Fuzzy Genetic Network

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Abstract

The goals of this paper are to introduce Fuzzy Genetic Network by means of SORE (Self Organizable and Regulating Engine) structure into the biocontrol system research community, to describe how it works and to explain why it could be a successful basic building block for a biocontrol system.

Keywords: Fuzzy logic , genetic network, Boolean network, fuzzy genetic network., modal logic

1. Introduction

In this paper we extend the traditional Boolean network used to study the behaviour of the genes and we introduce a special abstract model to know all possible behaviours and to solve the reverse engineering problem. The fuzzy genetic network is modelled by a population of genetic network with different functionality and with different initial conditions. Every gene expression is obtained by a contribution of a population of agents which give us the wanted fuzzy gene expression.

2. Specification Boolean Network

In a genetic network the total number of genes is represented by *N,* and *K* is the largest number of genes which regulates any one of the *N* genes in the genetic network. Based upon the theory of Stuart Kauffman, [12] a biological genetic network usually has a much smaller *K* for a more realistic model of a biological system. However It is in the condition $K = N$ that the network assumes its full strength for the best possible performance in a "biologizing" control system. This $K = N$ condition is what distinguishes SORE (Self Organizable and Regulating Engine)[8] from the standard BN [12].
We use SORE logic structure to introduce different levels of specification inside SORE. Given a SORE Boolean transformation as

$$T\begin{bmatrix} X \\ Y \\ Z \end{bmatrix} = \begin{bmatrix} Y \\ X+Y+Z \\ XZ \end{bmatrix} \qquad (1)$$

we can use the tautology $X + \overline{X} = 1$ to specify the transformation (1) in this way

$$T\begin{bmatrix} X \\ Y \\ Z \end{bmatrix} = \begin{bmatrix} YX + Y\overline{X} \\ XY + X\overline{Y} + Y\overline{X} + ZX + Z\overline{X} \\ XZY + XZ\overline{Y} \end{bmatrix} \qquad (2)$$

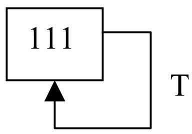
That can be written in this more specific way

$$\begin{bmatrix} YXZ + YX\overline{Z} + Y\overline{X}Z + Y\overline{X}\,\overline{Z} \\ XYZ + XY\overline{Z} + X\overline{Y}\,\overline{Z} + Y\overline{X}Z + Y\overline{X}\,\overline{Z} + ZX\overline{Y} + Z\overline{X}Y + Z\overline{X}\,\overline{Y} \\ XZY + XZ\overline{Y} \end{bmatrix}$$

When we isolate the term XYZ we have

$$T\begin{bmatrix} X \\ Y \\ Z \end{bmatrix} = \begin{bmatrix} XYZ \\ XYZ \\ XYZ \end{bmatrix}$$ so when XYZ = 1 we have

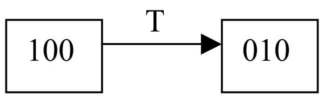
X = 1 , Y = 1 , Z = 1 , T(X) = 1,T(Y) = 1,T(Z) = 1 and we have the cycle $XYZ \to XYZ$



When we isolate $X\overline{Y}\,\overline{Z}$ we have

$$T\begin{bmatrix} X \\ Y \\ Z \end{bmatrix} = \begin{bmatrix} X\overline{Y}\,\overline{Z} \\ X\overline{Y}\,\overline{Z} \\ 0 \end{bmatrix}$$ so when $X\overline{Y}\,\overline{Z} = 1$ we have

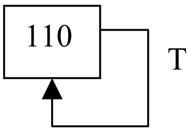
X = 1 , Y = 0 , Z = 0 ,T(X) = 0,T(Y) = 1,T(Z) = 0 and we have the transformation $X\overline{Y}\,\overline{Z} \to \overline{X}Y\overline{Z}$



When we isolate $XY\bar{Z}$ we have

$$T\begin{bmatrix} X \\ Y \\ Z \end{bmatrix} = \begin{bmatrix} XY\bar{Z} \\ XY\bar{Z} \\ 0 \end{bmatrix}$$ so when $XY\bar{Z} = 1$ we have

