A Metaheuristic Portfolio Selection Based on Soft Computing

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Abstract

We present a metaheuristic approach for portfolio selection. We have implemented two models. The first is a classic mean-variance model, which assumes that the distributions of probability on the asset returns are known. The second is a possibilistic model based in two issues: the approximation of the rates of return on securities by means of fuzzy numbers of trapezoidal form, and the perception that downside risk is a more realistic description of an investor's preferences. We use a data set from the Spanish stock market to illustrate the performance of our approach.

Keywords: Portfolio selection, Fuzzy returns, LR Fuzzy numbers, Possibilistic models, Genetic Algorithms.

1. Introduction

The portfolio selection problem deals with finding an optimal investment strategy to form a satisfying portfolio. This is not an easy task. It is difficult to decide which securities should be selected due to the uncertainty involved in the behaviour of the financial markets.

The first mathematical formulation of the portfolio selection problem was given by Markowitz (1959), which assumed that the return of each asset is a random variable. The probability distribution of the returns on the securities is assumed to be known. To model the behaviour of the economic agents the Markowitz theory combines probabilistic elements and ones from the optimization theory. There the goal of the investors is to minimize the risk of their investment, measured in terms of the variance of a portfolio, subject to the constraint that a given expected benefit should be achieved. This mean-variance analysis has been widely applied to asset allocation.

The main dissatisfaction with the use of variance as a measure of risk is that it makes no distinction between gains and losses. In order to measure only the downside risk of a portfolio it is possible to think of risk as the failure to achieve a target. Different measures of risk of a portfolio have been proposed which use either higher moments or lower partial moments. If the measure of risk used is the mean semi-absolute deviation, as proposed by Speranza (1993), the risk function can be easily evaluated, in contrast with the complexity of the task of finding the portfolios with minimum semivariance.

Fuzzy set theory has been widely used to solve many practical problems including financial risk management. In León et al (2002), we develop an interactive fuzzy method to solve the problem of getting a viable portfolio selection which respects the original will of the investor as much as possible. Now we present a fuzzy optimization scheme to select optimal portfolios assuming that the rates of returns on asset are modeled by means of a trapezoidal LR-fuzzy number which describes the possibility distribution of the return.

2. Investor profile

An investor wishes: to invest their wealth on n assets with returns R_i (i=1,...,n) variable, to select a set of securities and distribute their budget among them and to find the best trade-off between the risk and the expected return to obtain the highest benefits. He or she knows that in risky situations, it is convenient to diversify their investments, and should identify:

- The term of time of the investment,
- The criteria for determining the assets in their portfolio, and
- The goals of the investment.

Thus, the investors' problem is to find the portfolio $P(x):=\{x_1, x_2, \ldots,x_n\}$ that minimizes the risk to achieve a given level of mean return, where x_j represents the proportion of the j-th asset in the composition of the portfolio. The formulation of the optimization problems associated suffers from some limitations when it is applied for decision making in the real world, because it is usual to formulate them in terms of explicit functions whose coefficients and parameters must be exactly known.

When a small investor turns to an investment consultant, they can find, at least, two possibilities to approach the design of an optimal portfolio:

a) The consultant proposes a reduced number of assets, maybe four or five, classified according to the risk which the investor is prepared to assume.

b) The consultant determines the investor risk profile, and then decides which model should be used to obtain the portfolio.

In the second case for finding the optimal portfolio selection problem we have planned a heuristic method which allows us to consider all the elements involved. Figure 1 summarizes our proposal.

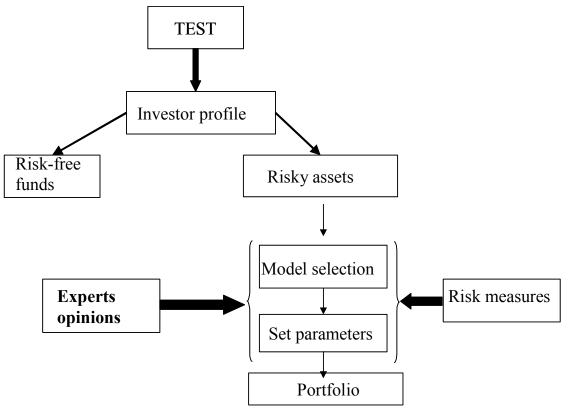

Fig. 1: Portfolio selection.

At the first step, we would determine the investor profile by means of a test consisting of 10 questions at most. Once this profile is known the procedure assigns a portfolio model by using a membership function which aggregates the risk aversion and the desired return. Although in this paper we only consider two portfolio selection models, it is easy to include other probabilistic or fuzzy models. Then we would choose the parameter values adequate to the investor profile by considering the experts opinion and the market situation.

