A Web-based Decision Support System for Linear Bilevel Multi-follower Problems without Shared Variables

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Abstract

Advanced web and database technology support remote data access and communication. It has opened new opportunities for decision support system builders to develop web-based DSS. This paper presents a web-based decision support system for linear bilevel multi-follower problems without shared variables. An extended Kuhn-Tucker method for linear bilevel multi-follower programming is implemented in this system. It can compute a global optimal solution for linear bilevel multi-follower programming problems without shared variables.

Keyword: Web intelligent, web and database technology, linear bilevel multi-follower programming, extended branch-bound method, decision support systems

1. Introduction

Information system researchers and technologists have built and investigated Decision Support Systems (DSS) for more than 35 years [1]. Web technology and database technology allow organizations to make decisions in a distributed environment that supports remote data access and communication. It also has opened new opportunities for DSS builders to develop network based DSS in the web [2]. Since the advance of Internet technology, which allows users fast and inexpensive access to an unprecedented amount of information provided by websites, web-based DSS have been proposed to extend the applications of traditional DSS to a global environment with a unified web platform. The computer facilities for utilizing DSS have been moving towards a more widespread use of the Internet with its graphical user interface, the web [3, 4].

Bilevel programming (BLP) was motivated by the game theory of Von Stackelberg [5, 6] in the context of unbalanced economic markets. The majority of research on BLP has centered on the linear version of the problem. There have been nearly two dozen algorithms, such as, the *K*-th best approach [7, 8], Kuhn-Tucker approach [9, 10, 11], complementarity pivot approach [12], penalty function approach [13, 14], proposed for solving linear BLP problems since the field being caught the attention of researchers in the mid-1970s.

Our previous work presented a new definition of solution and related theorem for linear BLP, thus overcame the fundamental deficiency of existing linear BLP theory [15]. We also described theoretical properties of linear BLP, developed an extended *K*th-best approach for linear BLP [16], an extended Kuhn-Tucker approach and its algorithm for linear BLP [17, 18]. In [19], we proposed a model and Kuhn-Tucker approach for linear bilevel multi-follower programming (BLMFP) problems in which there are not shared variables among followers except leader's variables. We addressed theoretical properties of linear BLMFP, developed a *K*th-best approach for linear BLMFP [20] and presented an extended branch and bound algorithm for linear BLMFP [21]. This paper proposes a web-based linear bilevel decision support system (WLBLDSS). Following the introduction, this paper overviews the extended Kuhn-Tucker method in Section 2. WLBLDSS architecture and implementation are addressed in Section 3. A conclusion and future work are given in Section 4.

2. The Extended Kuhn-Tucker Method for linear BLMFP

For $x \in X \subset R^n, y_i \in Y_i \subset R^{m_i}, f_i : X \times Y_i \to R^1$, $F : X \times Y_1 \times \ldots \times Y_K \to R^1$, and $i = 1,2,\ldots,K$, a linear BLMFP problem in which $K(K \geq 2)$ followers are involved and there is no shared information among them except the leader's is given [19]:

$$\min_{x \in X} F(x, y_1, \ldots, y_K) = cx + \sum_{s=1}^{K} d_s y_s \quad (1a)$$

subject to $Ax + \sum_{s=1}^{K} B_s y_s \le b$ (1b)

$$\min_{y_i \in Y_i} f_i(x, y_i) = c_i x + e_i y_i \quad (1c)$$

subject to $A_i x + C_i y_i \le b_i$, (1d)

where $c \in R^n$, $c_i \in R^n$, $d_i \in R^{m_i}$, $e_i \in R^{m_i}$, $b \in R^p$, $b_i \in R^{q_i}$, $A \in R^{p \times n}$, $B_i \in R^{p \times m_i}$, $A_i \in R^{q_i \times n}$, $C_i \in R^{q_i \times m_i}$, $i = 1,2,\ldots,K$.

We also presented and proved the following theorem to characterize the condition under which there is an optimal solution for a linear BLMFP problem in [19].

Let $u_i \in R^p$, $v_i \in R^{q_i}$ and $w_i \in R^{m_i}$ $(i = 1,\ldots,K)$ be the dual variables associated with constraints (1b), (1d) and $y_i \ge 0$ $(i = 1,\ldots,K)$, respectively. We presented and proved the following theorem in [19].

