An automated fuzzy finite element procedure for frequency response function analysis

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Abstract

This paper describes a non-probabilistic fuzzy finite element method for the calculation of fuzzy frequency response functions of structures with uncertain parameters. It also describes a procedure to perform such an analysis with the help of a standard deterministic FE solver. Finally, an analysis on a widely used benchmark problem proves the applicability and strength of the procedure.

1 Introduction

Simulation tools enable a very precise simulation of physical phenomena using numerical models. Especially in engineering applications, the finite element (FE) method has become a very popular tool for design validation and optimisation. Continuously growing computational capabilities allow for extremely detailed numerical models. In many cases however, the computational power could be of much greater value when used for the inclusion of uncertainties in the model rather than for modelling deterministic details.

A frequency response function (FRF) describes the deformation of a structure under a harmonic excitation force. It expresses the dependency between the amplitude and phase of a harmonic force applied locally on the structure and the resulting deformation at another location of the structure. This makes the FRF a powerful description of the dynamic behaviour of a structure, used both in numerical and experimental structural dynamics.

Determining the relation between uncertainties in the FE model and the variability in the resulting FRF enables a designer to incorporate and study the influence of these uncertainties, even in a very early stage of the design process. Because reliable probabilistic data is often unavailable or even not existing (e.g. for properties which are invariable in the actual product but unknown at an early design stage), non-probabilistic methods as the interval FE (IFE) method and the fuzzy FE (FFE) method are being developed.

The next sections describe these procedures for the calculation of envelope and fuzzy FRFs.

2 The FE FRF procedure

For undamped structures, the FRF between DOF (degree of freedom) j and DOF k of a FE model is obtained taking the jth component of $\{U\}$ satisfying the dynamic equilibrium equation

$$\left([K] - \omega^2 [M]\right) \{U\} = \left\{F^k\right\} \tag{1}$$

with $[K]$ and $[M]$ the FE system matrices, ω the pulsation and

$$F_i^k = \begin{cases} 1 & \text{if } i = k \\ 0 & \text{if } i \neq k. \end{cases} \tag{2}$$

The deterministic modal superposition concept states that, considering the first n modes, this FRF equals:

$$\mathrm{FRF}_{jk} = \sum_{i=1}^{n} \frac{1}{\hat{k}_i - \omega^2 \hat{m}_i} \tag{3}$$

with $\hat{k}_i$ and $\hat{m}_i$ the normalised modal stiffness and the normalised modal mass:

$$\hat{k}_i = \frac{\{\phi_i\}^{\mathrm{T}} [K] \{\phi_i\}}{\phi_{i_j} \phi_{i_k}} \tag{4}$$

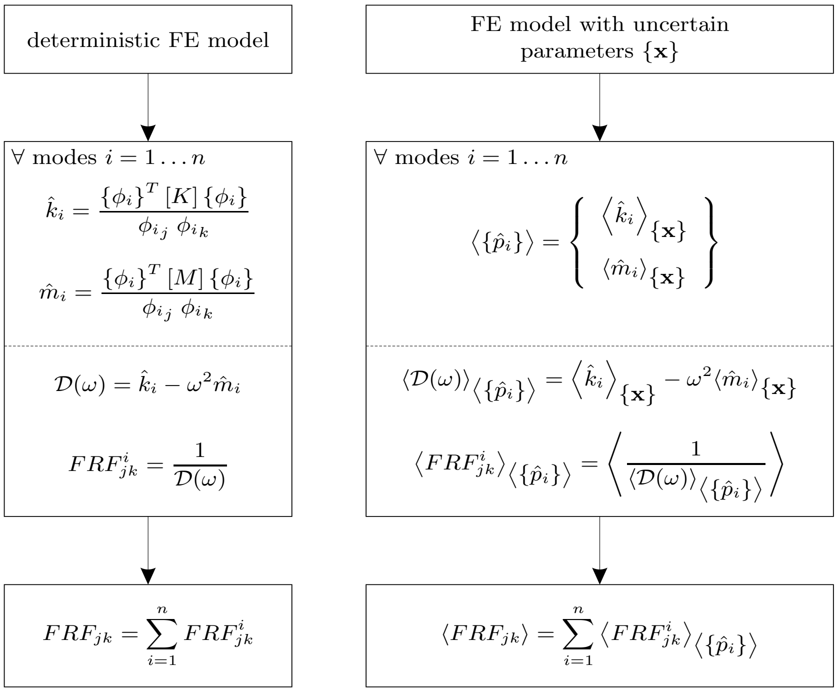
$$\hat{m}_i = \frac{\{\phi_i\}^{\mathrm{T}} [M] \{\phi_i\}}{\phi_{i_j} \phi_{i_k}} \tag{5}$$

with ϕ_i the i^{th} eigenvector of the system described by (1) and ϕ_{i_j} the j^{th} component of this eigenvector.