X = 1 , Y = 1 , Z = 0 ,T(X) = 1,T(Y) = 1,T(Z) = 0 and we have the cycle $XY\bar{Z} \to XY\bar{Z}$



So we have the possible transitions

$$\bar{X}\bar{Y}\bar{Z} \to \bar{X}\bar{Y}\bar{Z},\ X\bar{Y}\bar{Z} \to \bar{X}Y\bar{Z},$$
$$\bar{X}Y\bar{Z} \to X\bar{Y}\bar{Z},\ XY\bar{Z} \to XY\bar{Z}$$
$$\bar{X}\bar{Y}Z \to \bar{X}Y\bar{Z},\ X\bar{Y}Z \to \bar{X}YZ,$$
$$\bar{X}YZ \to XY\bar{Z},\ XYZ \to XYZ$$

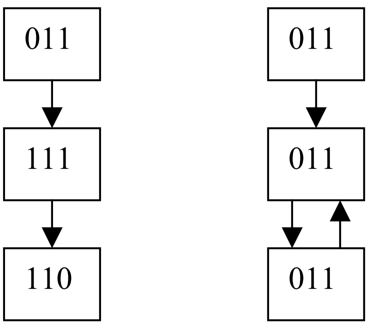
and the transition matrix M

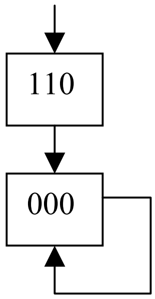
$$M = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

with the product of the matrix M we can determine the cycles of the SORE.[9] ,[10].[11]. In fact we have $M^3 = M^5$, $M^2 = M^4$.

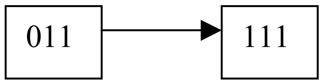
3. Reverse Engineering Problem

Given the Boolean network N_1





Because we have



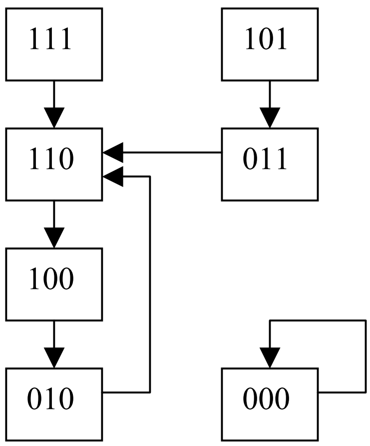
In the transformation that represents the network N_1 we have

$$T\begin{bmatrix} X \\ Y \\ Z \end{bmatrix} = \begin{bmatrix} \bar{X}YZ \\ \bar{X}YZ \\ \bar{X}YZ \end{bmatrix}$$

when we repeat the same process for all the network N_1 we obtain

$$T_1\begin{bmatrix} X \\ Y \\ Z \end{bmatrix} = \begin{bmatrix} \bar{X}YZ + XYZ + XY\bar{Z} + \bar{X}Y\bar{Z} \\ \bar{X}YZ + XYZ + \bar{X}\bar{Y}Z + X\bar{Y}Z \\ \bar{X}YZ + \bar{X}Y\bar{Z} \end{bmatrix} = \begin{bmatrix} Y \\ Z \\ \bar{X}Y \end{bmatrix}$$

Given the Boolean network N_2



We solve the Reverse Engineering problem with the position of the terms in this way

$$T_1\begin{bmatrix} X \\ Y \\ Z \end{bmatrix} = \begin{bmatrix} X_1 \\ Y_1 \\ Z_1 \end{bmatrix} = \begin{bmatrix} XYZ + XY\bar{Z} + \bar{X}Y\bar{Z} + \bar{X}YZ \\ XYZ + X\bar{Y}\bar{Z} + \bar{X}Y\bar{Z} + X\bar{Y}Z + \bar{X}YZ \\ X\bar{Y}Z \end{bmatrix}$$

