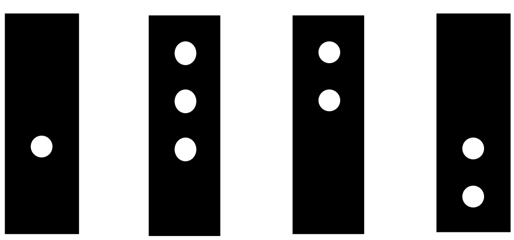

Fig. 5. Using the code that a logical 0 means no change in the beam's vertical position and logical 1 means deflection up, these masks in the ventral plane will produce AND, OR, XOR, and COINC respectively as the Boolean functions implemented.

5. Discussion

5.1. Universality

Obviously, all 16 Boolean logic gates can be built from a two-stage digital light deflector using the appropriate mask. This is very different from Fredkin or Toffoli gates. They are single gates that, if strung together properly, can implement any Boolean function. What I have described is a single gate whose functionality can be chosen – rather like any programmable electronic gate array.

5.2. Dimensionality

Obviously, we are not limited to Boolean functions of two variables. With an N-stage device, we can implement all the Boolean functions of N binary variables.

5.3. Adaptivity

Clearly, the functionality of such a unit is as fixed as its mask. If we use a SLM (Spatial Light Modulator) as the mask, such an element can change functionality quite rapidly. Nor is SLM contrast much of a problem. The attenuation must be enough to block the full beam if it is in the wrong position, but the brightness to be blocked is independent of the number of stages N.

5.4. Parallelism

One-dimensional arrays of such gates are possible in bulk optics. Two-dimensional arrays are unattractive, because one dimension is needed for deflection.

5.5. Speed/Bandwidth

For a fixed functionality gate, the controls are fixed and the mask is fixed, so the signal bandwidth is unchanged by the logic gate!

5.6. Energy Expenditure

For a fixed functionality gate, the logic is passive. It does not switch, so it requires no energy. This is allowed for conservative logic.

5.7. The Quantum Nature of These Gates

Note that the outputs can be written in terms of the input amplitudes A and B as follows:

$$C = (A + iB)/\sqrt{2}$$

and

$$D = (iA + B)/\sqrt{2}$$

These are clearly quantum superpositions of the two inputs or qubits (quantum bits).
To get the pattern of Table 1, we inserted an extra i in the B beam.

Going back to Table 1, the outputs are not restricted to 0 and 1 (or, to be more precise √2). Rather, they are 0 and one of the four points (1, i, -1, -i). This is what allows the reversibility.

Measurement, however, irreversibly destroys the phase information, so al four nonzero points reduce to 1. The irreversible loss of information on measurement is characteristic of quantum mechanics.

This means that the operations take place in the complex domain. That is why it is so well suited to coherent optics.

5.8. Physical Configuration

A reflective do nothing machine is actually easier to construct than the transmissive one, because the same deflector can be used for splitting and recombination as suggested in Fig. 6. A beamsplitter gives us access to the returned (phase conjugated) beam.

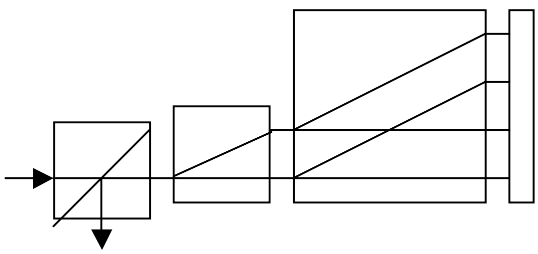

Fig. 6. This shows a two-layer reflective do nothing machine

5.9. Difference from Fredkin and Toffoli

Fredkin and Toffoli both assumed symbol conservation like that conservative arithmetic example I used earlier:

$$2 + 3 = 3 + 2$$

In contrast, our reversible gates have pure signals, A and B, in and quantum superpositions out.

6. The Flaw

The sole problem, in principle, is that direct cascading is impossible. The inputs are phases 0 and π and the outputs are signals of amplitudes 0 and 1. They are not the same.

It is possible to cascade these systems in a straightforward way. The output from one gate can modulate the phase on another beam using an electrooptic modulation driven by a voltage derived from a systems. Unfortunately, that operation does consume energy and does limit the bandwidth.

7. Conclusions

So long as we use only a single logic gate in each optical beam, we can have perfect optical logic (any gate, no energy loss, no bandwidth limitation, and rapidly reprogramable functions of two or more variables.

8. References

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