Figure 1(a) gives a graphical overview of this deterministic modal superposition procedure. It introduces the function $\mathcal{D}(\omega) = (\hat{k}_i - \omega^2 \hat{m}_i)$ to express the modal response denominator as a function of the frequency.

3 The IFE FRF procedure

For undamped structures with one or more interval inputs, the total envelope FRF can be calculated using a step by step interval translation of the deterministic superposition procedure, as shown in figure 1(b). The full mathematical description of this method, developed by Moens, can be found in [1]. It consists of three major steps:



(a) Deterministic algorithm

(b) Set translated algorithm

Figure 1: Deterministic and interval modal superposition algorithm.

Calculation of the range of modal parameters. The first step in the procedure consists of the calculation of the range of modal parameters for each mode, taking into account the ranges of the input uncertainties described by $\{\mathbf{x}\}$. Theoretically, the modal parameters are coupled through the global system. The exact range of the considered mode's modal parameters equals

$$\left\langle \hat{k}_i, \hat{m}_i \right\rangle = \left\{ \left(\hat{k}_i, \hat{m}_i \right) \,\middle|\, (\{x\} \in \{\mathbf{x}\}) \right\}. \tag{6}$$

This can be represented in the mode's modal parameter space, as illustrated by figure 2. The grey area represents a physically possible locus of $\langle \hat{k}_i, \hat{m}_i \rangle$ combinations. There is no general analytical description of the exact contour of this domain, but there are a number of (conservative) numerical approximations:

In the *modal rectangle method* (MR), the coupling between the modal stiffness and the modal mass is neglected. This means the bounds on the ranges can be calculated by minimising and maximising the modal parameters over the domain defined by the input uncertainties. Graphically, this means the $\langle \hat{k}_i, \hat{m}_i \rangle$ domain is approximated by a conservative rectangle.

The *modal rectangle method with eigenvalue interval correction* (MRE) aims at a less conservative approximation of the $\langle \hat{k}_i, \hat{m}_i \rangle$-domain through the introduction of the exact eigenvalue intervals $[\underline{\lambda_i}, \overline{\lambda_i}]$ in the procedure. These eigenvalue intervals can be obtained using a global optimisation approach similar to the modal parameter optimisation used in the MR method.

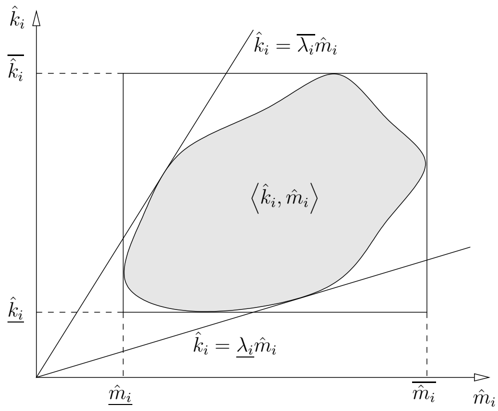


Figure 2: A $\langle \hat{k}_i, \hat{m}_i \rangle$-domain and its MR and MRE approximations.

Modal envelope FRF calculation. This step translates the range of the modal parameters in the vector $\langle\{\hat{p}_i\}\rangle$ into the modal envelope FRF, expressed as the range of the modal response function FRF_{jk}:

$$\langle\mathcal{D}(\omega)\rangle_{\langle\{\hat{p}_i\}\rangle} = \left\langle \hat{k}_i \right\rangle_{\langle\{\mathbf{x}\}\rangle} - \omega^2 \left\langle \hat{m}_i \right\rangle_{\langle\{\mathbf{x}\}\rangle} \tag{7}$$

$$\left\langle \mathrm{FRF}^i_{jk} \right\rangle_{\langle\{\hat{p}_i\}\rangle} = \left\langle \frac{1}{\langle\mathcal{D}(\omega)\rangle_{\langle\{\hat{p}_i\}\rangle}} \right\rangle \tag{8}$$