When we fuse the two networks into one we obtain

$$T_{1,2}\begin{bmatrix} X \\ Y \\ Z \end{bmatrix} = \begin{bmatrix} \overline{X}YZN_1 + \overline{X}YZN_2 + XYZ + XY\overline{Z} + \overline{X}Y\overline{Z}N_1 + \overline{X}Y\overline{Z}N_2 \\ XYZ + X\overline{Y}\overline{Z} + \overline{X}\overline{Y}\overline{Z} + X\overline{Y}Z + \overline{X}YZN_1 + \overline{X}YZN_2 + \overline{X}\overline{Y}Z \\ \overline{X}YZN_1 + \overline{X}Y\overline{Z} + X\overline{Y}Z + \overline{X}Y\overline{Z}N_2 \end{bmatrix}$$

where N_1 and N_2 with $N_1+N_2 = 1$ are Boolean variables the values of which solve the conflicting situation in the union of the two Boolean networks N_1 and N_2 . So when we have two transformations from the initial state , we use N_1 and N_2 to make a decision. So when $N_1 = 1$ and $N_2 = 0$, we use the transformation in N_1 , when $N_2 = 0$ and $N_1 = 1$, we choose the second Boolean Network.

4. Fuzzy Boolean Network

In the previous chapters every variable represents a gene in the genoma . We know that the value 1 of the gene X means that the gene is active and the value 0 means that the gene is not active. Because it is not realistic that the gene assumes only two possible values, for any gene X we have a Boolean vector $X = (X_1 , X_2 ,\ldots, X_p)$ of 1 and 0. Every component of the vector X is a different modality of the expression of the gene. Modality that can be expressed as a possible world in the modal logic [1]. Among the vectors we assume that there are these ordinary vector operations.

$$(X_1,X_2,\ldots,X_p)(Y_1,Y_2,\ldots,Y_p) = (Y_1X_1,Y_2X_2,\ldots,Y_pX_p)$$
$$(X_1,X_2,\ldots,X_p)+(Y_1,Y_2,\ldots,Y_p) = (Y_1+X_1,Y_2+X_2,\ldots,Y_p+X_p)$$
$$\overline{(X_1,X_2,\ldots,X_p)} = (\overline{X_1},\overline{X_2},\ldots,\overline{X_p})$$

That are the vector product , the vector sum and the vector negation. For any vector we can compute the membership value

$$\mu(X) = \frac{m_1X_1 + m_2X_2 + \ldots.. + m_nX_n}{m_1 + m_2 + \ldots + m_n}$$

where μ is the membership function and m_k k = 1 ,.., p are the weights of any component in the Boolean vector. As we introduce for the first time the interpretation of the μ by the modal logic [2].[3].[4].,[5] we can show that

when $\mu(Y) \le \mu(X)$ we have

$$\mu(Y_1X_1,\ldots,Y_pX_p) = \min[\mu(X),\mu(Y)] - \mu[\overline{X}Y]$$
$$\mu(Y_1+X_1,\ldots,Y_p+X_p) = \max[\mu(X),\mu(Y)] + \mu[\overline{X}Y]$$

because in the fuzzy set for the Zadeh rule we have

$$\mu[X\overline{X}] = \min[\mu(X),\mu(\overline{X})] > 0$$
$$\mu[X+\overline{X}] = \max[\mu(X),\mu(\overline{X})] < 1$$

the negation is not compatible with the other operations. So in the decomposition of the vector

$$\overline{(X_1,X_2,\ldots,X_p)} = (\widetilde{X_1},\widetilde{X_2},\ldots..,\widetilde{X_p})$$

the negation $\overline{X_k}$ is substituted by the negation

$\tilde{X} = \overline{X_k}\,\overline{S_k} + X_kS_k$ for which $\tilde{\tilde{X}} = X$.

we remark that we introduce a new logic variable S_k. The vector $S = (S_1,S_2,\ldots,S_p)$ is the inconsistent vector for which the Tautology is not always true and the contradiction is not always false as in the Zadeh rule. In fact we have for the contradiction expression

$$C = \tilde{X}X = (\overline{X_k}\,\overline{S_k} + X_kS_k)X_k = X_kS_k$$

where C is not always equal to zero.

