10-GHz Bandwidth Spectral-Hole-Burning Spectrum Analyzer

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Abstract

We propose and demonstrate a 10-GHz spectrum analyzer with >40-dB dynamic range based on spectral hole burning (SHB) technology. Illuminating an SHB crystal with an RF signal modulated onto an optical carrier excites ions in the crystal whose absorption frequencies match the spectral content of the incident signal, leaving holes burned in the crystal's absorption band that persist for 10 ms. Probing this altered absorption profile with a low-power, chirped laser while measuring the transmitted intensity produces a time-domain readout of the accumulated RF signal spectrum.

1 Introduction

Conventional RF spectrum analyzers use a frequency-swept local oscillator to mix the high bandwidth RF signals down to low intermediate frequencies and use narrow bandpass filters and power detectors to do the actual spectral estimation.[1] Because these scanning super-heterodyne analyzers sweep across the band of interest in sequential fashion, they can miss pulsed or hopping signals. In addition, increasing the signal bandwidth necessarily increases the sweep time for a given sweep rate; on the other hand, increasing the sweep rate results in a loss of resolution since the resolution is fundamentally limited by the inverse of the dwell time. Applications such as electronic warfare demand high probability of intercept and fine spectral resolution, making it undesirable to increase either the sweep time or the chirp rate. RF photonic techniques enable high bandwidth spectrum analysis with both unity probability-of-intercept (POI) and fine resolution.

Classic RF photonic techniques for signal processing typically transform RF modulation into spatial fringe patterns that can be analyzed using coherent optical methods. For example, acousto-optic spectrum analyzers use the traveling-wave nature of acoustic propagation to deflect an incident optical beam at an angle proportional to the RF frequency driving the acoustic wave. A lens Fourier transforms the deflection angles into spatial shifts to produce a spatial map of the RF signal spectrum that can be detected on a suitable 1-dimensional detector array.[2] However, acoustic wave generation and attenuation limit the bandwidth of these systems to a few GHz.

Spectral hole burning (SHB) crystals comprise rare earth ions lightly doped into crystal hosts such as YAG or YSO and support coherent optical signal processing techniques for RF signals with bandwidths $\leq$ 25 GHz. Cooling SHB crystals to cryogenic temperatures (4 K or below) causes the homogeneous absorption bands of the individual ions to shrink to sub-MHz linewidths. Differences in the ions' local environments shift the line centers of the absorption resonances so that the ensemble of Lorentzian resonances span several GHz. Illuminating a given region of the crystal with narrowband radiation excites only those ions which absorb at the radiation frequency. As the number of excited ions increases, the laser experiences a decrease in absorption that persists for the excited state lifetime, T_1; we refer to these spectrally-localized dips in the absorption bands as spectral holes. Using electro-optic modulators in conjunction with properly chosen SHB media would allow us to process RF signals whose bandwidths could potentially exceed 20 GHz using materials such as Er^{3+}:YAG or even wider bandwidths using recently developed disorder crystals such as Er^{3+}:$LiNbO_3$.[3]

Because SHB media allow multiplexing in four dimensions (three spatial and one spectral), they enable several different methods of spectrum analysis. For example, Lavielle et al. recently demonstrated a 3-GHz bandwidth spectrum analyzer that emulated the functionality of an angle-scanning acousto-optic device using an SHB crystal. They programmed a set of angle-multiplexed absorption gratings into an SHB crystal such that each grating diffracted only specific spectral components of an incident beam.[4]. In contrast, our method of spectrum analysis uses just a single spot in the SHB crystal to record a modulated spectrum in order to analyze 10 GHz of bandwidth with MHz resolution.

The SHB write/read process is analogous to the exposure/development process in color film, except with millions of colors. Illuminating a single spot in an SHB crystal with an RF signal modulated onto an optical carrier leaves a negative of the signal spectrum burned into the crystal's inhomogeneous absorption profile. Unlike conventional spectrum analyzers that sequentially scan a given band, all the ions in the SHB crystal are always recording as long as they are illuminated, guaranteeing intercept of any pulsed or hopping signal. Probing the crystal's altered inhomogeneous absorption profile with a chirped laser produces increased transmission at the peaks of the spectrum as the transmitted intensity of the chirp beam increases whenever the chirp scans through a spectral hole. The time-varying intensity of the output can be converted into a signal spectrum by simply multiplying the detected intensity's time coordinate by the chirp rate. Because the spectral holes persist for the excited state lifetime (T_1 = 10 ms), we can read 10 GHz of bandwidth on low bandwidth detectors at a chirp rate of 1 MHz/μs, giving MHz resolution. Our spectrum analyzer uses an SHB crystal to capture high bandwidth signals and stores the signals long enough to interpret them using low bandwidth electronics.

