STOCHASTIC ERROR DIFFUSION

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ABSTRACT

This paper presents a stochastic error diffusion approach and analyzes its effect on image quality and artifacts. Unlike traditional error diffusion techniques that use fixed or image-dependent error diffusion weights, stochastic error diffusion distributes image quantization errors to neighboring pixels based on a stochastic process whose expectation and entropy are controllable. A model is provided to derive diffusion weight expectations based on the principle of "uniform error propagation". The paper empirically compares the difference between random quantization thresholds and random diffusion weights and shows that stochastic error diffusion, unlike random dithering, does not generally produce images of degraded quality. On the contrary, by selecting appropriate expectation and entropy for diffusion weights, stochastic error diffusion may drastically reduce image artifacts with almost unperceivable loss of image sharpness.

1. INTRODUCTION

Originally introduced by Floyd and Steinberg [1], error diffusion is known as one of the most effective techniques to produce halftone images [2, 3]. Error diffusion techniques are widely used in image printing and display applications [4, 5]. Recently error diffusion is also extended to other research areas including image rendering [6, 7], watermarking [8, 9], and information hiding [10].

In the process of error diffusion, each pixel value in an image is quantized to a value of less levels using fixed or adaptive quantization, and the quantization error is then diffused non-uniformly to its neighboring pixels, altering their values. The original error diffusion algorithm [1] empirically selects diffusion weights. Although generally producing halftone images of high visual quality, it sometimes introduces artifacts to the resulting images [11]. Such image artifacts also exist in some other error diffusion techniques [2]. For Floyd-Steinberg error diffusion algorithm [1], the image artifacts usually appear around some special gray levels [12] such as $1/2$ and $3/4$ of the maximum gray level. These regular patterns usually have forms of extended continuous dark-pixel curves ("worms")[13], which adversely affect viewing experiences and printing quality.

Extensive research work has been done on improving image quality of error diffusion and on reducing image artifacts. Although there is still no theoretical conclusion on how image artifacts are formed, previous research [9] indicates that the artifacts are related to scanning process and to error diffusion weights. Therefore, there are improved error diffusion techniques including those using serpentine scanning path instead of raster scanning [14], those applying adaptive diffusion coefficients based on image properties [6], those selecting different sets of weights [13] or a package of weights [12], and those clustering pixel dots [2].

In this paper, we investigate how random diffusion weights affect error diffusion results. Focusing on FM (frequency modulated) halftoning, we introduce stochastic diffusion weights and show how they reduce image artifacts. We also model the stochastic process and analyze the expectation and entropy of the stochastic weights.

2. STOCHASTIC ERROR DIFFUSION

Suppose an image $I(x, y)$ has L gray levels. Each pixel has a value in the range of $[0, L)$. The output halftone image $I'(x, y)$ has less intensity levels. For a binary halftone image, the pixel value is either 0 or $L-1$. The original pixel values are quantized to values of less levels, and the quantization error of a pixel (x, y) is measured as:

$$e(x, y) = I'(x, y) - I(x, y).$$

The quantization error $e(x, y)$ is then diffused to the neighboring pixels of (x, y), changing their pixel values.

$$I(m, n) = I(m, n) + W_{m-x,n-y}\, e(x, y).$$

In the equation above, $W_{m-x,n-y}$ are the error distribution weights, which usually depend only on the relative spatial positions of pixel (x, y) and (m, n) (time-invariant process). The original Floyd-Steinberg error diffusion algorithm [1] distributes error $e(x, y)$ to its four immediate neighboring pixels $(x+1, y), (x-1, y+1), (x, y+1), (x+1, y+1)$ with weights $\{7/16, 3/16, 5/16, 1/16\}$. More recent research [6, 13] improves error diffusion results by using different diffusion weights or different sets of diffusion weights.

In our stochastic error diffusion approach, the error diffusion weights $W_{m-x,n-y}$ are no longer fixed values for all pixels. Instead, the diffusion weights are random variables and the error diffusion weights for all pixels form a stochastic process. The error distribution process is then changed to

$$I(m, n) = I(m, n) + \tilde{W}_{m-x,n-y}(t)e(x, y).$$

Where $\tilde{W}_{m-x,n-y}(t)$ is a stochastic process. The stochastic error diffusion processes pixels in the regular raster-scan order. The variable t here is related the spatial position of a pixel (x, y), representing the time that pixel is being processed. The process of stochastic error diffusion is illustrated in Figure 1.

