Soft-Resolution Method of Linguistic Hedges Lattice-Valued First-Order Logic *

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Abstract

In daily life, one usually judges a proposition using some linguistic hedges. It strengthens or weakens the degree of the truth value of the proposition. We consider the hedge operators using a qualitative method. Three kinds of qualitative values of the hedge variable with their qualitative operations are presented in this paper. Linguistic hedge of six lattice-valued first-order logic system is introduced and its soft-resolution is also presented. In the process of resolution, the hedge operators will be operated and the result with hedge operators are discussed.

Keywords: Hedge operator, Lattice-valued first-order logic, Soft-resolution

1. Introduction

It is well-known that a truth value of a traditional proposition is either true or false. In 1970s, Zadeh introduced and developed the theory of approximate reasoning based on the notions of a linguistic variable and fuzzy logic. From fuzzy logic point of view, the truth values are linguistic, e.g., of the form "true","very true","possible false", etc[1, 2]. According to Zadeh's rule for truth qualification, a proposition such as "Lucia is very young" is considered as being semantically equivalent with the proposition "Lucia is young is very true". In [3, 4] Linguistic hedges are defined as unary operators on fuzzy sets.
Lattice-valued logic system [5, 8] is an important case of multi-valued logic. It can be used to describe uncertain information that may be comparable or incomparable. Since resolution principle was introduced by Robinson [6] in 1965, automated reasoning based on the resolution principle has been extensively studied [7, 9]. Based on the above work, we deal with linguistic hedges using a qualitative method. The linguistic hedge latticed-valued first-order logic is introduced and its resolution method is discussed.

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2. Linguistic Hedges Lattice-Valued First-order Logic

2.1. Operations of the qualitative hedges

The set of linguistic hedges is denoted by $H = \{$*a little, a bit, slightly, some, almost, about, very, much, quite, highly, greatly, ...*$\}$. We analyze the hedges with a qualitative method. A hedge is called *strengthen operator* if it can increase the degree of the truth value of the proposition, such as *very*, *quite* and so on. Conversely, a hedge is called *weaken operator* if it can decrease the degree of the truth value of the proposition such as *a little*, *a bit* and so on.

Qualitative hedge variable set is obtained from the partition of hedge operator set due to the effect of the hedge operators to the proposition. We can get three classes of the hedge operators: strengthen operators, none, weaken operators. Denote the set of qualitative values by symbol $H = \{h^+, 0, h^-\}$. Let h be a qualitative hedge variable, its qualitative value $[h]$ is defined as follows:

$$[h] = \begin{cases} h^+ & \text{if } h \text{ is a strengthen operator,} \\ 0 & \text{if } h \text{ has no effect to the truth value} \\ & \text{or there is not hedge,} \\ h^- & \text{if } h \text{ is a weaken operator.} \end{cases}$$

There are three kinds of operations for the qualitative hedge. The operation $\oplus$ of qualitative hedge value is defined in Table 1.

Table 1: Operation $\oplus$ of qualitative hedge value

$\oplus$	h^+	0	h^-
h^+	h^+	h^+	0
0	h^+	0	h^-
h^-	0	h^-	h^-

The operation $\otimes$ of qualitative hedge value is defined in Table 2.

Table 2: Operation $\otimes$ of qualitative hedge value

$\otimes$	h^+	0	h^-
h^+	h^+	0	h^-
0	0	0	0
h^-	h^-	0	h^+

The qualitative value of negative of a linguistic hedge x, denoted it by $[x]'$, is defined in Table 3.

Table 3: Negative operation of qualitative hedge value

$[x]$	$[x]'$
h^+	h^-
0	0
h^-	h^+

When propositions do the operation conjunctive, disjunctive and negative their linguistic hedges should do the operation of $\oplus$, $\otimes$, $'$ respectively.

2.2. Linguistic hedges lattice-valued first-order logic system

Definition 2.1[5] Let $(L, \vee, \wedge, \prime, O, I)$ be a bounded lattice with universal boundaries O(the least element) and I(the greatest element) respectively, and "$\prime$" an order-reversing involution. If a mapping $\rightarrow$: $L \times L \rightarrow L$ satisfies for any x, y, $z \in L$,

(I_1) $x \rightarrow (y \rightarrow z) = y \rightarrow (x \rightarrow z)$,
(I_2) $x \rightarrow x = I$,
(I_3) $x \rightarrow y = y' \rightarrow x'$,
(I_4) if $x \rightarrow y = y \rightarrow x = I$, then $x = y$,
(I_5) $(x \rightarrow y) \rightarrow y = (y \rightarrow x) \rightarrow x$,
(I_6) $(x \vee y) \rightarrow z = (x \rightarrow z) \wedge (y \rightarrow z)$,
(I_7) $(x \wedge y) \rightarrow z = (x \rightarrow z) \vee (y \rightarrow z)$,

then $(L, \vee, \wedge, \prime, \rightarrow, O, I)$ is called a lattice implication algebra.