2 Theory of Operation

We can use third-order perturbation theory to describe the time domain response of SHB media to a set of time-ordered fields of amplitude $a_j(t_j)$ where $j = 1, 2, 3$.[5] However, we stand to gain more insight into the operation of an SHB spectrum analyzer by considering the interaction of these fields in the frequency domain. If we instead use the frequency domain representations of the interacting fields, $a_j(t_j) \rightleftharpoons A_j(\omega_j)$, we can write the time domain response of an SHB crystal, $a_{\rm out}(t)$, as [6]

$$a_{\rm out}(t) \propto \int_{-\infty}^{\infty} d\omega_3 \int_{-\infty}^{\infty} d\omega_2 \int_{-\infty}^{\infty} d\omega_1 A_3(\omega_3) A_2(\omega_2) A_1^*(\omega_1) \times \gamma_1(\omega_2 - \omega_1)\gamma_2(\omega_3 + \omega_2 - 2\omega_1) \times e^{i(\omega_3+\omega_2-\omega_1)t} \quad (1)$$

where

$$\gamma_1(\omega_2 - \omega_1) = \frac{1}{(1/T_1) + i(\omega_2 - \omega_1)} \quad (2)$$

$$\gamma_2(\omega_3 + \omega_2 - 2\omega_1) = \frac{1}{(2/T_2) + i(\omega_3 + \omega_2 - 2\omega_1)}. \quad (3)$$

The widths of the complex Lorentzians given in Equations 2 and 3 correspond to the material's excited state lifetime, T_1, and half the coherence time, T_2. The width of the Lorentzian in Equation 3 is $2/T_2$ rather than just $1/T_2$ because the coherence effect participates twice, once during the recording process between beams $a_1^*(t_1)$ and $a_2(t_2)$, and again after the readout beam, $a_3(t_3)$, generates the delayed echo, $a_{\rm out}(t)$.

We can apply this general result to an SHB spectrum analyzer by choosing $A_1(\omega) = A_2(\omega) = A(\omega)$. As the excited state lifetime is usually $T_1 = 10$ ms for materials such as Tm^{3+}:YAG and Er^{3+}:YSO, the linewidth of the Lorentzian γ_1 is roughly 100 Hz, meaning that we can reasonably approximate $\gamma_1(\omega_2 - \omega_1)$ as $\delta(\omega_2 - \omega_1)$. We can use a single sideband chirp, $a_3(t_3) = a_c \exp(i\beta t_3^2)$, to probe the resulting spectral holes, allowing us to write $A_3(\omega_3) \propto \exp(i\omega_3^2/4\beta)$ where β is the chirp rate with units of MHz/μs. For a Gaussian-apodized readout chirp of duration τ_r, $\beta = \pi b + i/\tau_r^2$, where b is the chirp rate in Mhz/μs. After performing these substitutions on Equation 1, we can apply the method of stationary phase [7] to get

$$a_{\rm out}(t) \propto \int_{-\infty}^{\infty} |A(\omega_2)|^2 \gamma_2(\omega_2 - 2\beta t) e^{i\beta t^2} d\omega_2 \quad (4)$$

$$\propto \left\{ |A(2\beta t)|^2 * \gamma_2(2\beta t) \right\} e^{i\beta t^2} \quad (5)$$

where $*$ denotes the convolution operation. The time domain output is the convolution of the signal's power spectrum with a Lorentzian of width $2/T_2$, multiplied by a quadratic phase factor. The signal power spectrum is scaled in the time domain by the chirp rate. The output field can be detected by heterodyne interference with another copy of the readout chirp. The cross term contains the signal spectrum information and can be written as

$$I_{\rm det}(t) \propto \left| \left\{ |A(2\beta t)|^2 * \gamma_2(2\beta t) \right\} \right|^2 e^{-t^2/\tau_r^2}. \quad (6)$$

The coherence time of both Tm^{3+}:YAG and Er^{3+}:YSO is roughly $T_2 = 10$ μs at cryogenic temperatures, limiting the resolution of this spectrum analyzer to 200 kHz, although spectral diffusion may further broaden the resolution to about 1 MHz. Recently developed spectral recovery techniques may help to avoid the 1-MHz resolution imposed by the dwell time of 1-MHz/μs chirp rates when analyzing 10 GHz of bandwidth within the 10-ms excited state lifetime of an SHB medium such as Tm^{3+}:YAG.[8]