3. Mathematical models

MV model

The classical mean-variance approach considers an investor who wants to allocate one unit of wealth among n assets offering random rates of return. Thus the vector of returns $R = (R_1,..., R_n)$ can be summarized by the average vector $E(R)$ and the covariance matrix Q. The optimization problem can be stated as follows:

$$
\begin{aligned}
\text{(MV) Min} \quad & x^{\mathrm{T}}Qx \\
\text{s.t.} \quad & \sum_{j=1}^{n} E(R_j)x_j \geq \rho \\
& \sum_{j=1}^{n} x_j = 1 \\
& x_j \geq 0 \quad \forall j
\end{aligned}
$$

where x_j is the proportion of the total investment fund devoted to asset i, $1 \leq i \leq n$. ρ is the parameter which indicates the minimum return expected by the investor.

A more general version of the model can be obtained by including diversification constraints, such constraints model the situation in which upper and/or lower bounds on the investments are given.

The MV model works under the assumption that the joint distribution of the random variables R_i is normal multivariate. Our proposal does not require the estimation of the joint distribution of asset returns, but instead we approach the decision problem by using some fuzzy tools.

Possibilistic model

We work on the assumption that the uncertainty in the returns is modeled by means of fuzzy quantities and we calculate the fuzzy downside risk for a given portfolio in order to build the possibilistic portfolio selection problem.

Let us assume that $\tilde{R}_j = \left(a_{lj}, a_{uj}, c_j, d_j\right)_{LR}$ is an LR-fuzzy number, whose possibility distribution is trapezoidal. Therefore, the fuzzy return can also be represented by its sets of level cuts:

$$
\begin{aligned}
\inf \tilde{R}_{j\alpha} &= a_{lj} - (1-\alpha)c_j \quad \alpha \in (0,1] \\
\sup \tilde{R}_{j\alpha} &= a_{uj} - (1-\alpha)d_j \quad \alpha \in (0,1]
\end{aligned}
$$

In 1987 Dubois and Prade introduced the mean value of a fuzzy number as a closed interval bounded by the expectations calculated from its lower and upper probability mean values. Thus, the mean value of a fuzzy quantity is defined as the interval whose endpoints are:

$E_*(\tilde{R}_j) = \int_0^1 (\inf \tilde{R}_{j\alpha})d\alpha$ and

$E^*(\tilde{R}_j) = \int_0^1 (\sup \tilde{R}_{j\alpha})d\alpha$, respectively.

Using the extension principle we can verify that the total fuzzy return on a given portfolio P(x) is the following LR-fuzzy number:

$$\sum_{j=1}^{n}\widetilde{R}_j x_j = \left(\sum_{j=1}^{n} a_{lj}x_j, \sum_{j=1}^{n} a_{uj}x_j, \sum_{j=1}^{n} c_j x_j, \sum_{j=1}^{n} d_j x_j\right)_{LR}$$

Hence, its mean interval using the definition provided by Dubois and Prade is:

$$\widetilde{E}(\sum_{j=1}^{n}\widetilde{R}_j x_j) = \left[\sum_{j=1}^{n} a_{lj}x_j - \frac{1}{2}\sum_{j=1}^{n} c_j x_j, \sum_{j=1}^{n} a_{uj}x_j + \frac{1}{2}\sum_{j=1}^{n} d_j x_j\right].$$

In the context of probability theory, Speranza (1993) proposed measuring the risk of a portfolio P(x) by means of the mean semi-absolute deviation, which penalized the negative deviations of the expected return. In a similar way we define the fuzzy downside risk with respect to the mean interval as follows:

$$w(P) = \tilde{E}\left(\mathrm{Max}\left\{0, \tilde{E}\left(\sum_{j=1}^{n}\tilde{R}_j x_j\right) - \sum_{j=1}^{n}\tilde{R}_j x_j\right\}\right).$$

In order to calculate this interval expectation we need first to specify the level-cuts of the negative deviation of the interval-valued mean. Then, using the extension principle and the arithmetic rules for trapezoidal fuzzy numbers we obtain that:

$$\tilde{E}\left(\mathrm{Max}\left\{0, \tilde{E}\left(\sum_{j=1}^{n}\tilde{R}_j x_j\right) - \sum_{j=1}^{n}\tilde{R}_j x_j\right\}\right) =$$

$$= \left[0,\ \sum_{j=1}^{n} a_{uj}x_j - \sum_{j=1}^{n} a_{lj}x_j + \sum_{j=1}^{n}\frac{1}{2}(c_j + d_j)x_j\right]$$

Note that the magnitude of the fuzzy risk is directly proportional to the amplitude of the cores and the spreads of the fuzzy returns weighted by the proportion devoted to each asset in the portfolio.

