The Integrated 3D Geometric Modeling for Woven Fabrics

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Abstract

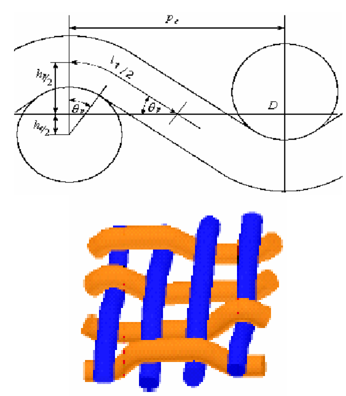
Taking the advantage of the integrated 3D geometric modeling, we developed a realistic physically based model covering different structures typical for woven fabrics. This paper summarily presents the relevant processing and methods of the geometric modeling for representations of fabric topological and geometrical structures, irregular yarn cross-sections and yarn paths in fabrics. This close-to-real model can be used to research, design, manufacture and wide applications especially for textile computer visualization, yarn consumptive estimation, calculation of textile physical parameters and mechanical analysis. Also this paper gave the conceptions of principal and secondary points with regard to fabric geometric structure and fabric modeling. By the new conception, the geometric modeling approach for fabric mentioned in this paper can be applied to other fields such as fabric/cloth drape deformation modeling and simulating.

Keywords: geometric modeling, virtual reality, woven fabric, computer graphics, computational geometry, topological structure, CAD.

1. Introduction

Textiles themselves are main material for apparel, decoration, household, etc. They also are important reinforcements for composites. Therefore, textile geometrical models are not only for textile visualization and estimating yarn consumption. The correct and realistic textiles geometrical models are quite important to calculate, predict and analysis physical parameters and mechanical properties of textiles and relevant composites for their research, design, manufacture and application. In this paper we focus on woven fabrics.

Many approaches and systems have been developed to represent the geometrical models of textiles, such as [1], [2], etc. But most of works consecutively adopted Peirce's assumptions set out in [3]. Those non-physically-based models have been widely applied to textiles and their applied field up to now for basic textile visualization and rough calculation of physical parameters. But those are not satisfied for technical requirements. Especially, in all of the models, yarns were compelled as columns with a uniform regular cross-sectional shape (circular, or lenticular, racetrack, etc) along its yarn path. The yarn path followed the cross-sectional shape of the other interlaced yarn at interlacement, or the yarn taken the shortest path: the straight-line segment between two adjacent interlacements. Fig. 1a showed the original Peirce's circular model and Fig. 1b represented a 2/2 fabric geometric model with circular cross-sectional shape for both warp and weft yarns based on Peirce's assumption. Experiments have shown that Peirce's assumption is not true. The effective physically based model is expected.



b.

Fig. 1: Peirce's geometric model and an existing model.

Woven fabrics consist of yarns with different complex textile structure. Therefore, the geometric modeling of textiles mainly concerns the representation of fabric structures, yarn cross-sections and yarn paths in fabrics. The integrated 3D geometric modelling is expected to cover different textile topological and geometric structures and to represent variable (including irregular) yarn cross-section and physically based yarn path. For these, we developed a comprehensive approach to perform the requirement above. The relevant software to support the approach has been created in [4]. This paper summarily explains the relevant processing and methods of geometric modeling, including:

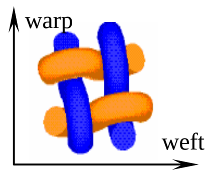
- The integrated model of fabric structure.
- Principal and secondary points
- Physically based yarn path.
- Irregular yarn cross-section.

- The creating of yarn.