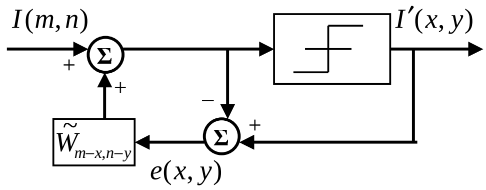

Figure 1: The stochastic error diffusion process. $\tilde{W}_{m-x,n-y}$ shows the weights generated by stochastic process.

In the stochastic error diffusion, the quantization error of a pixel (x, y) is diffused to its immediate unprocessed neighboring pixels, i.e., $\{(x + 1, y), (x - 1, y + 1), (x, y+1), (x+1, y+1)\}$. There are four stochastic processes in total: $\tilde{W}_{1,0}(t)$, $\tilde{W}_{-1,1}(t)$, $\tilde{W}_{0,1}(t)$, and $\tilde{W}_{1,1}(t)$, with only three of them independent because the restriction $\tilde{W}_{1,0}(t) + \tilde{W}_{-1,1}(t) + \tilde{W}_{0,1}(t) + \tilde{W}_{1,1}(t) \equiv 1$ is enforced to guarantee no more or less errors diffused.

The motivation of using stochastic weights instead of fixed weights or weight sets is to reduce image artifacts. Since quantization errors are distributed to neighboring pixels with probabilities, the chance of forming artifacts ("worms" or unwanted repeated textures) is greatly reduced. One explanation, though not a rigorous proof, is as the following. Suppose a definite error diffusion process generate a "worm" artifact of n continuous black pixels in a halftone image, for stochastic error diffusion, the output pixel has a probability of p of being a black pixel. For the convenience of discussion, we assume the probability of being a black pixel is the same, so the probability of forming such an artifact is only p^n. Image artifacts are drastically reduced in a probabilistic sense, as shown in Figure 4 and Figure 5 of the experimental results, although image sharpness is slightly affected, which is also indicated in these figures.

3. STOCHASTIC DIFFUSION WEIGHTS

In this paper, Gaussian process is used as the stochastic process of error diffusion weights $\tilde{W}_{p,q}(t)$ due to the fact that Gaussian distribution achieves the maximum entropy over all distributions for a given variance. The entropy function of a stochastic process with density function $p(x, t)$ is defined as

$$H(t) = -\int_{-\infty}^{\infty} p(x, t)\, log(p(x, t))dx.$$

The entropy of the stochastic process determines the amount of uncertainty in error diffusion. For a given entropy, the Gaussian distribution minimizes standard deviation. This entropy process is adjustable for the purpose of reducing image artifacts while avoiding introducing too much randomness in images.

Since the restrictions $0 \le \tilde{W}_{p,q}(t) \le 1$ and $\tilde{W}_{1,0}(t) + \tilde{W}_{-1,1}(t) + \tilde{W}_{0,1}(t) + \tilde{W}_{1,1}(t) \equiv 1$ hold for error distribution weights $\tilde{W}_{p,q}(t)$, the stochastic process is revised as windowed Gaussian distribution at epoch t. In the windowed Gaussian process, the expectation and variance need to be determined. We empirically select 0.05 as the value of standard deviation σ and compute the expectations of Gaussian distribution weights based on the principle of "Uniform Error Propagation", explained in detail in the following subsection.

3.1. Uniform Error Propagation

In our stochastic error diffusion process, the error is diffused to four neighboring pixels, as illustrated in Figure 2. Let μ_1, μ_2, μ_3 and μ_4 be the expectations of the four stochastic processes: $\mu_1 = E[\tilde{W}_{1,0}(t)], \mu_2 = E[\tilde{W}_{1,1}(t)], \mu_3 = E[\tilde{W}_{0,1}(t)], \mu_4 = E[\tilde{W}_{-1,1}(t)]$. Obviously, $\mu_1 + \mu_2 + \mu_3 + \mu_4 = 1$.