We will therefore state the possibilistic portfolio selection problem as follows:

$$\begin{aligned} &Min \quad \widetilde{w}(P(x)) \\ &s.t. \quad \widetilde{E}(\sum_{j=1}^{n}\widetilde{R}_j x_j) \geq \rho_0 \\ &\qquad \sum_{j=1}^{n} x_j = 1 \\ &\qquad l_j \leq x_j \leq u_j, \quad j = 1,\dots,n \end{aligned}$$

where ρ_0 is the given total return that must be achieved.

In order to obtain a crisp solution of the possibilistic model we choose to minimize the upper limit of the mean interval as objective function:

$$F(x_1, x_2, \dots, x_n) = \sum_{j=1}^{n} a_{uj}x_j - \sum_{j=1}^{n} a_{lj}x_j + \frac{1}{2}\sum_{j=1}^{n}(c_j + d_j)x_j$$

Thus the non-desired deviations will be minima. Concerning the fuzzy constraint:

$$\tilde{E}\left(\sum_{j=1}^{n}\tilde{R}_j x_j\right) \geq \rho_0$$

we decide to select the middle point of the mean interval of $\tilde{E}\left(\sum_{j=1}^{n}\tilde{R}_j x_j\right)$ to represent it at the crisp problem (see León *et al* (2003, 2004)).

4. CAOBA: A Decision Support System

In order to find optimal portfolios we have developed an interactive system, CAOBA, which comprises classical formulations based on probability theory and also fuzzy formulations.

CAOBA provides a consistent framework to analyze different investment strategies which takes into account the asset returns, the risk and the diversification. It has been implemented on the Visual Basic language. It consists on three different modules: data entry, active selection and storage of the portfolios proposed. In order to provide a set of reasonable options, the system allows the different model parameters to be modified.

CAOBA is user friendly and the different portfolios obtained by considering the different mathematical models can be easily inspected by the consultant. Therefore they will be able to select the most appropriate proposal. In practice, CAOBA can managed a great number of assets, for instance those included in the Spanish stock market, or firstly classifying the assets by sectors and then assigning them the corresponding percentages.

The use of genetic algorithms is also a suitable approach to the portfolio selection if the number of selectable assets is restricted to be an interval or a fixed number (Watada, 1997). The application of Soft Computing techniques has given rise to an important development of decision support systems, they usually provide solutions which do not appear with exact methods but may be advantageous to the decision maker (Verdegay, 2003).

We have made use of a data set from the

Spanish Stock Market (traded in the Madrid stock exchange) to check the performance of our DSS. In particular, we have considered the returns on 96 assets traded in the Madrid Stock Exchange. We have taken the observations of the Wednesday prices as an estimate of the weekly prices. Our data base covers the period from January 2001 to June 2004 and it has been used to check out the performance of the possibilistic model against the mean-variance model. For the probabilistic model we have estimated the average vector of returns and the elements of the covariance matrix over this period through historical data

On the other hand, if we consider the return observations as a sample, its percentiles inform us about the possibility distribution of the returns, then we have decided to set the core of the trapezoidal fuzzy number as an interval $[P_{25}, P_{75}]$ and the quantities P_{25}-P_5 and P_{95}-P_{75} as the left and right spreads, where P_k is the k-th percentile of the sample.

Let us describe one experiment that we have developed to make the comparisons. We have taken the weekly observations up to March 2004 (that is 170 observations) as a training set to estimate the sample means, variances and percentiles. We have kept the last twelve weeks of returns, between 07/04/2004 and 23/06/2004, as a test set in order to make an out-of-sample comparison of the profits.

We assume that the investor requires that 25% at most of the total fund must be invested in anyone asset. Besides, the future financial market at March 2004 was estimated pessimistically and the return parameters were set to their lowest values. The portfolios given by the MV and possibilistic models involve 27 and 4 assets, respectively. We have computed the 'a posteriori returns of these portfolios if the investment had been recovered during any of those twelve weeks. The results appear in Figure 2.

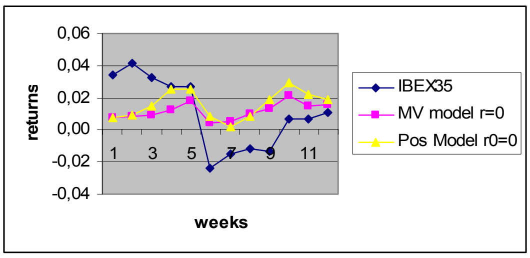

Fig. 2: A posteriori returns.

Notice that the return on the portfolios obtained by using the possibilistic model improves the returns on the MV model, except for the 7th and 8th weeks. The performance of the Spanish Index IBEX35 is also included for comparison.

Conclusions

We have introduced a Decision Support System for the optimal portfolio selection. The consultant can interact with the system to obtain, select and evaluate different proposals. We have obtained very promising results by applying our possibilistic model.

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