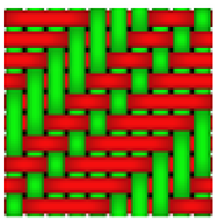
2. Fabric Structure

Topologic structure

The variation of topological structure of yarns interlacing in fabric leads different textile structures to optimize the performance of textiles themselves and relevant composites. X. Chen created an integrated matrix model to cover different textile topological structures in [5]. For 2D woven fabrics, it can be explained simply by examples in Fig 2.



$$\begin{bmatrix} -1 & 1 \\ 1 & -1 \end{bmatrix}$$



$$\begin{bmatrix} -1 & -1 & -1 & -1 & 1 & 1 & 1 & -1 & -1 & 1 \\ -1 & -1 & -1 & 1 & 1 & 1 & -1 & -1 & 1 & -1 \\ -1 & -1 & 1 & 1 & 1 & -1 & -1 & 1 & -1 & -1 \\ -1 & 1 & 1 & 1 & -1 & -1 & 1 & -1 & -1 & -1 \\ 1 & 1 & 1 & -1 & -1 & 1 & -1 & -1 & -1 & -1 \\ 1 & 1 & -1 & -1 & 1 & -1 & -1 & -1 & -1 & 1 \\ 1 & -1 & -1 & 1 & -1 & -1 & -1 & -1 & 1 & 1 \\ -1 & -1 & 1 & -1 & -1 & -1 & -1 & 1 & 1 & 1 \\ -1 & 1 & -1 & -1 & -1 & -1 & 1 & 1 & 1 & -1 \\ 1 & -1 & -1 & -1 & -1 & 1 & 1 & 1 & -1 & -1 \end{bmatrix}$$

Fig. 2: Matrix expression of fabric topologic structure.

Geometric structure

The binary matrix above for topologic structure can be extended to the general vector matrix H for geometric structure ([6]). The geometric structure model relating to weft yarns can be described by following vector matrix,

$$H_f = \begin{bmatrix} \vec{h}_{R_f,1} & \vec{h}_{R_f,2} & \cdots & \vec{h}_{R_f,i} & \cdots & \vec{h}_{R_f,R_w} \\ \vdots & \vdots & & \vdots & & \vdots \\ \vec{h}_{j,1} & \vec{h}_{j,2} & \cdots & \vec{h}_{j,i} & \cdots & \vec{h}_{j,R_w} \\ \vdots & \vdots & & \vdots & & \vdots \\ \vec{h}_{2,1} & \vec{h}_{2,2} & \cdots & \vec{h}_{2,i} & \cdots & \vec{h}_{2,R_w} \\ \vec{h}_{1,1} & \vec{h}_{1,2} & \cdots & \vec{h}_{1,i} & \cdots & \vec{h}_{1,R_w} \end{bmatrix},$$

where i is the float for i^{th} warp and j is the float for j^{th} weft (Notice that the float j is in descending order instead of ascending order to follow the textile industrial convenience). R_w and R_f describe the number of warp and the number of weft in one repeat of fabric. The element of matrix expresses the vector of yarn centerline position at interlacing points in fabric. The initial value of element

$$\vec{h}_{j,i} = \left(h_{j,i}{}^x, \quad h_{j,i}{}^y, \quad h_{j,i}{}^z\right)^T$$

is determined by fabric topologic structure, yarn thickness, yarn crimp (or half modular high) in fabric and fabric setting. Such one integrated geometric structure model covers different fabrics with different fabric and yarn parameters. Especially, this model is sufficient for the requirements of physical and mechanical analysis of textiles. As a matter of fact, when external force applied, the fabric undergoes deformation. The relevant variation on geometric structure is represented as the alteration of fabric settings, yarn crimp (or half modular high) and yarn thickness.

It should be point out that the geometric structure model relating to warp yarns H_w can be obtained directly from H_f and yarn cross-sectional parameters, vice versa.

Principal and secondary point

For fabric geometric modeling, we consider two stages: principal and secondary stage. The former represents the basic geometric skeleton. The latter describes the more internal geometric features.

The value of vector, element, in geometric structure model expressed the yarn centre point at interlacing points, which determine affinely the overall geometric structure of fabric without too many internal geometric features. We can call those points as ***principal point***. With the geometric structure it also can be applied to other fields such as the fabric/cloth drape modeling and simulating. All principal points are calculated based on initial fabric design, external force and condition. A separate work [7] has provided the relevant sample.

For internal geometric features, obviously they are determined by yarn in fabric. The yarn centrelines go through the relevant principal points and the overall points at yarns in fabric satisfy the differential equations of the deflection curves of yarns. We can call those points as ***secondary point***. Following sections explain how to determine them.