Let $P(x)$ be a predicate without any hedges such as "x is young". If there exists a hedge in the sentence such as "x is very young", put the hedge on the left of the predicate, denote it by $h^+TP(x)$. It is considered as being semantically equivalent to the proposition "x is young is very true".

When a hedge is added to the sentence $P(x)$, the truth value $V(P)$ of P will be strengthened or weakened, denoted it by $HV(P)$. Hence the truth value set is $V = \{h^+T, 0T, h^-T, h^+F, 0F, h^-F\}$, which represents *strong true, true, weak true, weak false, false and strong false.*

Let L=(V, $\vee$, $\wedge$, $'$, $\rightarrow$), its operation "$\vee''$ and "$\wedge$" shown in the Hasse diagram of L defined in Figure 1 and its operations " $\longrightarrow$" and " $'$ " defined in Table 4 and 5 respectively. Then $L = (V, \vee, \wedge, ', \rightarrow)$ is a lattice implication algebra.

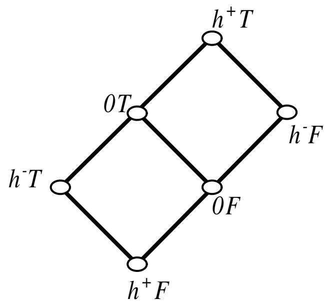

Figure 1: Hasse Diagram of V =$\{h^+T$,$0T$,h^-T, h^+F,$0F$, $h^-F\}$

We will discuss the lattice-valued first-order logic system with hedge operators (abbreviated to HLF).

Definition 2.2 Let P be a symbol of an atom, $h \in H$, hP is called a HLF atom. HLF atom is the fundamental element of HLF.

Table 4: Implication Operator of $L = (V, \vee, \wedge, ', \rightarrow)$

$\rightarrow$	h^+F	F	h^-F	h^-T	T	h^+T
h^+F	h^+T	h^+T	h^+T	h^+T	h^+T	h^+T
hF	hT	h^+T	hT	hT	h^+T	h^+T
h^-F	h^-T	hT	h^+T	h^-T	hT	h^+T
h^-T	h^-F	h^-F	h^-F	h^+T	h^+T	h^+T
hT	hF	h^-F	h^-F	hT	h^+T	h^+T
h^+T	h^+F	hF	h^-F	h^-T	hT	h^+T

Table 5: Complementary Operator of $L = (V, \vee, \wedge,', \rightarrow)$

v	h^+F	hF	h^-F	hT	h^+T
v	h^+T	hT	h^-T	hF	h^+F

Definition 2.3 The formula of HLF is defined recursively as follows
(1) HLF atom hP is a formula;
(2) If G is a formula, then (hG),$(\neg G)$are formulae, $h \in H$;
(3) If G, H are HLF formulae, then G, $(G \vee H)$, $(G \wedge H)$, $(G \longrightarrow H)$ and $(G \longleftrightarrow H)$ are formulas;
(4) If G is a formula and x is a free variable of G, then $(\forall x)G(x)$, $(\exists x)G(x)$, $(h\forall x)G(x)$, $(h\exists x)G(x))$ are formulae, where $h \in H$;
(5) All formulae are symbolic strings which use (1) to (4) finite times.

Proposition 2.1 For any $P(x)$,$Q(x) \in HLF$, the following properties holds good:
(1) $\neg(hP) \equiv h'P$;
(2) $\neg(hG) \equiv h(\neg G)$;
(3) $h(G \vee H) \equiv hG \vee hH$;
(4) $h(G \wedge H) \equiv hG \wedge hH$;
(5) $h((\forall x)G(x)) \equiv (\forall x)(hG(x))$;
$h((\exists x)G(x)) \equiv (\exists x)(hG(x))$;
$(h\forall x)G(x) \equiv h((\forall x)G(x))$;
$(h\exists x)G(x) \equiv h((\exists x)G(x))$;

Theorem 2.1 Let $J = \{h^+T, 0T, h^-T\}$ be a filter of L, and $hP(x)$ and $kQ(x)$ are formulae of HLF then

$$hP(x) \longrightarrow kQ(x) = \neg hP(x) \vee kQ(x)$$

3. Resolution Method of HLF

The definitions of literal and clause are the same as classical first-order logic too. We can convert a formula of HLF into a disjunctive normal form using the properties of HLF and applying Theorem 2.1 finite times.

Definition 3.1 The formula without $\exists$ is called h-skolem standard form if it is represented by a clause set S which every clause is the disjunctive of literals and every literal has the form hp.

Definition 3.2 Let $G \in HLF$, $J = \{h^+T, 0T, h^-T\}$ be a filter of HLF, the formula G is called J-satisfiable if there exists a interpretation I, such that $I(G) \in L$; G is called J-true, if for any interpretation I, such that $I(G) \in L$; G is called J-false, if for any interpretation I, such that $I(G) \notin L$.