As shown in Figure 2, the pixel (x, y) is the pixel being processed, the error generated at pixel (x, y) is diffused to $(x, y + 1)$, $(x + 1, y + 1)$, $(x, y + 1)$, and $(x - 1, y + 1)$. A "uniform error propagation" model is applied to compute μ_1, μ_2, μ_3, and μ_4 – the expectations of Gaussian distributions. The assumption of this model is that for the pixels with equal distance to (x, y), they receive equal overall effective portions of $e(x, y)$. As illustrated in Figure 2, the amount of error $e(x, y)$ received by pixel $(x, y + 1)$ should be the same as that received by $(x + 1, y)$, and the error received by $(x + 1, y + 1)$ should be the same as that received by $(x - 1, y + 1)$. However, this does not directly mean $\mu_1 = \mu_3$ and $\mu_2 = \mu_4$ since the error may affect one pixel both *directly* and *indirectly*.

The quantization error of of pixel (x, y) affects its neighboring pixels in two ways:

Direct Impact Shown as solid arrows in Figure 2, direct impact represents how much error distributed directly to its neighboring pixels. The value $\mu_1, \mu_2, \mu_3, \mu_4$ are the expectations of diffusion weights.

Indirect Impact Represented as dotted arrows in Figure 2, indirect impact refers to the amount of error that is diffused to neighboring pixels indirectly through other pixels. As shown in Figure 2, (x, y) may affect pixel $(x, y+1)$ indirectly through $(x-1, y+1)$ and through $(x+1, y)$.

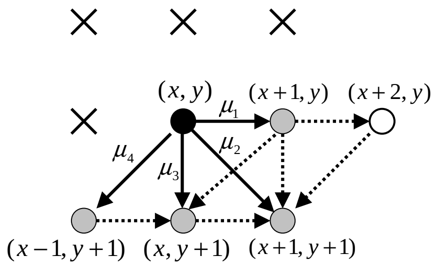

Figure 2: Direct and indirect impacts of diffusion errors. The crosses show the pixels already processed and the solid black circle is the pixel being processed. Direct impact is through the solid arrows, while indirect impact is through the path of solid and dotted arrows.

The "uniform error propagation" principle is to make total error diffusion impact, the sum of direct and indirect effect, equal for all equal-distance pixels. Indirect impact usually has complex processes but we simplify it by modelling indirect impact as the product of all diffusion weights in the error propagation path. For example, the indirect impact of (x, y) on $(x, y+1)$ may come from two paths: $(x, y) \rightarrow (x+1, y) \rightarrow (x, y+1)$ (with indirect impact $\mu_1\mu_4$) and $(x, y) \rightarrow (x-1, y+1) \rightarrow (x, y+1)$ (with indirect impact $\mu_4\mu_1$). So the total (direct and indirect) impact of error on pixel $(x, y+1)$ is $\mu_3 + \mu_1\mu_4 + \mu_4\mu_1$.

Under the principle of "uniform error propagation", the following constrains are derived:

$$\begin{cases} \mu_1 = \mu_3 + 2\mu_1\mu_4 \\ \mu_4 = \mu_2 + 2\mu_1\mu_3 + 2\mu_1^2\mu_4 \\ \mu_1 + \mu_2 + \mu_3 + \mu_4 = 1 \end{cases}$$

As easily seen from the equations above, there is still one free variable. Therefore, another constraint is added. That is, the overall impact of diffusion error is inversely proportional to the error propagation distance. In Figure 2, the total effect of (x, y) on $(x+1, y)$ is $\sqrt{2}$ times of that of (x, y) on $(x-1, y+1)$, i.e., $\mu_1 = \sqrt{2}\mu_4$. By introducing this new constraint, the equations above are solved: $\mu_1 = 0.47243$, $\mu_2 = 0.03672$, $\mu_3 = 0.15684$, and $\mu_4 = 0.33401$. This is the group of parameters used in the paper as expectations of stochastic weights.

Figure 3: Comparison of error diffusion results. Top row: Random quantization thresholds, Floyd-Steinberg method [1]; Bottom row:Shiau-Fan method[13], and stochastic error diffusion algorithm.