Theorem 1 A necessary and sufficient condition that $(x^*, y_1^*, \ldots, y_K^*)$ solves the linear BLMFP problem (1) is that there exist (row) vectors , $v_1^*, v_2^*, \ldots, v_K^*$ and $w_1^*, w_2^*, \ldots, w_K^*$ such that $(x^*, y_1^*, \ldots, y_K^*, u_1^*, \ldots, u_K^*, v_1^*, \ldots, v_K^*, w_1^*, \ldots, w_K^*)$ solves:

$$\min F(x, y_1, \ldots, y_K) = cx + \sum_{s=1}^{K} d_s y_s \quad (2a)$$

subject to $Ax + \sum_{s=1}^{K} B_s y_s \le b$ (2b)

$$A_j x + C_j y_j \le b_j, \ j = 1,\ldots,K \quad (2c)$$

$$u_i B_i + v_i C_i - w_i = -e_i, \ i = 1,2\ldots,K \quad (2d)$$

$$u_i(b - Ax - \sum_{s=1}^{K} B_s y_s) + v_i(b_i - A_i x - C_i y_i) + w_i y_i = 0, \ i = 1,2\ldots,K \quad (2e)$$

$$x \ge 0, y_j \ge 0, u_j \ge 0, v_j \ge 0, w_j \ge 0, \ j = 1,2,\ldots,K. \quad (2f)$$

3. WLBLDSS Architecture and Implementation

3.1. WLBLDSS Architecture

Figure 1 shows a framework of WLBLDSS. It is consists of a web server which resides a method base, a database sever which interacts with the web sever using an ODBC connection and clients which access the web server through Internet. The web sever manages the system and provides dynamic services.

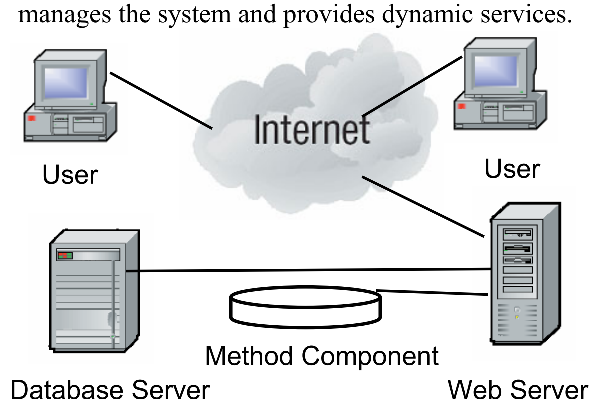

Fig.1: Architecture of WLBLDSS

3.2. Client Side Application Design

The interface design represents the central component for successful information delivery between a user and WLBLDSS. Interface design for web-based information systems is probably more a science than an art. We integrate intelligent knowledge about BLP together with advanced Javascript technology to design efficient. For example, client side application can guide a user to correctly set a decision problem data by using client-side data application software to validate user's form.

3.3. Sever Side Application Design

The web server (IIS 4.0) manages all the pages, traces user information and provides simultaneously services to multiple users through sessions, applications and coking facilities. Using the server side application program, the web server can manage and implement client tasks. For example, one user wants to play the WLBLDSS using a stored linear BLP decision problem data, the whole process is described as the follows:

1) The user logins the WLBLDSS by filling personal information form.
2) The web sever asks the database sever to confirm the user's information.
3) She/he sends the web server a decision task that uses a stored linear BLP decision problem, after being authorized to login.
4) The web sever fetches stored linear BLP decision problems from the database, display all the stored linear BLP decision problems, and asks the user to select one problem.

3.4 Method Design

A web-based linear bilevel DSS intends to provide necessary computerized assistance to different decision makers and various applications. Theorem 1 provides theoretical foundation for the extended branch and bound algorithm. The basic idea of the algorithm is to suppress the complementarity term and solve the resulting linear sub-problem. At each iteration, a check is made to see if (2e) is satisfied. If so, the corresponding point is in the inducible region and hence a potential solution to (1). If not, a branch and bound scheme is used to implicitly examine all combinations of complementarity slackness.

3.5. WLBLDSS Database Design

The strategy of design the WLBLDSS database is shared data information among users. The relational data mode technology is used to database design and management of WLBLDSS. There are three main entities in WLBLDSS database: users, resource (problem) data and solution data.