3. Physically Based Yarn Path

In [6], we provided the differential equation of the deflection curve of a yarn in a fabric repeat as following:

$$EI\frac{d^2w}{dx^2} = \sum_i M_i < x - a_i >^0 + \sum_j F_j < x - b_j >^1 + \sum_k \frac{1}{2} q_k < x - a_k >^2$$

where E, I are young's modulus, moment of inertia of area respectively. M_i, F_j, q_k are couples, concentrated forces, distributed forces with regard to the relevant actions from each other weft yarn and warp yarns. And $<x-a>^t$ are Macaulay functions, which are defined by the following expressions:

$$< x - a >^t = \begin{cases} (x-a)^t & (x \geq a) \\ 0 & (x < a) \end{cases}.$$

Analysing the solution structure of the differential equation above, if the distributed forces are not involved, the deflection curve of yarn in a fabric repeat can be described by the piecewise continuous cubic polynomial functions (cubic Spline), or piecewise continuous parabola functions (parabola Spline) for soft yarn in which the deflection of yarn with regard to F_j could be avoided. By the way, for some of sufficiently soft yarns, we could reasonably eliminate the influence of both forces and bending moments relating to deflection of yarn, and only consider simple problem of geometrical constrain.

There are two problems should be solved out:

- How to determine the boundary conditions of yarn paths?
- How to find out the yarn paths based on Spline curves?

Considering the continuity on repeat boundary in fabric, the periodic Spline curve should been found out for each yarn curve in fabric. Due to the high complexity of algebraic method to find out the Spline function [8], we created a high efficient new algorithm based on geometric reasoning to get both boundary condition and Spline function:

Firstly, we give the extending periodic geometric structure by extending the principal points. Fig. 3 showed an example with 1/1 fabric structure. Two principal points for a repeat are extended to four principal points for solving the yarn path of the repeat by parabola Spline. In this case, the repeat satisfies the periodic condition for both functions and first deferential coefficient.

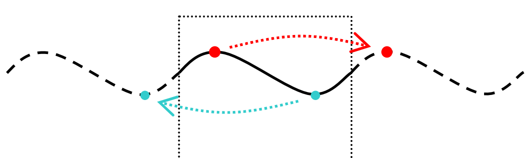


Fig. 3: The extension of principal point.

Then, the yarn paths can be found out by using the method of asymptotic iterative approximation. The method allows us avoid the high complexity of interpolation Spline algorithms and the non-controllable features of approximation methods.

Finally, the yarn paths going through the principal points will be obtained, which follows the principle of minimal energy and can be expressed as piecewise parameter type. Fig. 4 showed continued repeats (Fig. 4b) of one repeat fabric (Fig. 4a) with 2/1/3/1 structure.

By the way, for estimating yarn consumption in fabric and finding out the crimp, it is necessary to calculate the length of yarn, which can be obtained by the following formula

$$L = \sum_{i=1}^{N} L_i = \sum_{i=1}^{N} \int_0^1 \sqrt{x_i'(t) + y_i'(t) + z_i'(t)}dt$$

where $x_i(t)$, $y_i(t)$, $z_i(t)$ describe the project of yarn function on x-axis, y-axis and z-axis at i^{th} interval.

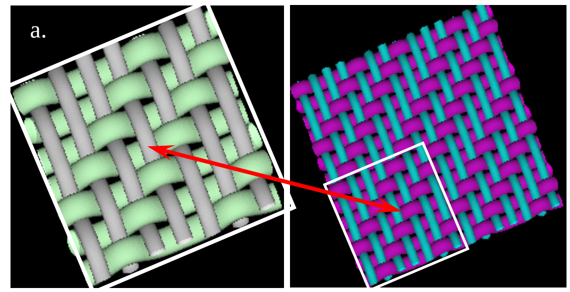


Fig. 4: The repeats of fabric with 2/1/3/1 structure.

