Development of High Performance Cylindrical Cams Using B-Spline and Bezier Functions

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Abstract

This paper describes a computer aided design and manufacturing system for design and machining of complex profiles for high performance cylindrical cams based on the B-spline and Bezier function representation of motion curves. The system graphically generates the cam profile on a cylinder after performing an analysis of kinematic performance for the prescribed follower motion using B-spline and Bezier functions and recommends the best cam profile. The paper concludes that adopted approach used in this system provides a faster and economical method of designing and manufacturing precision cylindrical cams.

Keywords: CAD/CAM, cylindrical cams, CNC machining, B-spline curve, Bezier curve

1. Introduction

Cylindrical cams are used for transmission of motion in a wide variety of machines such as packaging machines, textile machines, elevators, indexing devices and agricultural machines. In these cams, the cylinder rotates about its longitudinal axis and provides a translational or oscillatory displacement to the follower. Requirements of high kinematic performance of such machinery demand for efficient methods for design and manufacture of such cams. Conventional methods of designing and machining a cylindrical cam profiles within a given accuracy are tedious and time consuming. Even programming them on a computer numerical control (CNC) machine could be a difficult job because of complexity of cam profiles.

There has been a great deal of research and investigation carried out in the development of an efficient design and manufacturing system for cams and cam mechanisms. While most of such works are related to the plate cams or planar cams, very little work seems to have been done for cylindrical cams. For three dimensional cams, Dhande et al [1] developed a technique of generating the profiles of planar and spatial cams based on the concept that the common normal vector and the relative velocity vector are orthogonal to each other at the point of contact between the cam and the follower. Tsay and Wei [2] developed a CAD system for determining the cylindrical-cam profile equations with a translating conical follower using the theory of envelopes for a 1-parameter family of surfaces described in parametric form. Tsay and Lin [3] presented a procedure for the synthesis and analysis of the surface geometry of cylindrical cams with oscillating roller followers using B-splines. Lin and Liu [4] developed an analytical methodology, based on Denavit-Hartenberg notation, for NC data generation to produce and measure spatial cams on 5-axis machine tools. More recently, Tsay and Ho [5] considered the manufacturing parameters in the design of 3D cams and developed analytical expressions for the turret motion and indexing accuracy of intermittent grooved globoidal cam mechanisms. Little work appears to have been done in developing an interactive CAD/CAM system for high precision NC machining of cylindrical cams considering the synthesis and smoothing of specified follower motion curves for high performance cylindrical cams, which are so common in many engineering applications.

This paper describes the application of parametric B-spline and Bezier function representation for synthesising the motion curves of cylindrical cams with the aim of increasing the kinematic performance of cylindrical cams. This will be achieved by reducing the maximum acceleration and maximum velocities encountered during the operation of cam. Based upon this approach, an interactive CAD/CAM system, called CYLCAM, has been developed that recommends the best profile of the cam for user specified motion curves and within user specified desired tolerance values. The system performs graphical simulation of designed cam motion and generates the most economical and complete CNC program for the designed and recommended cylindrical cam. The system utilises the a unique half angle search algorithm developed by Masood and Lau [6] for linear interpolation CNC machining, which has proven to generate the most economical CNC programs and accurate cam profiles compared to other interpolation schemes.

2. Cam Follower Motion Curves

The follower motion curves play an important role in the performance of the cam mechanisms. Normally, the first task in the design of a cylindrical cam is to select a suitable follower motion curve that will satisfy the constraints of the application. Here the constraints mean the relationship of the displacement of the follower and the angle of rotation. The desirable characteristics of the motion curves are that they should give a continuous velocity, continuous acceleration, low velocity and low acceleration to the follower. These may not be important when a cam is operating at low speed, but will be crucial at high speeds since it can result in high vibrations and may lead to the short life span of cam and other mechanism in the machine. In addition, a concern of the designer must be the type and the magnitude of loading to be resisted by the cam and the roller surfaces. These forces include working loads, inertia forces, impact forces and friction forces

To analyse the action of a cam it is therefore necessary to study its displacement-time diagram and its associated velocity and acceleration curves. There are many follower motions that can be used for the rises and the returns. For this research, we present the following eight traditional curves for designing cam profile.

1. Constant velocity Motion
2. Parabolic Motion
3. Simple Harmonic Motion
4. Cycloidal Motion
5. 3-4-5 Polynomial Motion
6. Modified sinusoidal Motion
7. Modified Trapezoidal Motion
8. 4-5-6-7 Polynomial Motion

3. Sysnthesis of Motion Curves

When the kinematic and dynamic performances of the cam-follower mechanisms are to be considered, the follower motion must be carefully synthesised. For example, to reduce the amplitude of the inertial force, the maximum value of the acceleration curve of the follower motion must be as small as possible. One method of improving the follower motion curves would be to represent the curves by parametric Bezier or B-spline curves.