Theorem 3.1 Let $G \in HLF$ and G^* be a Skolem standard form of G. Then G is J-false if and only if G^* is J-false.

Definition 3.3 Let xL and $y(\neg L)$ be two literals on HLF, they are called J-complementary literals and:

(1) If $x = 0T$, $y = 0T$ then xL and $y(\neg L)$ are called complementary literals;

(2) If $x = y = h^+T$ then xL and $y(\neg L)$ are called strong-complementary literals;

(3) If $x = h^+T$, $y = 0T, h^-T$ or $x = 0T, y = h^+T, h^-T$ or $x = h^-T, y = h^+T, 0T, h^-T$ then xL and $y(\neg L)$ are called weak-complementary literals.

Definition 3.4 (Soft-resolution principle) Let C_1 and C_2 be two clauses in HLF, xL and $y(\neg L)$ be disjunction terms in C_1 and C_2 respectively, where xL and $y(\neg L)$ are J-complementary literals. C_1^r and C_2^r are obtained from C_1 and C_2 which delete xL and $y(\neg L)$ respectively, $C_1^r \vee C_2^r$ is called J-resolvent of C_1 and C_2. Also

(1) If xL and $y(\neg L)$ are strong-complementary literals, then $C_1^r \vee C_2^r$ is strong-resolvent of C_1 and C_2 , denote it by $h^+T(C_1^r \vee C_2^r)$;

(2) If xL and $y(\neg L)$ are complementary literals, then $C_1^r \vee C_2^r$ is resolvent of C_1 and C_2, denoted it by $0TC^{(}r_1 \vee C_2^r)$;

(3) If xL and $y(\neg L)$ are weak-complementary literals, then $C_1^r \vee C_2^r$ is weak-resolvent of C_1 and C_2, denoted it by $h^-T(C_1^r \vee C_2^r)$.

Theorem 3.2 Let $G \in HLF$ be a formula , G is J-false if and only if there exists a deduction from G which can deduce J-null clause(denoted by $J - \square$). Here we give the resolution method of HLF:

Step 1 :Convert the formula into Skolem normal form;

Step 2 :We deal with the formula on J. If the formula is not in J then we consider its negative. Namely, if there exists $h^{+}F$, 0F, $h^{-}F$ then use Proposition 2.1 to convert it into J;

Step 3 : If there exists a J-false clause then the theorem is proved, stop; Otherwise goto Step 4;

Step 4 : Resolution of J-complementary literals, the hedge operator of resolvent is the result of the operation $\oplus$ between the former hedge.

Step 5 : If we get J-null clause, then the theorem is proved, stop. Otherwise goto Step 4.

Example If the profit of a company is high, then its manager must have ability or it must be a good enterprise and have a good year of business. Now the fact is that the profit of a company is very high; it is a little bad year of business.
Prove there is a very good manager in the company.
Proof.
A=the profit of a company is high;
B= the company has a good manager;
C= the company must be a good enterprise;
D=there is a good year of business.
The fact is $h^{+}A$, $h^{-}T(\neg D)$, we will prove $h^{+}TB$.
We can use the formulae to represent the information above:

$$\begin{aligned}
&P_1 : A \longrightarrow (B \vee (C \wedge D)) \\
&= \neg A \vee (B \vee (C \wedge D)) \\
&= \neg A \vee ((B \vee C) \wedge (B \vee D)) \\
&= (\neg A \vee B \vee C) \wedge (\neg A \vee B \vee D)) \\
&P_2 : A \\
&P_3 : \neg D \\
&P_4 : \neg B
\end{aligned}$$

Then this clause set holds:

(1) $h^{+}T(\neg A) \vee h^{+}TB \vee ?C$

(2) $h^{+}T(\neg A) \vee h^{+}TB \vee h^{-}D$

(3) $h^{+}TA$

(4) $h^{-}T(\neg D)$

(5) $h^{+}T(\neg B)$

(6) $((h^{+} \oplus h^{+})TB) \vee ((h^{+} \oplus h^{-})TD) = h^{+}TB \vee 0TD$ ((2) and (3))

(7) $(h^{-} \oplus h^{+})TB = 0TB$ ((4) and (6))

(8) $(h^{+} \oplus 0)T\Box = h^{+}T\Box$ ((5) and (7)

4. Conclusions

We discussed Linguistic Hedge of six lattice-valued first order logic. A resolution method is introduced which can deal with linguistic hedges. It is an extension of the six lattice-valued propositional logic. It is also shown that a formula can be proved with layered-resolution, so that we can deduce the degree that the formula holds.

We considered only three qualitative hedge values in this paper. We also can partition the hedge operators into five values: *more strengthen operators, strengthen operators, none, weaken operators, more weaken operators.* Hence the truth value set is ten values. Further work would be to extend the number of hedge values to ten in HLF and then find its resolution method.

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