When $S = (S_1,S_2,\ldots,S_p) = (0,0,..,0)$ we come back to the classical negation and $C_k = 0$ is always true. For the tautology we have

$$T = \tilde{X} + X = (\overline{X_k}\,\overline{S_k} + X_kS_k) + X_k = \overline{X_k}\,\overline{S_k} + X_k$$

that is not always equal to one. In fact when $S_k = 1$ and $X_k = 0$, we have that $T_k = 0$.
For example the transformation

$$T\begin{bmatrix} X \\ Y \end{bmatrix} = \begin{bmatrix} Y \\ \overline{X} \end{bmatrix} = \begin{bmatrix} YX + Y\overline{X} \\ \overline{X}Y + \overline{X}\,\overline{Y} \end{bmatrix} \quad \text{and}$$

$$T\begin{bmatrix} X \\ Y \end{bmatrix} = \begin{bmatrix} Y \\ \overline{X} \end{bmatrix} = \begin{bmatrix} YX \\ 0 \end{bmatrix} + \begin{bmatrix} Y\overline{X} \\ Y\overline{X} \end{bmatrix} + \begin{bmatrix} 0 \\ \overline{X}\,\overline{Y} \end{bmatrix} \quad \text{and we}$$

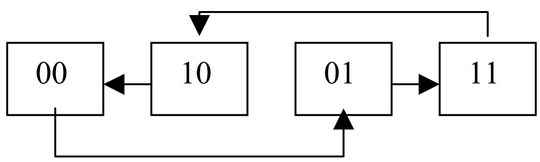
have the transformations

$$\overline{X}\,\overline{Y} \to \overline{X}Y, X\overline{Y} \to \overline{X}\,\overline{Y}, \overline{X}Y \to XY, XY \to X\overline{Y}$$

the transition matrix is

$$M = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \end{bmatrix}$$

for which we have $M^4 = M$, so we have the cycle.



For the vector representation the transformation T becomes

$$T\begin{bmatrix}(X_1,X_2,...,X_p)\\(Y_1,Y_2,...,Y_p)\end{bmatrix}=\begin{bmatrix}Y_k\\ \widetilde{X}_k\end{bmatrix}=\begin{bmatrix}Y_k\\ \overline{X}_k\overline{S}_k+X_kS_k\end{bmatrix}$$

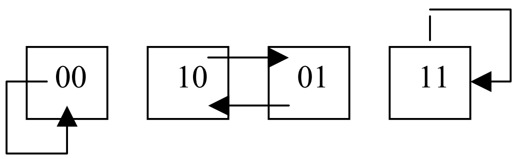
When $S_k = 0$ we have

$$T_1\begin{bmatrix}X_k\\Y_k\end{bmatrix}=\begin{bmatrix}Y_k\\ \overline{X}_k\end{bmatrix}\quad k = 1,2,..n$$

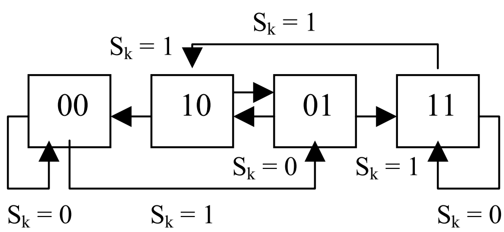
When $S_k = 1$ we have

$$T_2\begin{bmatrix}X_k\\Y_k\end{bmatrix}=\begin{bmatrix}Y_k\\X_k\end{bmatrix}\quad k = 1,2,..n$$

and its transition diagram is



In the fuzzy system in every component of X or Y we have the complex transition diagram or system where S is the input and the (X ,Y) are the states.



The fuzzy logic opens the genetic network and introduces the input S_k for every component of the vector. So if $S_k = 0$ or $S_k=1$ for every k and in every time we have n biological machines (Boolean network) with different initial conditions. But when in different k we have different values of S_k , we have a population of Boolean networks (agents) that synchronically can assume the operator T_1 or the operator T_2 with different initial conditions.

5. Conclusion

In this paper we give the logic and mathematical model to study the fuzzy genetic network . We introduce abstract matrices M to find the behaviour of the genetic network. The fuzzy genetic network is modelled by a population of simple Boolean networks one for every component . The collective contribution of parallel and synchronic agents with different behaviour and initial conditions gives us the membership functions together with the traditional fuzzy logic rules as t-norm and t-conorm.

6. References

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