Bezier curve uses a control polygon and is approximated by a polynomial curve, whose degree is one less than the number of polygon vertices (known as control points or track points). The Bezier control points provide an easier way of controlling the shape of the polynomial than the tangent vectors of the Hermite formulation.

Given an input set of n+1 control points p_i, with i varying from 0 to n, the points on the approximating Bezier curve are given by the expression

$$p(u) = \sum_{i=0}^{n} \frac{n!}{(n-i)!i!} u^i (1-u)^{n-i} p_i \qquad 0 \le u \le 1$$

It is to be noted that Bezier curves approximate points without, in general, passing through them (apart from the first and last in a set, as noted). Bezier curves are also only capable of being globally modified and their blending functions are non-zero in the whole range $0 < u < 1$. Therefore moving one of the points in the control polygon affects every

$$+\left[\frac{u}{2}\left(-\frac{3}{2}u^2 + 2u\right) + \frac{u^2}{6}(3-u)\right]p_2 + \frac{u^3}{6}p_3$$

position on the curve (apart from the end points).

B-splines are a class of spline curves that cannot only interpolate a given set of control points, but also allow localised modifications to be made easily without greatly affecting other parts of the curve. Another advantage of B-spline is that any number of control points can be specified by a designer without increasing the degree of the curve. A cubic curve could then be used for many different curve shapes, without the need to piece curve segments together. Any number of control points can be added or modified to manipulate the curve shape.

For B-spline representation, given an input set of n+1 control points p_i, with i varying from 0 to n, then the points on the approximating B-spline curve are given by

$$p(u) = \Sigma\, p_i\, N_{i,k}(u)$$

where $N_{i,k}(u)$ is a defined as a blending function, which is a polynomial of degree k-1. A method for setting up the polynomial form of the blending functions is to define them recursively over various subintervals of the range for parameter u. This range now depends on the number of control points n and the choice for k, so that u varies from 0 to n-k+2 (instead of 0 to 1).

In cam design, once the basic follower motion curve has been specified, it can then be approximated by B-spline or Bezier functions. The following steps illustrate the general procedure used in this study.

1. Create the basic curve of the selected follower motion (parabolic, simple harmonic, etc).
2. Divide the angle interval of the curve into five pates. Each part has the same angle interval. This will create 6 points $p_0, p_1, p_2, p_3, p_4,$ and p5 lying on the curve.
3. These six points are interpolated by considering the constraints of n=5 and k = 3 and using the parametric equations of B-spline or Bezier as given below.

B-spline equation

For $0 \le u \le 1$,

$$p_{x,y} = (1-u)^3 p_0 + \left[u(1-u)^2 + \frac{1}{2}(2-u)\left(-\frac{3}{2}u^2 + 2u\right)\right] p_1$$

For $1 \le u \le 2$,

$$p_{x,y} = \frac{1}{4}(2-u)^3 p_1 + \left[\frac{u}{4}(2-u)^2 + \frac{(3-u)}{3}\left(-u^2 + 3u - \frac{3}{2}\right)\right] p_2$$

$$+ \left[\frac{u}{3}\left(-u_2 + 3u - \frac{3}{2}\right) + \frac{1}{4}(3-u)(u-1)^2\right] p_3 + \frac{1}{4}(u-1)^3 p_4$$

For $2 \le u \le 3$,

$$p_{x,y} = \frac{1}{6}(3-u)^3 p_2 + \left[\frac{u}{6}(3-u)^2 + \frac{3-u}{2}\left(\frac{-3}{2}u^2 + 7u - \frac{15}{2}\right)\right] p_3$$

$$+\left[\left(\frac{u-1}{2}\right)\left(\frac{-3}{2}u^2 + 7u - \frac{15}{2}\right) + (3-u)(u-2)^2\right] p_4 + (u-2)^3 p_5$$

Bezier equation

For $0 \le u \le 1$,

$$p_{x,y} = (1-u)^5 p_0 + 5u(1-u)^4 p_1 + 10u^2(1-u)^3 p_2 + 10u^3(1-u)^2 p_3$$
$$+ 5u^4(1-u)p_4 + u^5 p_5$$

where p_x and p_y are the x- and y-coordinates of the points on the curve.

Using the above procedure, each motion curve is converted into an equivalent B-spline and Bezier curve. Both the new curves may look approximately the same as the former basic curve but it will have differences in the smoothness and maximum velocity and acceleration. These are the factors that affect the kinematic performance of the cams. The velocity and acceleration diagram of various cams can be examined using the CAD/CAM system, called CYLCAM, which has been developed for design, manufacturing and synthesis of cylindrical cams.