4. EXPERIMENTAL RESULTS

First, we compare the result of error diffusion using random quantization thresholds (random dithering) with that using stochastic error diffusion. Figure 3 shows the results of four methods: random dithering, Floyd-Steinberg method [1], Shiau-Fan method [13], and our stochastic error diffusion method. It is already known [2] that random dithering method produces no better results than Floyd-Steinberg error diffusion method [1] does. Since random dithering maximizes image entropy, it disrupts the intrinsic correlation of image pixels with random quantization thresholds, resulting in images with more randomized pixels. Stochastic error diffusion, however, is drastically different from random dithering. In stochastic error diffusion process, the coherence and relations of pixels are well preserved through distributed errors. The random error distribution weights prevent regular patterns and image artifacts being formed but they have little effect on image quality. As seen in Figure 3, the image of stochastic error diffusion is very close in quality to that of definite error diffusion process with barely noticeable loss of image sharpness.

As seen from Figure 4, the image artifacts appear in the definite process of error diffusion(Floyd-Steinberg method [1] and Shiau-Fan method [13]). They occur at places around the middle of gray levels. While in our stochastic error diffusion process, the stochastic weights decrease the occurrence of such repetitive patterns, reducing image artifacts.

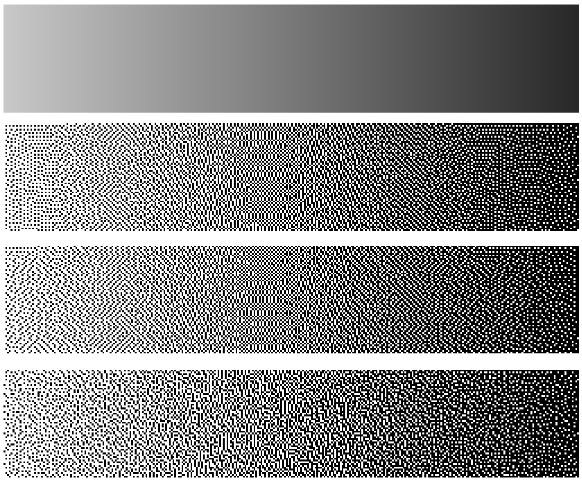

Figure 4: Error diffusion results using different algorithms. From top to bottom: the original image, Floyd-Steinberg method [1], Shiau-Fan method [13], and stochastic error diffusion. The belt has gray levels gradually changing from 200 to 40.

Figure 5 shows the results of a natural scene image. As usual, image artifacts occur in the areas of small gradient change. Disturbing image artifacts (strips) show in the flat sky areas. Since halftone image quality is affected by image size, resolution, and viewing distance, for the results in Figure 5, image artifacts are more salient when printed out than viewed on screen. The stochastic error diffusion generates an image with significantly less artifacts and with little loss of image sharpness. For the application of image printing, stochastic error diffusion may improve image quality when regular error diffusion techniques produce salient artifacts.

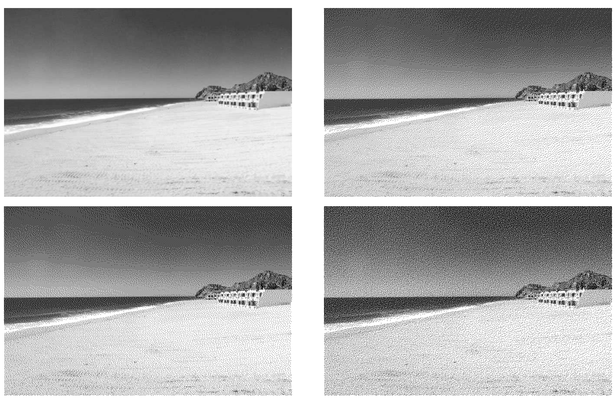

Figure 5: Comparison of image results of three error diffusion methods on a scenery picture. Top row: original image and result of Folyd-Steinberg method; Bottom row: the results of Shiau-Fan method [13] and stochastic error diffusion.

5. CONCLUSIONS

This paper presented a novel probabilistic approach to error diffusion. By introducing randomness in error diffusion weights, image artifacts are dramatically suppressed with little loss of image sharpness. Future research includes a more in-depth study of the relation between the entropy of stochastic weight process and the image quality. A hybrid approach of stochastic error diffusion with considerations of image-dependent features such as image regions and gradient is also possible.

6. REFERENCES

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