Total envelope FRF calculation. The final step for the computation of the total envelope FRF consists of the summation of all modal envelope FRFs resulting from the previous step:

$$\langle \mathrm{FRF}_{jk} \rangle = \sum_{i=1}^{n} \left\langle \mathrm{FRF}^{i}_{jk} \right\rangle_{\langle \{\hat{p}_i\} \rangle} \tag{9}$$

Thus, the three step algorithm results in a hybrid procedure: in the first step, the $\langle \hat{k}_i, \hat{m}_i \rangle$-domain is approximated using a global optimisation approach; in the second and third step, the modal and total envelope FRFs are calculated using interval arithmetic.

4 The FFE FRF procedure

The FFE procedure is implemented as a sequence of IFE analyses, using the *α-level* technique. This technique subdivides the membership range into a number of α-levels. At each level, the intersection with the membership function of the input uncertainties results in an interval. Based on these input intervals for all uncertain parameters, an interval analysis can be performed. This results in an output interval at the considered α-level. The total fuzzy solution is assembled from these interval results at all α-levels.

5 Implementation of the FFE FRF procedure

The calculation of the range of modal parameters is the core of the IFE and FFE FRF procedures. In this step, the ranges of the modal parameters ($\hat{k}_i$, $\hat{m}_i$ and for the MRE method also λ_i) are determined using a global optimisation approach. This part is implemented in the program FuzzyFEM. Figure 3 shows a simplified block diagram. The topmost block represents the input to FuzzyFEM, the bottommost block represents the output from FuzzyFEM and the remaining blocks in the middle represent the analysis itself.

The analysis control block and the non-linear optimiser block only use information about the input uncertainties. For these blocks, the FE model is a 'black box' function with the input uncertainties as inputs, and the modal parameters (λ_i, $\hat{k}_i$, $\hat{m}_i$) as outputs.

The analysis control block knows how to perform an analysis in terms of 'black box' optimisations. Every analysis type (MR, MRE,...) has a dedicated analysis control block. To get the required results, the analysis control block communicates with the non-linear optimiser block: it sends the objective function and the constraints and receives the optimised results. These results

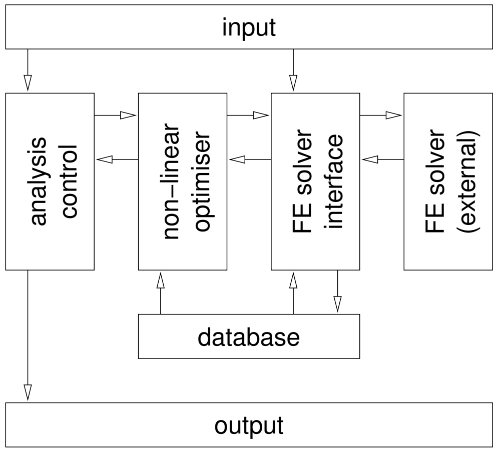


Figure 3: Block diagram of FuzzyFEM.

are written to the output files for further processing.

The non-linear optimiser block optimises (minimises or maximises) an output of the 'black box' FE model. It uses the FE solver interface block to calculate the modal parameters based on deterministic values for the input uncertainties.

The input uncertainties are the only model data used so far. The FE solver interface block translates actual deterministic values for these input uncertainties together with the deterministic FE model and the relations linking the input uncertainties to this model to an FE solver input file, suitable for the deterministic FE solver. This module also translates the output from the deterministic FE solver to an FE solver independent format. Every deterministic FE solver requires a dedicated FE solver interface block.

The FE solver block itself is an external deterministic FE solver. At this moment MSC.Nastran is used, but because all FE solver dependent code is concentrated in the FE solver interface block, it is easy to add support for other FE solvers.

Because the function evaluations (FE analyses) can be very expensive, all results are stored in a database. This database is used by the non-linear optimiser to find start vectors for optimisations and by the FE solver interface to check if the requested results are already calculated in a previous optimisation or run.

6 Application: the Garteur benchmark problem

The aircraft model of the Garteur benchmark problem, designed and manufactured by the Garteur Action Group, has been used extensively to test model updating methods and model error localisation procedures.