For machining the cam on a CNC machine, the cutter location data required to machine the specified cam profile should be minimised. This will result in a shorter programming time and more economical CNC programs. The system calculates the cutter location data required for the cam profile or the basis of the basic curve as well as the interpolated B-spline or Bezier curve.

4. The CYLCAM System

The CYLCAM system for cylindrical cam is an interactive program written in Visual Basic 6.0 programming language using a graphical user interface. The program requires the user to specify the cam speed, the desired tolerance on cam profile, the type of basic motion curves for each segment of motion curve (from the given list), the direction of follower (lift/fall), and angle interval for each motion segment. After executing, the system then provides the comparative mechanical performance results of three different profiles based on Normal, B-spline, and Bezier parametric equation and recommend the best cam profile.

The program is designed to be highly interactive with graphic screens, buttons and menus (not shown in this paper due to space constraints). The main features of the program are as follows:

- Cam profile based on user specified tolerance
- Choice of three cam profiles based on normal, B-spline, Bezier functions
- Recommendation of best profile based on kinematic performance
- Graphical display of profile for each cam
- Animated graphical simulation of designed cam
- Graphical and table data display of each cam (displacement, velocity, acceleration diagram)
- Graphical comparison of maximum values of displacement, velocity, and acceleration for cam profiles
- Graphical comparison of displacement, velocity, acceleration diagram for three cam profiles
- Cutter Location data table for each cam profile
- Graphical comparison of NC cutter points for the three cam profiles.
- Generation of complete CNC program for each cam profile.

For CNC part program generation, the system employs the half angle search algorithm [6]. This provides a systematic searching method to determine the minimum number of NC points required for machining the cam profile to within the user specified tolerance.

As an example of using the CYLCAM system, the input data for design of a cylindrical cam with follower motion consisting of two rise segments, a dwell, and two return segments are shown in Table I. For this example, the cam diameter is 100mm, cam speed is 80 rpm, cam groove depth is 3 mm, and desired tolerance is 0.03 mm. The results of the desired profile from the CYLCAM system are shown in Table II.

Table I Input Data for Cylindrical Cam

Motion Segment	Height (cm)	Interval (degrees)
Rise1 Parabolic	10	0-120
Rise2 SHM	10	120-210
Dwell	20	210-260
Return 1 Modified Sine	-10	260-310
Return 2 3-4-5 Poly	-10	310-360

For this example, as shown in Table II, the CYLCAM system recommended the best motion profile to be Bezier profile for lowest maximum velocity and B-spline profile for lowest maximum acceleration. The number of cutter location NC points required for each profile is also shown.

Table II Results summary of desired profile

Compared Values	Normal Profile	B-spline Profile	Bezier Profile
Maximum displacement	20 mm	17.10	13.74
Maximum velocity	179.98 m/sec	97.75	64.42
Maximum acceleration	5319.8 m/sec^2	602.22	618.98
Cutter location Points (NC)	251	60	36

Using the CYLCAM system, the CNC programs were generated for each of the cam profiles based on the basic curve, the B-spline modified curve and the Bezier modified curve. For this exercise, the input cam data used were: cam diameter 35 mm, cam speed 60 rpm, cam groove depth 6 mm and desired tolerance 0.01 mm. The follower motion used a rise-dwell-return profile with parabolic motion curve for rise and modified sine motion curve for return motion. The programs were run on a Mitsui Seiki horizontal machining centre using aluminium cylindrical blocks. Figure 1 shows the three machined cylindrical cams. The CNC machining parameters used were: spindle speed 2000 rpm, feed rate 100 mm/min, and the tool diameter was 6 mm. The CYLCAM system also recommends the length of the cylindrical cam based on lowest acceleration and lowest velocity. In this case the recommended length of cam was 40 mm for the lowest acceleration and the recommended length was 38 mm for lowest velocity.

5. Conclusions

The CYLCAM system presented in this paper provides the designer with the capability of generating a cam profile defined by a set of several follower motion curves and improving the cam profile by parametric function representation. Application of the parametric equations has demonstrated marked improvement in the cam profile design and the cam performance. As a result, the velocity and the acceleration peak values have been improved. Results have shown that both Bezier and B-spline replacement of follower motion curves give better cam performance over the traditional method. The CYLCAM system also provides the features of producing the cams within user specified desired tolerance values and generating the most economical CNC part program using the half angle search algorithm.

Figure 1 Three cylindrical cams machined with CNC programs generated by CYLCAM

6. References

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