3 SHB Spectrum Analyzer

We constructed a 1-D spectrum analyzer using an electro-optic modulator (EOM) to burn RF signals into the absorption profile of a 0.05% Tm:YAG crystal cooled to 4 K in a Helium bath cryostat, as shown in Figure 1. We used a tunable external-cavity diode laser (ECDL) to drive the EOM such that the output power of the unmodulated EOM when biased

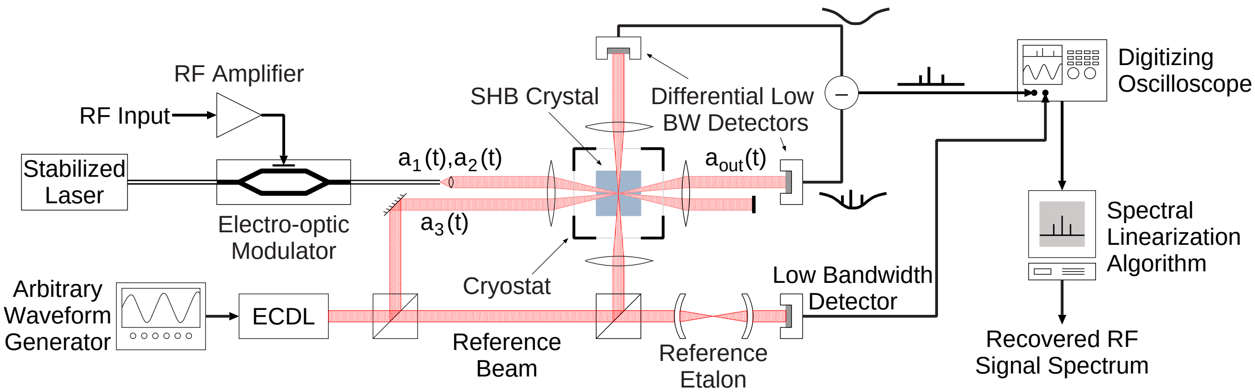

Figure 1: High bandwidth spectrum analyzer based on spectral hole burning (SHB). An RF signal drives an EOM that in turn illuminates a single spot in an SHB crystal cooled to 4 K. A chirp beam reads the spectrum engraved into the spot to illuminate one port of a differential detector. A second copy of the chirp probes a different spot in the crystal before illuminating the differential detector's second port. A third copy of the chirp passes through a reference etalon to produce a reference signal for correcting any distortion in the time-domain map of the RF spectrum produced by the differential detector.

at quadrature was approximately 3 mW. We stabilized this ECDL to a spectral feature located in a volume of the SHB crystal (not shown) that was well-separated from the volume that we used to perform the spectrum analysis.[9] Without stabilization, the laser frequency jitters and drifts, blurring the holes burned into the crystal's absorption band by the spectral components of the RF modulation. We focused the collimated output of the EOM to a 150-μm spot inside the crystal using a 300-mm focal length lens. We probed the region illuminated by the EOM output with a 150-μW chirp beam in order to measure the signal spectrum as described above. The chirp beam was focused through the same 300-mm focal length lens to produce a slightly tilted 150-μm spot that almost completely overlapped the programming spot. We produced the chirp by driving the piezoelectric tuning part of a second ECDL with a 36-Hz sinusoid. The linear portion of the sinusoid drove the ECDL frequency across the band of interest while the gentle transitions at the peaks and valleys of the sinusoid prevented the piezo from experiencing sharp changes in drive voltage or unwanted mode hops. We didn't stabilize the chirped read ECDL, so its center frequency varied from shot to shot. In addition, the imperfect response characteristics of the read ECDL's piezo drive distorted the linearity of the chirp in a random fashion. Rather than attempt to compensate these errors, both of which distort the output of the spectrum analyzer, we employed a novel system for measuring chirp non-linearities in order to remove them during post-processing.[10] We transmitted a portion of the chirp through a reference etalon of fixed finesse and recorded the transmitted intensity during the read-out period. A perfectly linear chirp propagating through the etalon would produce an transmitted intensity that varies periodically. Because our chirp is non-linear, the transmitted output looks like a phase-modulated signal. However, the time increments between adjacent zero crossings in the transmitted intensity correspond to identical frequency increments of the frequency scan. The intensity transmitted through the etalon is distorted in exactly the same way as the time-domain read-out of the spectrum. We can therefore scale the time bins in the raw signal in proportion to the time increments between zero crossings in the reference signal when converting the time-domain map of the RF signal spectrum to the frequency domain.