It is a much simplified small aircraft model with a length of 1.5 m, a wing span of 2 m and a mass of 44 kg. The fuselage, wings and tail are aluminium plates. The wings are connected to the fuselage through an intermediate steel plate. Wingtips are connected rigidly at the end of both wings. The tail is connected rigidly to the fuselage. A part of the wings is covered with a visco-elastic layer. Three concentrated masses are connected to the structure: one at each wingtip and one on the fuselage.

Figure 4 shows the FE model, created based on the description of the physical model and data of the FE model made by the Garteur Action Group. It consists of about 1000 shell elements and has about 20000 DOFs. The input and output DOFs j and k used in this analysis are shown on the figure too.

The model contains some inherent uncertainties, some due to a lack of knowledge of the physical model, some due to modelling uncertainties. Three uncertainties are considered in this analysis. The uncertainty about the damping characteristics of the visco-elastic layer and about the quality of the glue connection to the wings is modelled as an uncertain thickness of the visco-elastic layer (0.1 mm to 1.6 mm, nominal value 1.1 mm). The uncertainty about the connection between the wings and the fuselage is modelled as the stiffness of the interconnecting springs (10^5 N/m to 10^{11} N/m, nominal value 10^8 N/m). The third uncertainty is the E-modulus of the wing (67.5 GPa to 68.5 GPa, nominal value 68.0 GPa).

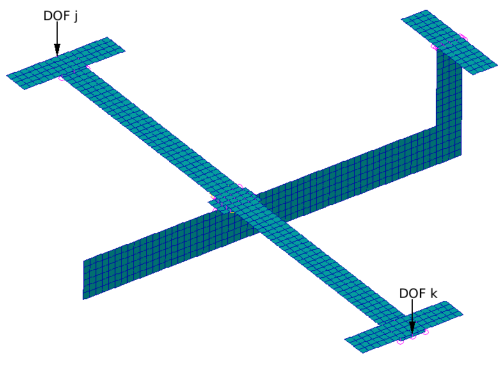


Figure 4: FE model of the benchmark model

The damping is modelled using proportional damping, with coefficients $\alpha_K = 5 \cdot 10^{-5}$ and $\alpha_M = 2$. The proportional damping is introduced in the calculation of the modal envelope FRF (the second step in the IFE procedure described in section 3, as described by Moens [2]. The first and third steps of the procedure remain unchanged.

Figure 5 shows the amplitude of the damped fuzzy FRF between the DOFs j and k. This fuzzy FRF is the result of interval analyses at 11 α-levels.

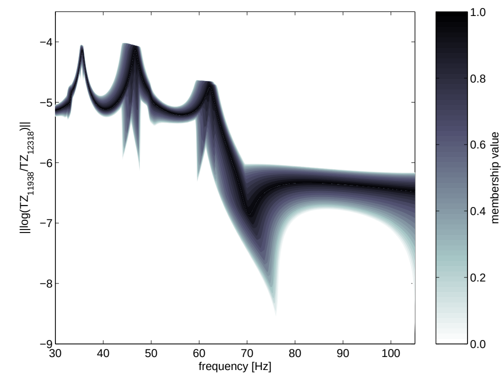


Figure 5: Fuzzy FRF of the benchmark model

This example is described detailed by De Gersem [3], including the modelling procedure, the influence of the separate uncertainties and the influence of the proportional damping.

7 Conclusions

This paper shows the strength and applicability of the IFE and FFE procedures for envelope and fuzzy FRF calculation of structures with uncertain parameters. The interval analysis is transformed to a series of optimisations of a ‘black box’ FE model (analysed by a standard deterministic FE solver) and some interval arithmetic to assemble the total envelope FRF. The fuzzy analysis is implemented as a series of interval analyses.

References

[1] D. Moens and D. Vandepitte. An interval finite element approach for the calculation of envelope frequency response functions. *International Journal for Numerical Methods in Engineering*, 61:2480–2507, 2004.

[2] D. Moens and D. Vandepitte. A fuzzy finite element procedure for the calculation of uncertain frequency response functions of damped structures: Part I - procedure. *Journal of Sound and Vibration*, accepted for publication.

[3] H. De Gersem, D. Moens, W. Desmet and D. Vandepitte. A fuzzy finite element procedure for the calculation of uncertain frequency response functions of damped structures: Part II - numerical case studies. *Journal of Sound and Vibration*, accepted for publication.