4. Yarn Cross-Section

Over the years, many works have been carried out attempting to define more accurately the cross-section models of fabrics. But those models always adopt the Peirce' regular mathematical description and its extensions, such as circular, race-track, lenticular, etc. Based on idealized assumptions, those cannot represent the complicated and irregular (but following the principle of physics and mechanics) variety yarn cross-section on the fabric (especially on the non-plain woven fabric) as the yarns on the fabrics press on each other. The most important duty is to develop an approach for creating the irregular cross-sectional model that should enable the changes to be made easily.

In addition to the regular cross-sectional shapes including super-ellipse, we also developed a method for creating irregular yarn cross-sectional shapes. The method employed both polar coordinate system and spline technique. Fig. 5 presented an irregular cross-section by our irregular generator.

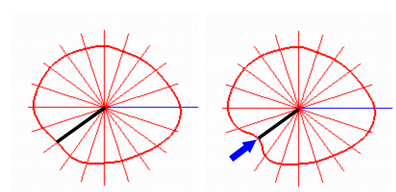


Fig. 5: The irregular cross-section.

5. Creating Yarn

Once the centerline of a yarn path and cross-section are determined, the cross-section can be swept along the centerline of warp or weft yarn. The cross-sections in a yarn are not parallel to each other by simply translating a cross-section (Fig. 6a). At each point on the centerline, it should be created on the normal plane of centerline that is perpendicular to the centerline

(Fig. 6b). Each normal plane of yarn centerline can be derived from the tangent vector of the centerline at the relevant point below:

$$\vec{g}'(t_0) = \frac{\vec{g}(t_1) - \vec{g}(t_{-1})}{2 \cdot \Delta t}$$

$$\vec{g}'(t_N) = \frac{\vec{g}(t_{N+1}) - \vec{g}(t_{N-1})}{2 \cdot \Delta t}$$

$$\vec{g}'(t_i) = \frac{\vec{g}(t_{i+1}) - \vec{g}(t_{i-1})}{2 \cdot \Delta t}$$

where

$$\vec{g}'(t_{-1}) = \vec{g}'(t_{N-1}), \ \vec{g}'(t_{N+1}) = \vec{g}'(t_1).$$

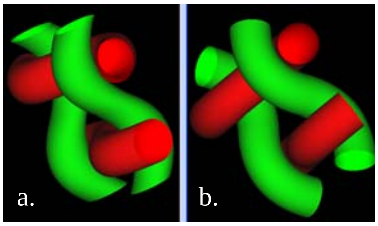


Fig. 6: Differential geometry for yarn modeling.

6. Examples of 3D Models

A software package has been developed to represent computer 3D graphic with comprehensive functions. The input parameters include fibre densities of yarn, yarn parking densities, yarn linear densities, yarn modular heights/crimps, yarn cross sectional shapes, fabric setting, fabric pattern, etc. The warp yarn and weft yarn can have different parameters subject to design requirement. The software then automatically calculates derivative parameters and fabric physical properties. Following mechanical analysis and calculation, the graphics will be changed to describe the alteration of mechanical parameters.

Fig. 7 shows a pair of 2/3/2/3 fabric. Fig. 7a shows the model based on Peirce's assumption. Fig. 7b represents one used minimal energy principle.

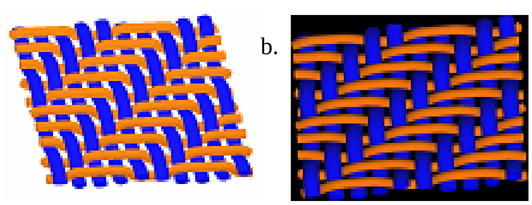
a.  b.

Fig. 7: Comparing previous model based on Peirce's assumption with our model based on minimal energy principle.

The geometric model we developed can cover different structures typical for woven fabrics, even irregular structures by giving variable yarns and fabrics parameters. Fig. 8 showed a funny fabric example.

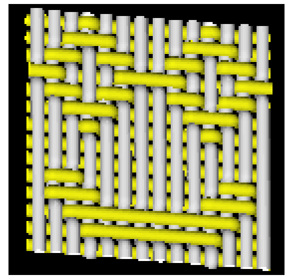


Fig. 8: A funny fabric example.

7. Acknowledgements

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8. References

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