Although we can correct any distortions in the chirp, we can't compensate for drift in the center frequency of the read ECDL with just a single etalon (although using 2 or 3 etalons, absolute calibration is possible). We collect our data on a shot-to-shot basis and perform any required averaging after spectral calibration. We can improve the spectrum analyzer's dynamic range by employing differential heterodyne detection techniques. Differentially detecting another copy of the chirp beam that probes a 150-μm spot in a different volume of the crystal with the same power as the read spot suppresses noise terms present in both beams, thereby increasing the system's sensitivity and dynamic range.

Figure 2 shows the post-processed spectra for a set of 420-MHz square waves riding on multi-GHz carriers as measured by our spectrum analyzer. We detected the signals on a balanced 10-MHz differential detector and recorded 50,000-point, single-shot traces on a digitizing oscilloscope. Because the start frequency of the read ECDL's chirp varies with time,

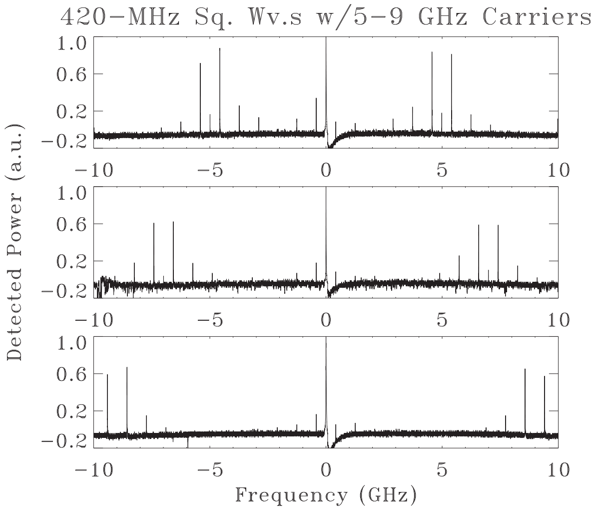

Figure 2: 420-MHz square waves on 5-, 7-, and 9-GHz RF carriers as measured by the SHB spectrum analyzer. The DC peak is clipped to facilitate viewing. The lowest sidelobes are approximately 30 dB below main peaks of the square waves.

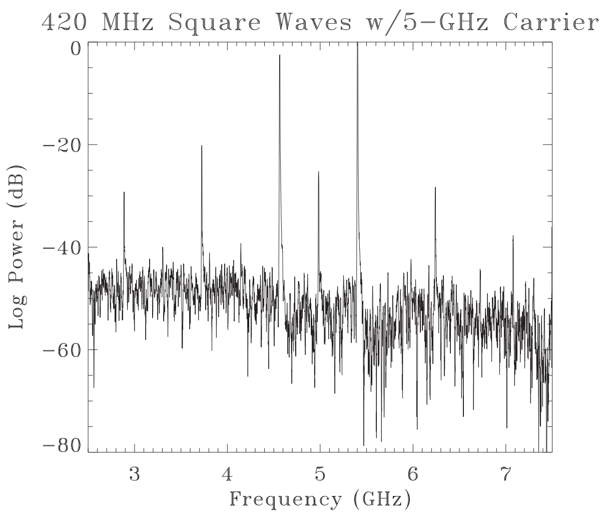

Figure 3: Close up of a 420-MHz square wave on a 5-GHz RF carriers as measured by the SHB spectrum analyzer showing dynamic range in excess of 40 dB.

we had to center the spectra about a known frequency - we used the DC peak from the EOM, so Figure 2 shows both sidebands as modulated onto the optical carrier by the EOM. After centering the spectra, we mapped the signals into the frequency domain with the distortion correction algorithm described above. The square waves were approximately -12 dBm when measured with a conventional spectrum analyzer, so Figure 3 shows that our SHB spectrum has sensitivity approaching -50 dBm, as limited by the oscilloscope bit depth.

4 Conclusion

We have successfully measured the spectra of signals with bandwidths up to 10 GHz using an SHB crystal to transform a high bandwidth signal into a low bandwidth signal suitable for processing with conventional electronics. We have demonstrated greater than 10-GHz instantaneous unity probability-of-intercept spectrum analysis with MHz resolution and spectral update times of 10 ms. We expect to further increase the system bandwidth to 20 GHz or more, while simultaneously achieved resolution well below 1 MHz are possible. In addition, we expect much larger dynamic range when operating in the non-linear saturation regime of the SHB material.

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