3.6. Guide of WLBLDSS

The WLBLDSS works to cope with a task through the following steps.

1) A user first fills the personal information form. The WLBLDSS client application program checks the form and the web sever stores the user information into the database server.

2) The user can enter a new decision problem using the data enter page, or choose an existing (stored) decision problem. For simplification, we use the second situation to discuss the process steps.

3) The web server displays all the stored linear bilevel decision problems in the system, and the user chooses one of them. Figure 2 shows one example.

4) When a decision problem is determined, related information will be stored in the database. The web server, with application programs, will run the method

5) After one problem data is selected and confirmed, the web server gives this information to method management component and waits for an answer.
6) Once the web server gets a solution, it lets the database sever to store the solution.
7) Finally, the web server constructs a page including the solution details, and sends it to the user.

to obtain solutions and display it to the user. Figure 3 shows the results of the example.

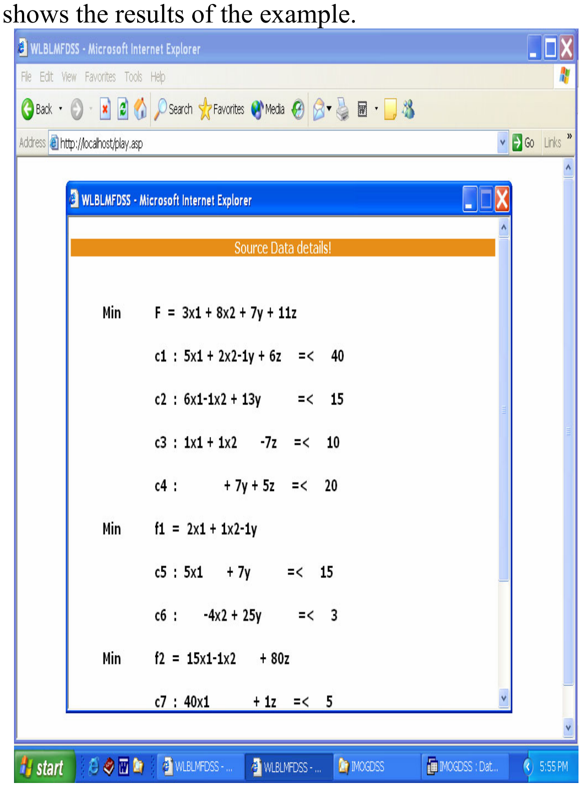

Fig. 2: Example of linear bilevel multi-follower problems

4. Conclusions and Future Work

The WLBLDSS is developed as an online decision-making tool to support linear bilevel multi-follower decision-making. The system is interactive, flexible and easy to use for various linear bilevel multi-follower decision problems. The system is expected to be applied in practical bilevel decision problems such as product planning, resource management, research project funding, and network transportation. We hope to get feedback from our users and to make the system widely available. We are intending to enhance the systems to support bilevel multi-follower decision-making with shared variables.

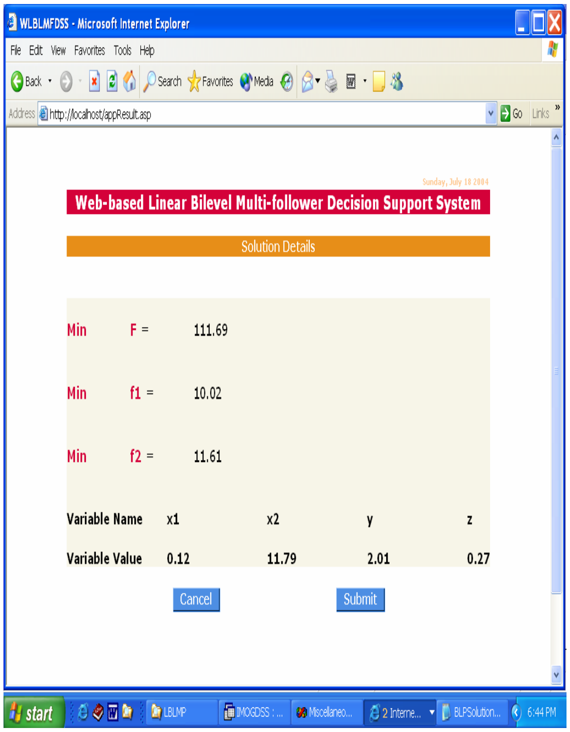

Fig. 3: Results of the example

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